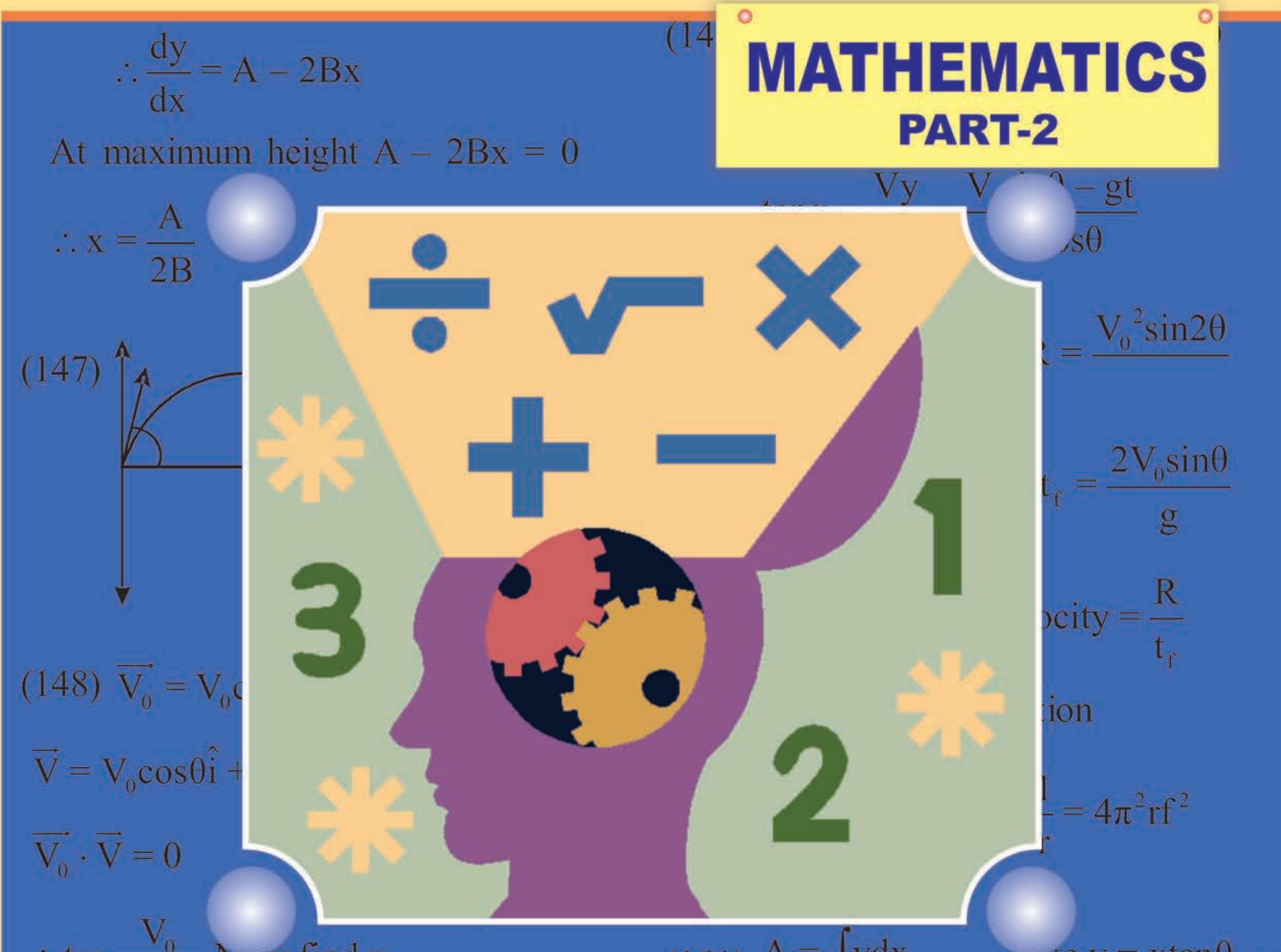
Gujarat Secondary and Higher Secondary Education Board, Gandhinagar

MATHEMATICS
PART-2



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QUESTION BANK
MATHEMATICS

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Contribution

1	Dr. Hasmukh Adhiya (IAS)	Principal Secretary , Education Department Gandhinagar
2	Shri R. R. Varsani (IAS)	Chairman , G.S&H.S.E. Bord, Gandhinagar
3	Shri H. K. Patel (G.A.S)	Dy. Chairman, G.S&H.S.E. Bord, Gandhinagar
4	Shri M. I. Joshi (G.E.S)	Secretary , G.S&H.S.E. Bord, Gandhinagar

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PREFACE

Uptil now , the Students had to appear in various entrance examinations for engineering and medical courses after std-12. The burden of examinations on the side of the students was increasing day-by-day. For alleviating this difficulty faced by the students, from the current year, the Ministry of Human Resource Development , Government of India, has Introduced a system of examination covering whole country. For entrance to engineering colleges, JEE(Main) and JEE(Advanced) examinations will be held by the CBSE. The Government of Gujarat has except the new system and has decided to follow the examinations to be held by the CBSE.

Necessary information pertaining to the proposed JEE (Main) and JEE(Advanced) examination is available on CBSE website www.cbse.nic.in and it is requested that the parents and students may visit this website and obtain latest information – guidance and prepare for the proposed examination accordingly. The detailed information about the syllabus of the proposed examination, method of entrances in the examination /centers/ places/cities of the examinations etc. is available on the said website. You are requested to go through the same carefully. The information booklet in Gujarati for JEE(Main) examination booklet has been brought out by the Board for Students and the beneficieries and a copy of this has been already sent to all the schools of the state. You are requested to take full advantage of the same also However, it is very essential to visit the above CBSE website from time to time for the latest information – guidance . An humble effort has been made by the Gujarat secondary and Higher Secondary Education Boards, Gandhinagar for JEE and NEET examinations considering the demands of the students and parents , a question bank has been prepared by the expert teachers of the science stream in the state. The MCQ type Objective questions in this Question Bank will provide best guidance to the students and we hope that it will be helpful for the JEE and NEET examinations.

It may please be noted that this "Question Bank" is only for the guidance of the Students and it is not a necessary to believe that questions given in it will be asked in the examinations. This Question Bank is only for the guidance and practice of the Students. We hope that this Question Bank will be useful and guiding for the Students appearing in JEE and NEET entrance examinations. We have taken all the care to make this Question Bank error free, however, if any error or omission is found, you are requested to refer to the text – books.

	M.I. Joshi	**R.R. Varsani (IAS)**
Date: 02/01/2013	**Secretary**	**Chairman**

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PART - II

Unit - 10

Differential Equation

Important Points

Differential Equation :

"y = f(x) and the derivatives of w.r.t. x are $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$,........... then the functional equation F(x, y, $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ ....) = 0 is called an ordinary differential equation."

Example, (1) $x^2\left(\frac{d^3y}{dx^3}\right)+y\left(\frac{dy}{dx}\right)=\log\frac{d^2y}{dx^2}$ (2) $\frac{dy}{dx}+\ell og\frac{d^2y}{dx^2}=xy$

Order of a differential equation :

"Order of the highest order derivative of the dependent variable with respect to the independent variable occurring in a given differential equation is called the order of differential equation."

Example, (1) order of $\left(\frac{d^3y}{dx^3}\right)^2+x\left(\frac{dy}{dx}\right)^5+y=o$ is 3

(2) order of $e^{\frac{dy}{dx}}+\frac{d^2y}{dx^2}$ is 2

Degree of a differential equation :

"When a differential equation is in a polynomial form in derivatives, the highest power of the highest order derivative occurring in the differential equation is called the degree of the differential equation."

Note : (1) The degree of a differential equation is a positive integer.

(2) If the differential equation cannot be expressed in a polynomial form in the derivatives, the degree of the differential equation is not defined.

Example : (1) The degree of $\left(\frac{dy}{dx}\right)^3=y+\frac{d^2y}{dx^2}$ is 1

(2) The degree of $x\frac{d^2y}{dx^2}+\sin\frac{dy}{dx}$ =o is not defined.

Differential Equation of first order and first degree :

f(x, y) dx + g(x, y) dy = 0 **OR** $\frac{dy}{dx}$ =F(x.y) is form of first order and first degree differential equation.

(1) Differential Equation of variables separable :

$\rightarrow$ p(x).dx + q(y).dy = 0 equation is said to be in variables separable form.

$\rightarrow$ solution : p(x).dx + q(y).dy= 0

$\Rightarrow \int p(x)dx + \int q(y)dy = c$ is the general solution (c is an arbitrang constant)

(2) Homogeneous differential equation :

$\rightarrow$ If in a differential equation f(x, y) dx + g(x, y) dy = 0, f(x, y) and g(x, y) are homogeneous functions with same degree, then this defferential equation is called homogeneous differential equation.

The homogenous differential equation be in the form of $\frac{dy}{dx} = \phi\left(\frac{y}{x}\right)$

$\rightarrow$ Solution : Let $\frac{y}{x} = v$

$\Rightarrow y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$

$\therefore$ Differential equation,

$\Rightarrow v + x\frac{dv}{dx} = \phi(v)$

$\Rightarrow \frac{dv}{\phi(v) - v} = \frac{dx}{x}$ (variable separable form)

$\Rightarrow \int \frac{1}{\phi(v) - v} dv = \int \frac{1}{x} dx$

$\Rightarrow \int \frac{1}{\phi(v) - v} dv = \ell og\,|x| + c$

This is the general solution of a homogeneous differential equation.

(3) Linear Differential Equation :

$\rightarrow$ If p(x) and q(x) are functions of variable x, then the differential equation

$\frac{dy}{dx}$ +P(x).y=Q(x) is called a linear differential equation.

$\rightarrow$ <u>Solution :</u>

If we multiply both sides by I.F. = $e^{\int p(x).dx}$.

We get, $\frac{dy}{dx}e^{\int p(x).dx} + p(x)ye^{\int p(x).dx} = \phi(x)e^{\int p(x).dx}$

$\Rightarrow \frac{d}{dx}\left[y.e^{\int p(x).dx}\right] = \phi(x)e^{\int p(x).dx}$

$\Rightarrow y.e^{\int p(x).dx} = \int \phi(x)e^{\int p(x).dx}$

This is the general solution of a linear differential equation.

Application in geometry :

Let y = f(x) is a given curve. Slope of the tangent at the point (x_0, y_0) is = $\left(\frac{dy}{dx}\right)_{(x_0, y_0)}$.

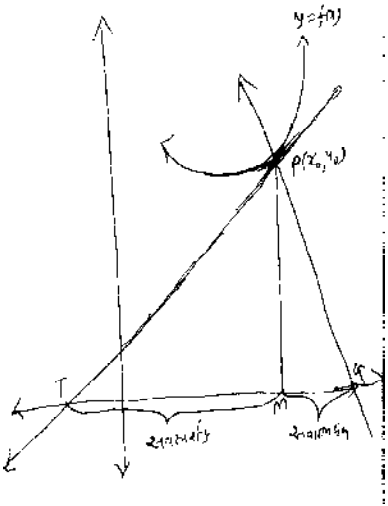
$\rightarrow$ The equation of the tangent to the curve at point (x_0, y_0) is $y - y_{0=}\left(\frac{dy}{dx}\right)_{(x_o, y_0)}(x - x_0)$.

$\rightarrow$ The equation of the normal to the curve at point (x_0, y_0) is $y - y_0 = \left(\frac{dx}{dy}\right)_{(x_0, y_0)}(x - x_0)$.

$\rightarrow$ Any point,

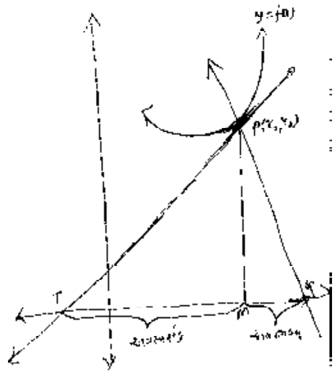
(1) Length of the tangent $PT = \left|\frac{y\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\frac{dy}{dx}}\right|$.

(2) Length of the normal $\mathrm{PG}=\left|\mathrm{y}\sqrt{1+\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2}\right|$



(3) Length of subtangent $TM=\left|\frac{y}{\frac{dy}{dx}}\right|$

(4) Length of subnormal $MG=\left|y\frac{dy}{dx}\right|$



QUESTION BANK

(1) The degree of the differential equation is $y_2^{\frac{3}{2}} - y_1^{\frac{1}{2}} + 1 = 0$ ________.

(A) 6 (B) 3 (C) 2 (D) 4

(2) The order of the differential equation whose general solution is given by

$y = c_1 e^{x+c_2} + (c_3 + c_4) . \sin (x + c_5)$,

where c_1, c_2, c_3, c_4, c_5 are arbitrary constant is ________.

(A) 5 (B) 4 (C) 3 (D) 2

(3) The degree of the differential equation of all curves having normal of constant length c is.

(A) 1 (B) 2 (C) 3 (D) none of these

(4) The degree of the differential equation $\frac{d^3y}{dx^3} + 7\left(\frac{d^2y}{dx^2}\right)^3 = x^2 . \log \frac{d^2y}{dx^2}$ is is :

(A) 2 (B) 3

(C) 1 (D) degree doesn't exist

(5) The degree of the differential equation satisfying

$\sqrt{1+x^2} + \sqrt{1+y^2} = k\left[x\sqrt{1+y^2} - y\sqrt{1+x^2}\right]$ is :

(A) 4 (B) 3 (C) 1 (D) 2

(6) If m and n are order and degree of the equation

$$\left(\frac{d^2y}{dx^2}\right)^5 + 4\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 . - 1, \quad \text{then :}$$

(A) m = 3, n = 2 (B) m = 3, n = 3 (C) m = 3, n = 5 (D) m = 3, n = 1

(7) The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively.

(A) 1, 2 (B) 3, 2 (C) 2, 3 (D) 2, 1

(8) The differential equation representing the family of curves $y^2 = 2c\left(x+\sqrt{c}\right)$, where c is a positive parameter, is of order and degree as follows.

(A) order 1, degree 1 (B) order 1, degree 2

(C) order 2, degree 2 (D) order 1, degree 3

(9) The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of.

(A) second order and second degree (B) first order and first degree

(C) first order and second degree (D) second order and first degree

(10) Order and degree of differential equation of all tangent lines to the parabola $y^2 = 4ax$ is ________ .

(A) 2, 2 (B) 3, 1 (C) 1, 2 (D) 4, 1

(11) The order of differential equation of all parabola with it's axis paralled to y-axis and touch x-axis is.

(A) 2 (B) 3 (C) 1 (D) none of these

(12) Which of the following differential equation has the same order and degree ________ .

(A) $\frac{d^4y}{dx^4} + 8\left(\frac{dy}{dx}\right)^6 + 5y = e^x$ (B) $5\left(\frac{d^3y}{dx^3}\right) + 8\left(1+\frac{dy}{dx}\right)^2 + 5y = x^8$

(C) $y = x^2\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ (D) $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$

(13) The differential equation of all conics having centre at the origin is of order.

(A) 2 (B) 3 (C) 4 (D) 5

(14) The order of the differential equation of family of circle touching a fixed straight line passing through origin is.

(A) 2 (B) 3 (C) 4 (D) none of these

(15) The order and degree of the differential equation $y^2 = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ are (respectively)

(A) 2, 1 (B) 2, 2 (C) 2, 3 (D) 2, 6

(16) Which of the following equations is a linear equation of order 3 ?

(A) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}.\frac{dy}{dx} + y = x$ (B) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + y^2 = x^2$

(C) $x.\frac{d^3y}{dx^3} + \frac{d^3y}{dx^3} = e^x$ (D) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \log x$

(17) Integrating factor of differential equation $\frac{1}{\cos x}.\frac{dy}{dx} + \frac{1}{\sin x} y = 1$ is.

(A) sec x (B) cos x (C) tan x (D) sin x

(18) The integrating factor of the differential equation $\frac{dy}{dx}.(x \log x) + y = 2\log x$ is :

(A) e^x (B) log x (C) log(logx) (D) x

(19) Integrating factor of differential equation $x \frac{dy}{dx} + y\log x = x. e^x. x^{-\frac{1}{2}\log x}$; $x \neq o$ is :

(A) $x^{\log x}$ (B) $\left(\sqrt{e}\right)^{(\log x)^2}$ (C) e^{x^2} (D) $x^{\log\sqrt{x}}$

(20) If sin x is an Integrating factor of $\frac{dy}{dx} + p.y = Q$ then p is :

(A) sin x (B) log sin x (C) cot x (D) log cos$x$

(21) Integrating factor of differential equation $(1+x)\frac{dy}{dx} - x.\ y = 1 - x$ is :

(A) 1 + x (B) log (1 + x) (C) $e^{-x}(1 + x)$ (D) x . e^{x}

(22) The order and degree of differential equation $\sqrt{1-y^2}\,dx + \sqrt{1-x^2}\,dy = o$ is________ .

(A) order 1, degree 1 (B) order 1, degree 2

(C) order 2, degree 1 (D) order and degree doesn't exist

(23) The degree of differential equation $(y_2)^2 - \sqrt{y_1} = y^3$ is ________ .

(A) $\frac{1}{2}$ (B) 2 (C) 3 (D) 4

(24) The order and degree of the differential equation $\left[1+3\frac{dy}{dx}\right]^{\frac{2}{3}} = 4.\frac{d^3y}{dx^3}$ are (respectively) ________ .

(A) 1, $\frac{2}{3}$ (B) 3, 1 (C) 3, 3 (D) 1, 2

(25) The Integrating factor of the differential equation $(1-y^2)\frac{dx}{dy} - yx = 1$ is :

(A) $\frac{1}{\sqrt{1-y^2}}$ (B) $\sqrt{1-y^2}$ (C) $\frac{1}{1-y^2}$ (D) $1-y^2$

(26) $y^2 = (x - c)^3$ is general solution of the differential equation : (where c is arbitrary constant).

(A) $\left(\frac{dy}{dx}\right)^3 = 27y$ (B) $2\left(\frac{dy}{dx}\right)^3 - 8y = 0$

(C) $8\left(\frac{dy}{dx}\right)^3 = 27y$ (D) $8\frac{d^3y}{dx^3} - 27y = 0$

(27) $y = ae^{2x} + be^{-3x}$ is general solution of differential equation :

(A) $\frac{d^2y}{dx^2}+\frac{dy}{dx}=6y$ (B) $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=6y$

(C) $\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$ (D) $x\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$

(28) The differential equation of family of curves $y = Ax + \left(\frac{B}{x}\right)$ is :

(A) $y\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}-y=o$ (B) $y\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}+y=o$

(C) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$ (D) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

(29) Family of curves $y = e^x$ (A cos x + B sin x) represents the differential equation : ________ . (where A and B are arbitrary constant)

(A) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=o$ (B) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}-2y=o$

(C) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}-y=0$ (D) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

(30) The differential equation of family of parabolas with focus at origin and x-axis as axis is :

(A) $y\left(\frac{dy}{dx}\right)^2-2x\frac{dy}{dx}=y$ (B) $y\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}=y$

(C) $y\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}=y$ (D) $y\left(\frac{dy}{dx}\right)^2+2x\frac{dy}{dx}=y$

(31) The differential equation of all parabolas having the directrix parallel to x-axis :

(A) $\frac{d^3x}{dy^3}=0$ (B) $\frac{d^3y}{dx^3}=0$ (C) $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=o$ (D) $\frac{d^2y}{dx^2}=o$

(32) The differential equation of all parabolas having axis parallel to y-axis :

(A) $\frac{d^3x}{dy^3} = 0$ (B) $\frac{d^3y}{dx^3} = 0$ (C) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = o$ (D) $\frac{d^2y}{dx^2} = o$

(33) The differential equation of family of hyperbolas with asymptotes x + y = 1 and x - y = 1 is :

(A) $yy_1 = x - 1$ (B) $yy_1 + x = 0$ (C) $yy_2 = y_1$ (D) $y_1 + xy = 0$

(34) The differential equation of family of circles of radius 'a' is :

(A) $a^2y_2 = [1 - y_1^3]^2$ (B) $a^2y_2 = [1 - y_1^2]^3$

(C) $a^2 (y_2)^2 = [1 + y_1^3]^2$ (D) $a^2 (y_2)^2 = [1 + y_1^2]^3$

(35) Family $y = Ax + A^3$ of curves is represented by the differential equation of degree :

(A) 1 (B) 2 (C) 3 (D) 4

(36) The differential equation of all non-vertical lines in a plane is :

(A) $\frac{dy}{dx} = 0$ (B) $\frac{d^3x}{dy^3} = 0$ (C) $\frac{d^2y}{dx^2} = 0$ (D) $\frac{dx}{dy} = 0$

(37) The differential equation of the family of circles with fixed radius 5 units and centeres on the line y = 2 is :

(A) $(y-2)^2\left(\frac{dy}{dx}\right)^2 = 25-(y-2)^2$ (B) $(y-2)\left(\frac{dy}{dx}\right)^2 = 25-(y-2)^2$

(C) $(x-2)\left(\frac{dy}{dx}\right)^2 = 25-(y-2)^2$ (D) $(x-2)^2\left(\frac{dy}{dx}\right)^2 = 25-(y-2)^2$

(38) The differential equation of all circles passing through the origin and having their centres on the x-axis is :

(A) $y^2 = x^2 + 2xy \frac{dy}{dx}$ (B) $y^2 = x^2 - 2xy \frac{dy}{dx}$

(C) $x^2 = y^2 + xy \frac{dy}{dx}$ (D) $x^2 = y^2 + 3xy \frac{dy}{dx}$

(39) The differential equation of all circles passing through the origin and having their centres on the y-axis is :

OR

The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is :

(A) $(x^2 - y^2)\, y' = 2xy$ (B) $2(x^2 - y^2)\, y' = xy$

(C) $2(x^2 + y^2)\, y' = xy$ (D) $(x^2 + y^2)\, y' = 2xy$

(40) The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1 and c_2 are arbitarary constants, is :

(A) $y' = y^2$ (B) $y'' = y'y$ (C) $yy'' = (y')^2$ (D) $yy'' = y'$

(41) The general solution of the differential equation $x(1 + y^2)\, dx + y(1 + x^2)\, dy = 0$ is :

(A) $(1 + x^2)(1 + y^2) = 0$ (B) $(1 + y^4)\, c = (1 + x^2)$

(C) $(1 + x^2)(1 + y^2) = c$ (D) $(1 + x^2) = c(1 + y^2)$

(42) The solution of $\frac{dy}{dx} = \frac{ax+b}{cy+d}$ represents a parabola if.

(A) a = 1, b = 2 (B) a = 0, $c \neq 0$ (C) a = 0, c = 0 (D) a = 1, c = 1

(43) Solution of differential equation $\frac{dy}{dx} + ay = e^{mx}$ is :

(A) $y = e^{mx} + c.e^{-ax}$ (B) $(a + m)y = e^{mx} + c$

(C) $(a + m)\, y = e^{mx} + c.e^{-ax}$ (D) $y.e^{ax} = m.e^{mx} + c$

(44) The curve for which the slop of the tangent at any point equals the ratio of the abscissa to the ordinate of the point is:

(A) a circle (B) an ellipse

(C) a rectangular hyperbola (D) none of these

(45) A particle moves in a straigth line with a velocity given by $\frac{dx}{dt} = x + 1$ (x is the distance described) the time taken by a particle of transverse a distance of 99 meters is :

(A) $2 \log_e 10$ (B) $\log_{10}^e$ (C) $2 \log_{10}^e$ (D) none of these

(46) If y = y(x) and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x$, y(0) = 1, then $y\left(\frac{\pi}{2}\right)$ equal :

(A) $\frac{-1}{3}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 1

(47) Solution of $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, y(0) = 0 is :

(A) $y = \tan(c + x + x^2)$

(B) $y = \tan\left(x + \frac{x^2}{2}\right)$

(C) $y^2 = \exp\left(x + \frac{x^2}{2}\right) - 1$

(D) $y^2 = 1 + c \cdot \exp\left(x + \frac{x^2}{2}\right)$

(48) The solution of xdy - ydx = 0 represents :

(A) parabola having vertex at (0, 0)

(B) circle having centre at (0, 0)

(C) a st. line passing through (0, 0)

(D) a rectangular hyperbola

(49) The differential equation $y\frac{dy}{dx} + x = a$ ('a' being a constant) represents :

(A) set of circles with centres on y-axis

(B) set of circles with centres on x-axis

(C) set of parabolas

(D) set of ellipses

(50) The solution of $\frac{d^2y}{dx^2} = o$ represents :

(A) a point (B) a st. line (C) a parabola (D) a circle

(51) The general solution of the equation $\frac{dy}{dx} = \frac{x^2}{y^2}$ is :

(A) $x^3 + y^3 = c$ (B) $x^3 - y^3 = c$ (C) $x^2 + y^2 = c$ (D) $x^2 - y^2 = c$

(52) The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ is : y = ________ .

(A) $\frac{1}{4}e^{-2x} + cx + d$ (B) $\frac{1}{4}e^{-2x}$ (C) $\frac{1}{4}e^{-2x} + cx^2 + d$ (D) $\frac{1}{4}e^{-2x} + cx + d$

(53) If $\frac{dy}{dx}$ = y +3 > 0 and y(0) = 2, then y (log 2) is equal to.

(A) -2 (B) 5 (C) 7 (D) 13

(54) The curves whose subtangents are proportional to the abscissas of the point of tangency (the proportionality factor is equal to k) is :

(A) $y^k = cx^2$ (B) $y^k = cx$ (C) $y^{\frac{k}{2}} = cx^3$ (D) none of these

(55) An equation of the curve in which subnormal varies as the square of the ordinate is (k is constant of proportinaliting)

(A) $\frac{y^2}{2} + kx = A$ (B) $y^2 + kx^2 = A$ (C) $y = e^{kx}$ (D) $y = Ae^{kx}$

(56) Solution of $\frac{d^2y}{dx^2}$ = log x is :

(A) $y = \frac{1}{2} x^2 \log x - \frac{3}{4} x^2 + c_1x + c_2$ (B) $y = \frac{1}{2} x^2 \log x + \frac{3}{4} x^2 + c_1x + c_2$

(C) $y = -\frac{1}{2} x^2 \log x - \frac{3}{4} x^2 - c_1x + c_2$ (D) None of these

(57) Solution of $\frac{d^2y}{dx^2} = xe^x + 1$ is :

(A) $y = (x - 2) e^x + \frac{1}{2} x^2 + c_1x + c_2$ (B) $y = (x - 1) e^x + \frac{1}{2} x^2 + c_1x + c_2$

(C) $y = (x + 2) e^x + \frac{1}{2} x^2 + c_1x + c_2$ (D) None of these

(58) If $y = \left(x+\sqrt{1+x^2}\right)^n$, then $\left(1+x^2\right).\frac{d^2y}{dx^2}+x.\frac{dy}{dx}=$ ________ .

(A) -y (B) $2x^2y$ (C) n^2y (D) $-n^2y$

(59) $\frac{dy}{dx} = e^{x+y} + x^2 e^y$ has the particular solution for x = y = 0 :

(A) $e^x - e^{-y} + \frac{x^3}{3} = 2$ (B) $e^x + e^{-y} + \frac{x^3}{3} = 2$

(C) $e^{x-y} + \frac{x^3}{3} = 2$ (D) $e^{y-x} - \frac{x^3}{3} = 2$

(60) The equation of a curve passing through $\left(2, \frac{7}{2}\right)$ and having gradient $1-\frac{1}{x^2}$ at (x, y) is :

(A) xy = x + 1 (B) $y = x^2 + x + 1$ (C) $xy = x^2 + x + 1$ (D) none of these

(61) A particular solution of $\log \frac{dy}{dx} = 3x + 4y$, y(0) = 0 is :

(A) $3e^{3x} + 4e^{4y} = 7$ (B) $4.e^{3x} - e^{-4y} = 3$ (C) $e^{3x} + 3e^{-4y} = 4$ (D) $4e^{3x} + 3e^{-4y} = 7$

(62) Solution of differential equation : dy - sin x . sin y dx = 0 is :

(A) $e^{\cos x} . \tan \frac{y}{2} = c$ (B) cos x . tan y = c (C) $e^{\cos x} . \tan y = c$ (D) cos x . sin y = c

(63) The curve passing through the point (0, 1) and satisfying the equation $\sin\left(\frac{dy}{dx}\right) = a$ is :

(A) $\cos\left(\frac{y+1}{x}\right) = a$ (B) $\sin\left(\frac{y-1}{x}\right) = a$ (C) $\cos\left(\frac{x}{y+1}\right) = a$ (D) $\sin\left(\frac{x}{y-1}\right) = a$

(64) The particular solution of the differential equation $y^1 - y = 1$; y(0) = 1 is y(x) = ________ .

(A) -1 (B) -exp(-x) (C) -exp(x) (D) 2 exp(x) - 1

(65) The particular solution of $(1 + y^2)\, dx + (x - e^{-\tan^{-1} y})dy = 0$ with intial condition y(0) = 0 is :

(A) $xe^{\tan^{-1} x} = \cot^{-1} x$

(B) $x.e^{\tan^{-1} y} = \tan^{-1} y$

(C) $x.e^{\tan^{-1} y} = \cot^{-1} y$

(D) $x.e^{\cot^{-1} y} = \tan^{-1} y$

(66) The equation of the curve passing through $\left(1, \frac{\pi}{4}\right)$ and having the slope $\left(\frac{\sin 2y}{x + tauy}\right)$ at (x, y) is :

(A) x = tan y

(B) y = 2tan x

(C) y = tan x

(D) x = 2tan y

(67) The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$ is :

(A) $x \,.\, e^{\tan^{-1} y} = \tan^{-1} y + k$

(B) $x \,.\, e^{2\tan^{-1} y} = e^{-\tan^{-1} y} + k$

(C) $2x \,.\, e^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

(D) $(x - 2) = k \,.\, e^{\tan^{-1} y}$

(68) Solution of the differential equation cos x . dy = y (sin x - y) dx, $0 < x < \frac{\pi}{2}$ is :

(A) y tan x = sec x + c

(B) tan x = (sec x + c) y

(C) y sec x = tan x + c

(D) sec x = (tan x + c) y

(69) If $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$ then,

(A) $y = -\cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$

(B) $y = \cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$

(C) $y = -\cos x - \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$

(D) None of these

(70) The solution of $x^2y - x^3 \frac{dy}{dx} = y^4 \cos x$; y(0) = 1 is :

(A) $x^3 = y^3 \sin x$ (B) $x^3 = 3y^3 \sin x$ (C) $y^3 = 3x^3 \sin x$ (D) none the these

(71) The solution of $\frac{dy}{dx} = \frac{1}{2x - y^2}$ is :

(A) $x = c \cdot e^{2y} + \frac{1}{2}y^2 + \frac{1}{2}y + \frac{1}{4}$ (B) $x = c \cdot e^{-y} + \frac{1}{4}y^2 + \frac{1}{4}y + \frac{1}{2}$

(C) $y = c \cdot e^{-2x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4}$ (D) $x = c \cdot e^{y} + \frac{1}{4}y^2 + y + \frac{1}{2}$

(72) The solution of the equation $x + y\frac{dy}{dx} = 2y$ is :

(A) $xy^2 = c^2 (x + 2y)$ (B) $y^2 = c(x^2 + 2y)$

(C) $\log (y - x) = c + \frac{x}{y - x}$ (D) $\log \left(\frac{x}{x - y}\right) = c + y - x$

(73) The solution of intial value problem $x\frac{dy}{dx} = x + y$; y(1) = 1 is y = ________ .

(A) x log x - 1 (B) x log x + 1 (C) x (log x + 1) (D) none of these

(74) The slope of the tangent at (x, y) to a curve passing through $(1, \frac{\pi}{4})$ is given by $\frac{y}{x} - \cos^2 \frac{y}{x}$, then the equation of the curve is :

(A) $y = \tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$ (B) $y = x \cdot \tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$

(C) $y = x \tan^{-1}\left(\frac{x}{e}\right)$ (D) none of these

(75) If $x\frac{dy}{dx}$ =y (log y - log x + 1), then the solution of the equation is :

(A) $x\log\left(\frac{y}{x}\right)$ = cy (B) $\log\left(\frac{y}{x}\right)$ = cx (C) $\log\left(\frac{x}{y}\right)$ = cy (D) $y\,.\log\left(\frac{x}{y}\right)$ = cx

(76) The solution of the differential equation $\frac{dy}{dx}=\frac{x+y}{x}$ satistying the condition y(1) = 1 is :

(A) y = x lnx + x (B) y = lnx + x (C) y = x lnx + x^2 (D) y = x . e^{x-1}

(77) The general solution of $\left(x\frac{dy}{dx}-y\right)e^{\frac{y}{x}}$ = x^2 cosx is :

(A) $e^{\frac{x}{y}}$ = cos x + c (B) $e^{\frac{x}{y}}$ = sin x + c (C) $e^{\frac{y}{x}}$ = sin x + c (D) $e^{\frac{y}{x}}$ = cos x + c

(78) The solution of differential equation x sin $\frac{y}{x}$ dy = (y sin $\frac{y}{x}$ - x) dx is :

(A) log y = $\cos\left(\frac{y}{x}\right)$ + c (B) log x = $\cos\left(\frac{x}{y}\right)$ + c

(C) log x = $\cos\left(\frac{y}{x}\right)$ + c (D) log y = $\cos\left(\frac{x}{y}\right)$ + c

(79) If the slope of tangent af (x, y) to the curve passing through (2, 1) is $\frac{x^2+y^2}{2xy}$ The equation of the curve is :

(A) $2(x^2 - y^2) = 6y$ (B) $2(x^2 - y^2) = 3x$ (C) $x(x^2 + y^2) = 10$ (D) $x(x^2 - y^2) = 6$

(80) Solution of $\frac{y}{x}\cos\frac{y}{x}\left(\frac{dy}{dx}-\frac{y}{x}\right)+\sin\frac{y}{x}\left(\frac{dy}{dx}+\frac{y}{x}\right)=o$; $y(1)=\frac{\pi}{2}$ is :

(A) $y\sin\frac{y}{x}=\frac{\pi}{2x}$ (B) $y\sin\frac{y}{x}=\frac{\pi}{x}$ (C) $y\sin\frac{y}{x}=\frac{\pi}{3x}$ (D) none of these

(81) The solution of the differential equation y dx + (x + x^2y) dy = 0 is :

(A) $\frac{1}{xy}$ + log y = c (B) -$\frac{1}{xy}$ + log y = c (C) -$\frac{1}{xy}$ = c (D) log y = cx

(82) The solution of y^5x + y - x $\frac{dy}{dx}$ = 0 is :

(A) $\left(\frac{x}{y}\right)^5 + \frac{x^4}{4}$ = c (B) $(xy)^4 + \frac{x^5}{5}$ = c (C) $\frac{x^5}{y} + \frac{1}{4}\left(\frac{x}{y}\right)^4$ = c (D) $\frac{x^4}{y} + \frac{1}{5}\left(\frac{x}{y}\right)^5$ = c

(83) The solution of $\frac{x}{x^2+y^2}dy = \left(\frac{y}{x^2+y^2} - 1\right)dx$ is :

(A) y = x tan (c - x) (B) y = x cot (c - x) (C) $\cos^{-1}\frac{y}{x}$ = -x + c (D) $\frac{y^2}{x^2}$ = x tan (c - x)

(84) The solution of the differential equation $x^2\frac{dy}{dx}$ - xy = 1 + cos $\frac{y}{x}$ is :

(A) tan $\frac{y}{x}$ = c + $\frac{1}{x}$

(B) tan $\frac{y}{2x}$ = c - $\frac{1}{2x^2}$

(C) cos $\frac{y}{x}$ = 1 + $\frac{c}{x}$

(D) x^2 = (c + x^2) . tan $\frac{y}{x}$

(85) A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = O$ is :

(A) y = 2 (B) y = 2x^2 - 4 (C) y = 2x (D) y = 2x - 4

(86) The solution of $\frac{dy}{dx}$ = 4x + y + 1 is : _________ .

(A) 4x + y + 1 = c . e^x

(B) 4x + y + 5 = e^x + c

(C) 4x + y + 5 = c . e^x

(D) none of these

(87) If the general solution of $\frac{dy}{dx} = \frac{y}{x} + f\left(\frac{x}{y}\right)$ is $y = \frac{x}{\log|cx|}$, then $f\left(\frac{x}{y}\right)$ is given by :

(A) $\frac{x^2}{y^2}$ (B) $\frac{y^2}{x^2}$ (C) $\frac{-x^2}{y^2}$ (D) $\frac{-y^2}{x^2}$

(88) If f and g are twice differentiable on [0, 2] satisfying $f^{11}(x) = g^{11}(x)$, $f^1(1) = 4$, $g^1(1) = 1$, f(3) = 3, g(3) = 9, then f(x) - g(x) at x = 5 is :

(A) 0 (B) -30 (C) 30 (D) none of these

(89) Integral curve satisfying $\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 - y^2}$, y(1) = 1 has the slope at point (1, 0) of the curve, equal to :

(A) $\frac{5}{3}$ (B) $\frac{-5}{3}$ (C) 1 (D) -1

(90) If integrating factor of : $x(1 - x^2)\, dy + (2x^2y - y - ax^3)\, dx = 0$ is $e^{\int p.dx}$, then p is equal to :

(A) $2x^2 - 1$ (B) $\frac{2x^2 - 1}{x\left(1 - x^2\right)}$ (C) $\frac{2x^2 - ax^3}{x\left(1 - x^2\right)}$ (D) $\frac{2x^2 - 1}{ax^3}$

(91) The solution of the equation (2x + y + 1) dx + (4x + 2y - 1) dy = 0 is :

(A) log |2x + y - 1| + x + 2y = c (B) log (2x + y + 1) + x + 2y = c

(C) log |2x + y - 1| = c + x + y (D) log (4x + 2y - 1) = c + 2x + y

(92) If f(x) and g(x) are two solutions of the differential equation $q\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + y = e^x$, then f(x) - g(x) is the solution of :

(A) $q\frac{d^2y}{dx^2} + y = e^x$ (B) $q^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^x$

(C) $q^2\frac{d^2y}{dx^2} + y = e^x$ (D) $q\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + y = o$

(93) Differential equation of the curves having the subnormal with $\frac{7}{2}$ units and passes through (0, 0) is :

(A) $x^2 = 7y$ (B) $y^2 = 7x + c$, ßÞk $c \neq 0$

(C) $y^2 = 7x$ (D) None of these

(94) Let m and n be respectively the degree and order of the differential equation of the family of circles touching the lines $y^2 = x^2$ and lying in the 1^{st}, 2^{nd} quadrant. Then

(A) m = 1, n = 1 (B) m = 1, n = 2 (C) m = 2, n = 2 (D) none of these

(95) The solution of (3x + 2y + 1) dx - (3x + 2y - 1) dy = 0 is :

(A) $\frac{5}{2}$ (x - 2) + log (15x) = c (B) log (15x + 10y - 1) = c

(C) log (15x + 10y - 1) + $\frac{5}{2}$ (x - y) = c (D) none of these

(96) The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is :

(A) $\phi\left(\frac{y}{x}\right) = kx$ (B) $\phi\left(\frac{y}{x}\right) = ky$ (C) $x.\phi\left(\frac{y}{x}\right) = k$ (D) $y.\phi\left(\frac{y}{x}\right) = k$

(97) The family passing through (0, 0) and satisfying the differential equation $\frac{y_2}{y_1} = 1\left(\text{where } y_n = \frac{d^n y}{dx^n}\right)$ is :

(A) y = k (B) y = kx (C) $y = k(e^x - 1)$ (D) $y = k(e^x + 1)$

(98) If $\sin(x+y)\dfrac{dy}{dx} = 5$ then

(A) $5\int \dfrac{dt}{5+\sin t} = t + x$(where t=x+y)

(B) $5\int \dfrac{dt}{5+\sin t} = t - x$(where t=x+y)

(C) $\dfrac{dt}{5+\cos ect} = dx$(where t=x+y)

(D) $\dfrac{dt}{5s\text{in}\,t+1} = dt$(where t=x+y)

(99) The solution of $x^3 \dfrac{dy}{dx} + 4x^2 . \tan y = e^x . \sec y$ satisfying y(1) = 0 is :

(A) $\sin y = e^x (x-1) x^{-4}$

(B) $\tan y = (x-1)e^x . x^{-3}$

(C) $\sin y = e^x (x-1) x^{-3}$

(D) $\tan y = (x-2)e^x . \log x$

(100) The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{1}{xy\left[x^2 \sin y^2 + 1\right]}$ is :

(A) $x^2 (\cos y^2 - \sin y^2 - e^{-y^2}) = 4$

(B) $y^2 (\cos x^2 - \sin y^2 - 2c . e^{-y^2}) = 2$

(C) $x^2 (\cos y^2 - \sin y^2 - 2c\, e^{-y^2}) = 2$

(D) none of these

Assertion - Reason Type Questions :

Each question has four choices (a), (b), (c) and (d) out of which only one is correct. Write (a), (b), (c) and (d) according to the following rules.

(a) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True, Statement-2 is **not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

(101) Statement-1 : Curve satisfying the differential equation $y' = \dfrac{y}{2x}$ passing through (2, 1) is a parabola with $\text{Focus}\left(\dfrac{1}{8}, 0\right)$.

Statement-2 : The differential equation $y' = \dfrac{y}{2x}$ is variable separable.

(102) Statement-1 : Curve satisfying the differential equation $\frac{dy}{dx} = \frac{y}{2x}$ passing through (2, 1) is a parabola with $\underline{\text{Focus}\left(\frac{1}{4}, 0\right)}$.

Statement-2 : The differential equation $\frac{dy}{dx} = \frac{y}{2x}$ is variable separable.

(103) Let $(xy^2 + x)\,dx + (y - x^2y)\,dy = 0$ satisfy $y(0) = 0$

Statement-1 : The curve represented by the solution of the given differential equation is a circle.

Statement-2 : It is circle with radius 1 and centre (0, 0)

(104) Statement-1 : The differential equation of all circles in a plane must have maximum be of order 3.

Statement-2 : There is only one circle passing through three non-collinear points.

(105) Let $\frac{dy}{dx} + \sin\frac{x+y}{2} = \sin\frac{x-y}{2}$.

Statement-1 : A solution satisfying $y(0) = \pi$ is periodic function with period 4π.

Statement-2 : y can be explicitly represented in terms of x.

Hints

(2) $y = c_1.e^{c_2}.e^x + (c_3 + c_4).\sin(x + c_5)$

$y = Ae^x + B \sin(x + C)$

where, A, B, C are three arbitrary constant.

$\therefore$ order = 3

(3) Length of the normal = $y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$.

(4) The differential equation cannot be expressed in a polynomial form.

(5) Let x = tan α and y = tan β,

equation is,

$\sec\alpha + \sec\beta = k(\tan\alpha \sec\beta - \tan\beta \sec\alpha)$

$$\Rightarrow \cot\left(\frac{\alpha-\beta}{2}\right) = k$$

$$\Rightarrow \alpha - \beta = 2\cot^{-1} k$$

$$\Rightarrow \tan^{-1} x - \tan^{-1} y = 2\cot^{-1} k$$

$$\Rightarrow \frac{1}{1+x^2} - \frac{1}{1+y^2}\frac{dy}{dx} = O$$

$\therefore$ degree = 1

(7) Family of all parabolas, $y^2 = 4a(x - h)$, where a, h arbitrary constants.

(10) Equation of all tangent lines to the parabola

$y^2 = 4ax$ is, $y = mx + \frac{a}{m}$,

a = constants

m = arbitrary constants.

(11) Equation of all parabola,

$(x - h)^2 = 4by$, where h, b is arbitrary constants.

(13) Equation of all conics,

$ax^2 + 2hxy + by^2 = 1$. Where a, h, b is arbitrary constants.

(14) According to the condition, equation of family of circle has two arbitrary constants.

(16) (c) and (d) is linear differential equation but (d) is differential equation of order 2.

(20) I.F. $= e^{\int p(x)dx} = \sin x = e^{\log_e \sin x}$

$\Rightarrow \int P(x)dx = \log \sin x$

$\Rightarrow P(x) = \cot x$

(25) Differential equation, $\dfrac{dx}{dy} - \dfrac{y}{(1-y^2)}x = \dfrac{1}{1-y^2}$

I.F. = $e^{\int p(y)dx} = e^{-\int \frac{y}{1-y^2}dy}$

(26) $y^2 = (x - c)^3$ ...(1)

$\Rightarrow 2yy_1 = 3(x - c)^2$ ...(2)

$\dfrac{(2)}{(1)} \Rightarrow x - c = \dfrac{3y}{y_1}$

(27) The differential equation whose general solution is,

$y = Ae^{\alpha x} + B.e^{\beta x}$

is $(D - \alpha)(D - \beta)y = 0$.

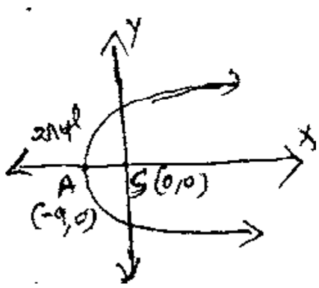
$\therefore (D - 2)(D + 3)y = 0$ $(\because \alpha = 2, \beta = -3$

$\Rightarrow (D^2 + D - 6)y = 0$

$\Rightarrow \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 6y = 0$

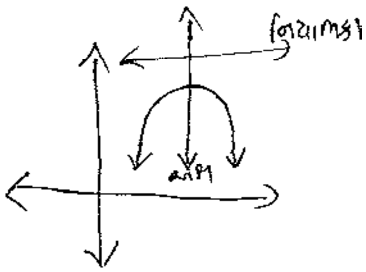
(30) The equation of family of parabolas,

$y^2 = 4a(x + a)$, where a is arbitrary constants.



(31) The equation of family of parabolas,

$(x - h)^2 = 4b (y - k)$, where, h, k, b arbitrary constants.



(32) The equation of family of parabolas,

$(x - h)^2 = 4b (y - k)$, where, h, k, b arbitrary constants.

(33) Asymptotes are mutually perpendicular

$\therefore$ curve is rectegular hyperbola with centre (1, 0)

$\therefore$ equation is

$$\frac{(x-1)^2}{a^2} - \frac{(y-o)^2}{a^2} = 1$$

where a = arbitrary constant.

(34) Equation of family of circle of radius 'a' is,

$(x - h)^2 + (y - k)^2 = a^2$, where, h, k arbitrary constant and a = constant.

(36) Equation of family of lines,

$y = mx + c$, where m, c arbitary constant.

(37) Equation of family of circles,

$\left[\text{center}\left(\text{h}, 2\right), \text{radius} = 5\right]$

$(x - h)^2 + (y - 2)^2 = 25$,

where h = arbitary constant.

(38) Equation of family of circles,

$(x - a)^2 + y^2 = a^2$,

where a = arbitary constant.

(39) Equation of family of circles,

$x^2 + (y - a)^2 = a^2$,

where a = arbitary constant.

(41) Differential equation, $\frac{x}{1+x^2}\,dx + \frac{y}{1+y2}\,dy = 0$

is variables separable form.

(42) Differential equation,

(cy + d) dy = (ax + b) dx

$\Rightarrow c\frac{y^2}{2} + dy = a\frac{x^2}{2} + bx + k$

is represent a parabola then, c = 0, $a \neq 0$ **OR** $c \neq o$, a = 0

(43) $\frac{dy}{dx} + a.y = e^{mx}$ is linear differential equation.

$\therefore$ I.F. = $e^{\int P(x).dx} = e^{\int a.dx} = e^{ax}$

(44) Here, $\frac{dy}{dx} = \frac{x}{y}$,

$\Rightarrow$ y.dy = x.dx ,

is variables separable form.

(45) Here, $\frac{dx}{dt} = x+1$, is variables separable form.

Now, x = 99 m then t = ?

(46) Differential equation,

$$\frac{1}{y+1}dy = \frac{-\cos x}{2+\sin x}\,dx$$

$$\Rightarrow \ell og(y+1) = -\ell og\,|2+\sin x| + \log|c|$$

$$\Rightarrow y+1 = \frac{c}{2+\sin x}$$

(47) $\frac{dy}{dx} = (1+x)\,(1+y^2)$

$$\Rightarrow \frac{1}{1+y^2}dy = (1+x)dx$$

(48) Differential equation, $\frac{1}{y}dy - \frac{1}{x}dx = o$

(49) Differential equation, y . dy = (a - x) . dx

(50) $\frac{d^2y}{dx^2} = 0$

$\Rightarrow \frac{dy}{dx} = c$

y = cx + k, is represent line.

(52) $\frac{d^2y}{dx^2} = e^{-2x}$

$\Rightarrow \frac{dy}{dx} = \int e^{-2x}dx$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}.e^{-2x} + c$

$\Rightarrow y = \frac{-1}{2}\int e^{-2x}\, dx + \int c\, d\, x$

(53) Differential equation,

$\frac{1}{y+3}dy = dx$

(54) $y\frac{dx}{dy} \propto x$

$\Rightarrow y\frac{dx}{dy} = kx$

(55) $y\frac{dy}{dx} \propto y^2$

$\Rightarrow y\frac{dy}{dx} = ky^2$

(56) $\frac{d^2y}{dx^2} = \log x$

$\Rightarrow \frac{dy}{dx} = \int \log x.dx$

(59) Differential equation, $\frac{dy}{dx} = e^y \left[e^x + x^2\right]$

$\Rightarrow \frac{1}{e^y} dy = \left(e^x + x^2\right)dx$, is variable separable form.

(60) $\frac{dy}{dx} = 1 - \frac{1}{x^2}$

$dy = \left(1 - \frac{1}{x^2}\right)dx$

(61) Differential equation,

$\frac{dy}{dx} = e^{3x+4y}$

$\Rightarrow \frac{dy}{dx} = e^{3x} \ e^{4y}$

$\Rightarrow e^{-4y} dy = e^{3x} dx$

(63) Differential equation, $\frac{dy}{dx} = \sin^{-1} a$

$\Rightarrow \int 1.\ dy = \int \sin^{-1} a.dx$

(64) Differential equation, $\frac{dy}{dx} - y = 1$

$\Rightarrow \frac{1}{y+1} dy = dx$

(65) Differential equation,

$$\left(1+y^2\right)\frac{dx}{dy}+x=e^{-\tan^{-1}y}$$

$$\Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2}\, x = \frac{e^{-\tan^{-1}y}}{1+y^2}$$

is linear differential equation.

$$\text{I. F.} \quad = e^{\int p(y)dy}$$

$$= e^{\int \frac{1}{1+y^2}} = e^{\tan^{-1}y}$$

(66) $\frac{dy}{dx} = \frac{\sin 2y}{x+\tan y}$

$$\Rightarrow \frac{dx}{dy} = \frac{x+\tan y}{\sin 2y}$$

$$\Rightarrow \frac{dx}{dy} - \frac{1}{\sin 2y}x = \frac{1}{2\cos^2 y}$$

is linear differential equation.

$$\text{I. F.} \quad = e^{-\int \frac{1}{\sin 2y}} = e^{-\frac{1}{2}\log\tan y}$$

(68) $\frac{dy}{dx} = y\tan x - \frac{1}{\cos x}y^2$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \frac{1}{y}\tan x - \sec x$$

$$\Rightarrow \frac{-dt}{dx} = t.\tan x - \sec x \qquad \left(\because \frac{1}{y}=t \Rightarrow \frac{1}{y^2}\frac{dy}{dx}=\frac{-dt}{dx}\right)$$

$\Rightarrow \frac{dt}{dx} + \tan x.t = \sec x$, is linear differential equation.

$\text{I. F.} = e^{\int \tan x.\, dx}$

(69) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

$y + x\frac{dy}{dx} + y = x(\sin x + \log x)$

$\frac{dy}{dx} + \frac{2}{x}.y = \sin x + \log x$, is linear differential equation.

(70) Differential equation,

$$\frac{1}{x}.\frac{1}{y3} - \frac{1}{y4}\frac{dy}{dx} = \frac{1}{x3}\cos x \; (\therefore \; x^3y^4 \neq o$$

by taking $\frac{1}{y3} = t$, it will be a linear differential equation.

(71) $\frac{dy}{dx} = 2x - y^2$

$\Rightarrow \frac{dx}{dy} - 2.\, x = - y^2$

$\text{I. F.} = e^{\int -2\, .dy}$

$= e^{-2y}$

(72) Differential equation,

$$\frac{dy}{dx} = \frac{2y-x}{y}$$, is , is Homogeneous differential equation.

take $y = v x$

(74) $\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x}$,

is Homogeneous differential equation.

(75) Differential equation,

$$\frac{dy}{dx} = \frac{y}{x}\left[\log \frac{y}{x}+1\right], \text{ is}$$

Homogeneous differential equation.

(77) Take $\frac{y}{x} = v$

$$\Rightarrow \frac{x\frac{dy}{dx} - y}{x^2} = \frac{dv}{dx}$$

$$\Rightarrow x\frac{dy}{dx} - y = x^2.\frac{dv}{dx}$$

$\therefore$ differential equation,

$$x^2.\frac{dv}{dx}.e^v = x^2.\cos x$$

$$\Rightarrow \int e^v.dv = \int cosx.dx$$

(79) $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$, is

Homogeneous differential equation.

(80) $$\frac{dy}{dx} = \frac{\frac{y^2}{x^2}\cos\frac{y}{x} - \frac{y}{x}\sin\frac{y}{x}}{\frac{y}{x}\cos\frac{y}{x} + \sin\frac{y}{x}}$$

is Homogeneous differential equation.

(81) Differential equation, y dx + x dy = - x^2 . y dy

$$\Rightarrow \frac{y.dx + xdy}{x^2.y^2} = -\frac{1}{y}dy$$

take xy = t.

(82) Differential equation,

y^5 x dx + y dx - x dy = 0

$$x^4.dx + \frac{x^3}{y^3}\left(\frac{y.dx\text{-}xdy}{y^2}\right) = o \quad \text{(mulfi pul by } \frac{x^3}{y^5}$$

take $\frac{x}{y} = v$

(83) differential equation,

$$\frac{xdy - ydx}{x^2 + y^2} = -dx \Rightarrow \frac{\frac{xdy-ydx}{x^2}}{1 + \frac{y^2}{x^2}} = -dx$$

take $\frac{y}{x} = \nu$

(84) Differential equation,

$$x^2 \frac{dy}{dx} - xy = 2\cos^2\left(\frac{y}{2x}\right)$$

$$\Rightarrow \frac{1}{2}\sec^2\left(\frac{y}{2x}\right)\left[\frac{x\frac{dy}{dx} - y}{x^2}\right] = \frac{1}{x^3}$$

$$\Rightarrow \frac{d}{dx}\left[\tan\left(\frac{y}{2x}\right)\right] = \frac{1}{x^3}$$

(85) Take $\frac{dy}{dx} = p$,

p^2 - xp + y = 0

$\Rightarrow$ y = xp - p^2 ... (1)

$$\Rightarrow \frac{dy}{dx} = (x - 2p)\frac{dp}{dx} + p$$

$$\Rightarrow p = (x - 2p)\frac{dp}{dx} + p$$

$\Rightarrow \frac{dp}{dx} = o$

$\Rightarrow$ p=constant

$\therefore$ from (1), $y = x \cdot c - c^2$, here c = 2

(86) Take $4x + y + 1 = v$.

(87) Take $\frac{y}{x} = v$, then $\frac{x}{y} = \frac{1}{v}$

(88) $f''(x) = g''(x) \Rightarrow f'(x) = g'(x) + c$

(91) Take $2x + y = v$

(92) $af''(x) + x^2 f'(x) + y = e^x$ **and** $ag''(x) + x^2 g'(x) + y = e^x$

$\Rightarrow a[f'' - g''] + x^2 [f' - g'] + [f - g] = 0$

$\Rightarrow a\frac{d^2y}{dx^2} + x^2 \frac{dy}{dx} + y = O$

(93) $y \frac{dy}{dx} = \frac{7}{2}$

(94) Equation of the family of circles, $(x - 0)^2 + (y - b)^2 = r^2$, where b, r arbitrary constant.

(95) Take $3x + 2y = v$

(96) Take $\frac{y}{x} = v$

(97) $\frac{y_2}{y_1} = 1$

$\Rightarrow \frac{d}{dx}[\log y_1] = 1$

(98) Differential equation,

$\frac{dy}{dx} = 5\text{cosec}\,(x+y)$

take x + y = t

(99) Both side multiply by xcos y,

$$x^4 . \cos y \frac{dy}{dx} + 4x^3 . \sin y = x . e^x$$

$$\Rightarrow \frac{d}{dx}\left(x^4 \sin y\right) = x.e^x$$

(100) Differential equation,

$$\frac{dy}{dx} = xy\,[x^2 \sin y^2 + 1]$$

$$\Rightarrow \frac{1}{x^3}\frac{dx}{dy} - \frac{1}{x^2}\, y = y \sin y^2$$

take $-\frac{1}{x^2} = t$,

Differential equation,

$$\frac{dt}{dy} + \underline{2y}.t = 2y \sin y^2$$

$$\text{I.F.} = e^{\int 2y.dy} = e^{y^2}$$

(101) For the given differential equation, $\frac{dy}{dx} = \frac{y}{2x}$

$$\Rightarrow 2\frac{1}{y}\, dy = \frac{1}{x}\, dx \text{ (separable variable form)}$$

$$\Rightarrow 2 \log |y| = \log |x| + \log |c|$$

$$\Rightarrow y^2 = \frac{1}{2}x$$

$\therefore$ co-ordinates of focus point are $\left(\frac{1}{8}, o\right)$ and statement-2 satisfy the statment-1.

(103) Solution of the given differential equation exists as the equation $x^2 + y^2 = 0$ which is point - circle.

(104) If the circle passes through three non-collinear points, then the equation of a circle consists three arbitary constants.

(105) Differential equation,

$$\frac{dx}{dy} = \sin\frac{x-y}{2} - \sin\frac{x+y}{2}$$

$$\Rightarrow \frac{dy}{dx} = -2\sin\frac{y}{2}\cos\frac{x}{2}$$

It solution,

$$y = 4\tan^{-1}\left[e^{-2\sin\frac{x}{2}}\right]$$

is periodic fuction with period 4π

Answers									
1-A	2-C	3-B	4-D	5-C	6-A	7-A	8-D	9-D	10-C
11-A	12-D	13-B	14-A	15-B	16-C	17-D	18-B	19-B	20-C
21-C	22-A	23-D	24-C	25-B	26-C	27-A	28-C	29-D	30-D
31-B	32-B	33-A	34-D	35-C	36-C	37-A	38-A	39-A	40-C
41-C	42-B	43-C	43-C	45-A	46-B	47-B	48-C	49-B	50-B
51-B	52-A	53-C	54-B	55-D	56-A	57-A	58-C	59-B	60-C
61-D	62-A	63-B	64-D	65-B	66-A	67-C	68-D	69-A	70-B
71-A	72-C	73-C	74-B	75-B	76-A	77-C	78-C	79-B	80-A
81-B	82-C	83-A	84-B	85-D	86-C	87-D	88-A	89-C	90-B
91-A	92-D	93-C	94-B	95-C	96-A	97-C	98-B	99-A	100-C
101-A	102-D	103-C	104-A	105-B					

Unit-11 Lines

1. The equation of line equidistant from the points $A(1, -2)$ *and* B(3,4) and making congruent angles with the coordinate axes is . . .

   (a) $x + y = 1$ (b) $y - x + l = 0$ (c) $y - x - 1 = 0$ (d) $y - x = 2$

2. The equation of line passing through the point (–5,4) and making the intercept of length $\frac{2}{\sqrt{5}}$ between the lines $x + 2y - 1 = 0$ and $x + 2y + 1 = 0$ is . . .

   (a) $2x - y + 4 = 0$ (b) $2x - y - 14 = 0$ (c) $2x - y + 14 = 0$ (d) None of these

3. The equation of line containing the angle bisector of the lines $3x - 4y - 2 = 0$ and $5x - 12y + 2 = 0$ is . . .

   (a) $7x + 4y - 18 = 0$ (b) $4x - 7y - 1 = 0$ (c) $4x - 7y + 1 = 0$ (d) None of these

4. The equation of line passing through the point of intersection of the lines $3x - 2y = 0$ and $5x + y - 2 = 0$ and making the angle of measure $\tan^{-1}(-5)$ with the positive direction of x – *axis* is . . .

   (a) $3x - 2y = 0$ (b) $5x + y - 2 = 0$ (c) $5x + y = 0$ (d) $3x + 2y + 1 = 0$

5. If for $a + b + c \neq 0$, the lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent, then . . .

   (a) $ab + be + ca = 0$ (b) $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 1$ (c) $a = b$ (d) $a = b = c$

6. The equation of line passing through the point (1,2) and making the intercept of length 3 units between the lines $3x + 4y = 24$ *and* $3x + 4y = 12$, is . . .

   (a) $7x - 24y + 41 = 0$ (b) $7x + 24y = 55$ (c) $24x - 7y = 10$ (d) $24x + 7y - 38 = 0$

7. If (a, a^2) lies inside the angle between the lines $y = \frac{x}{2}$, $x > 0$ *and* $y = 3x$, $x > 0$,

   then $a \in$ . . .

   (a) $\left(-3, -\frac{1}{2}\right)$ (b) $(3, \infty)$ (c) $\left(-\frac{1}{2}, 3\right)$ (d) $\left(0, \frac{1}{2}\right)$

8. If P(–1,0), Q(0,0) and $R(3, 3\sqrt{3})$, then the equation of bisector of $\angle PQR$ is . . .

   (a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \frac{\sqrt{3}}{2}y = 0$ (c) $\sqrt{3}x + y = 0$ (d) $x + \sqrt{3}y = 0$

9. If the non zero numbers a, b,c are in harmonic progression, then the line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ passes through the point . . .

(a) (1,–2) (b) (–1,–2) (c) (–1,2) (d) $\left(1, \frac{1}{2}\right)$

10. A line passing through 0(0,0) intersect the parallel lines 4x + 2y = 0 *and 2x* + y + 6 = 0 at P and Q respectively, then in what ratio does *0* divide $\overline{PQ}$ from P ?

(a) 1 : 2 (b) 3 : 4 (c) 2 : 1 (d) 4 : 3

11. The points on the line $3x - 2y - 2 = 0$, which are 3 units away from the line 3x + 4y – 8 = 0 are . . .

(a) (3, −3), $\left(3, -\frac{1}{3}\right)$ (b) $\left(3,\frac{7}{2}\right),\left(-\frac{1}{3},-\frac{3}{2}\right)$ (c) $\left(\frac{7}{2},3\right),\left(-\frac{1}{3},3\right)$ (d) (3,1),(1,3)

12. If A(1, –2), 5(–8,3), *A–P–B* and 3 $AP = 7AB$, then P = . . .

(a) $\left(22,-\frac{41}{3}\right)$ (b) $\left(-22,\frac{41}{3}\right)$ (c) not possible (d) None of these

13. For the collinear points $P - A - B$, $AP = 4AB$, then P divides $\overline{AB}$ from A in the ratio.....

(a) 4 : 5 (b) – 4 : 5 (c) –5 : 4 (d) –1 : 4

14. If the length of perpendicular drawn from (5,0) on kx + 4y = 20 is 1, then k = . . .

(a) 3, $\frac{16}{3}$ (b) 3, $-\frac{16}{3}$ (c) –3, $\frac{16}{3}$ (d) –3, $-\frac{16}{3}$

15. If the lengths of perpendicular drawn from the origin to the lines $x\cos\alpha - y\sin\alpha = \sin 2a\alpha$ and $x\sin\alpha + y\cos\alpha = \cos 2\alpha$ are p *and* q respectively, then $p^2 + q^2$ = . . .

(a) 4 (b) 3 (c) 2 (d) 1

16. The points onY – *axis* at a distance 4 units from the line x + 4y = 12 are . . .

(a) $(3+\sqrt{14}, 0)$ $(3-\sqrt{14}, 0)$ (b) $(-3-\sqrt{17}, 0)$ $(3+\sqrt{17}, 0)$

(c) $(0, 3+\sqrt{17})$ (d) $(0, -3-\sqrt{17})$ $(0, -3+\sqrt{17})$

17. A base of a triangle is along the line $x = b$ and its length is $2b$. If the area of triangle is b^2, then the vertex of a triangle lies on the line . . .

(a) $x = -b$ (b) x = 0 (c) x = $\frac{b}{2}$ (d) $x = b$

18. Shifting origin at which point the transformed form of $x^2 + y^2 - 4x - 8y - 85 = 0$ would be $x^2 + y^2 = k$?

(a) (2,4) (b) (–2, –4) (c) (2, –4) (d) (–2,4)

19. A(1,0) and B(–1,0), then the locus of points satisfying $AQ - BQ = \pm 1$ is . . .

(a) $12x^2 + 4y^2 = 3$ (b) $12x^2 - 4y^2 = 3$ (c) $12x^2 - 4y^2 = -3$ (d) $12x^2 + 4y^2 = -3$

20. A rod having length $2c$ moves along two perpendicular lines, then the locus of the mid point of the rod is . . .

(a) $x^2 - y^2 = c^2$ (b) $x^2 + y^2 = c^2$ (c) $x^2 + y^2 = 2c^2$ (d) None of these

21. Consider a square ΔPQR having the length of side a, where O(0,0). The sides $\overline{OP}$ and $\overline{OR}$ are along the positive X – *axis* and *Y – axis* respectively. If *A and B* are the mid points of $\overline{PQ}$ and $\overline{QR}$ respectively, then the angle between $\overline{OA}$ and $\overline{OB}$ would be... . .

(a) $\cos^{-1}\frac{3}{5}$ (b) $\tan^{-1}\frac{4}{3}$ (c) $\cos^{-1}\frac{3}{4}$ (d) $\sin^{-1}\frac{3}{5}$

22. $\sqrt{3}x + y = 2$ is the equation of line containing one of the sides of an equilateral triangle and if (0,–1) is one of the vertices, then the length of the side of the triangle is . . .

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

23. If the point $\left(1+\frac{t}{\sqrt{2}}, 2+\frac{t}{\sqrt{2}}\right)$ lies between the two parallel lines $x + 2y = 1$ and $2x + 4y = 15$, then the range of t is . . .

(a) $0 < t < \frac{5}{6\sqrt{2}}$ (b) $-\frac{4\sqrt{2}}{3} < t < 0$ (c) $-\frac{4\sqrt{2}}{3} < t < \frac{5\sqrt{2}}{6}$ (d) None of these

24. If two perpendicular lines passing through origin intersect the line $\frac{x}{a} + \frac{y}{b} = 1, a \neq 0, b \neq 0$ at A and B, then $\frac{1}{OA^2} + \frac{1}{OB^2} =$ ....

(a) $\frac{1}{a^2} - \frac{1}{b^2}$ (b) $\frac{ab}{a^2 + b^2}$ (c) $\frac{a^2 + b^2}{a^2 b^2}$ (d) None of these

25. The equation of a line at a distance $\sqrt{5}$ units from the origin and the ratio of the intercepts on the axes is 1 : 2, is . . .

(a) $2x + y \pm 5 = 0$ (b) $2x + y \pm 5 = 0$ (c) $x - 2y \pm 5 = 0$ (d) None of these

26. For any values of *p and q*, the line $(p + 2q)x + (p - 3q)y - p - q$ passes through which fixed point ?

(a) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (b) $\left(\frac{2}{5}, \frac{2}{5}\right)$ (c) $\left(\frac{3}{5}, \frac{3}{5}\right)$ (d) $\left(\frac{2}{5}, \frac{3}{5}\right)$

27. If $A(x_1,y_1)$, $B(x_2,y_2)$ and $P(tx_2 + (1 - t)x_1,\ ty_2 + (1 - t)y_1)$ where $t < 0$, then P divides $\overline{AB}$ from A in the ratio . . .

(a) $1 - t$ (b) $\frac{t-1}{t}$ (c) $\frac{t}{1-t}$ (d) $t - 1$

28. $A(1,2)$, $B(5,7)$ and $P(x,y) \in \overrightarrow{AB}$, then $y - x - 1$ is . . .

(a) < 0 (b) > 0 (c) $= 0$ (d) -3

29. $A(2,3)$, $B(4,7)$ and $P(x,y) \in \overline{AB}$, then the maximum value of $3x + y$ is . . .

(a) 19 (b) 9 (c) –19 (d) –9

30. $A(-2,5)$, $5(6,2)$, then $\overleftrightarrow{AB} - \overline{AB} =$ ....

(a) $\{(8t - 2,\ 5 - 3t \ /\ t < 0)$ (b) $\{(8t - 2,\ 5 - 3t)\ /\ 0 \leq t \leq 1\}$

(c) $\{(8t - 2,\ 5 - 3t)\ /\ t \in R - [0, 1]\}$ (d) $\{(8t - 2,\ 5 - 3t)\ /\ t > 1\}$

31. The $p - \alpha$ form of the line $x + \sqrt{3}y - 4 = 0$ is

(a) $x\cos\frac{\pi}{6} + y\sin\frac{\pi}{6} = 2$ (b) $x\cos\frac{\pi}{3} + y\sin\frac{\pi}{3} = 2$

(c) $x\cos\left(-\frac{\pi}{3}\right) + y\sin\left(-\frac{\pi}{3}\right) = 2$ (d) $x\cos\left(-\frac{\pi}{6}\right) + y\sin\left(-\frac{\pi}{6}\right) = 2$

32. The length of side of an equilateral triangle is a. There is circle inscribed in a triangle. What is the area of a square inscribed in a circle ?

(a) $\frac{a^2}{3}$ (b) $\frac{a^2}{6}$ (c) $\frac{a^2}{\sqrt{3}}$ (d) $\frac{a^2}{\sqrt{2}}$

33. If the lines $x + 2ay + a - 0$, $x + 3by + 3 = 0$ *and* $x + 4cy + c = 0$ are concurrent, then a, b, c are in . . .

(a) A.P. (b) H.P. (c) G.P. (d) A.G.P

34. The foot of perpendicular drawn from (2,3) to the line $4x - 5y - 34 = 0$ is . . .

(a) (6,–2) (b) $\left(\frac{246}{41}, \frac{82}{41}\right)$ (c) (–6,2) (d) None of these

35. The equation of a line passing through (4,3) and the sum of whose intercepts is –1 is.....

(a) $\frac{x}{2} + \frac{y}{3} = 1,\ \frac{x}{2} + \frac{y}{1} = 1$ (b) $\frac{x}{2} + \frac{y}{3} = -1,\ \frac{x}{-2} + \frac{y}{1} = 1$

(c) $\frac{x}{2} + \frac{y}{3} = -1,\ \frac{x}{-2} + \frac{y}{1} = -1$ (d) $\frac{x}{2} - \frac{y}{3} = 1,\ \frac{x}{-2} + \frac{y}{1} = 1$

36. A line intersects $X - axis$ and $Y - axis$ at A and B respectively. If AB = 15 and $\overleftrightarrow{AB}$ makes a triangle of area 54 units with coordinate axes, then the equation of $\overleftrightarrow{AB}$ is . . .

(a) $4x \pm 3y = 36$ *or* $3x \pm 4y = 36$ (b) $4x \pm 3y = 24$ *or* $3x \pm 4y = 24$

(c) $-4x \pm 3y = 24$ *or* $-3x \pm 4y - 24$ (d) $-4x \pm 3y = 12$ *or* $-3x \pm 4y - 12$

37. The angle between the lines $x\cos85° + y\sin85° = 1$ and $x\cos40° + y\sin40° = 2$ is :

(a) 90° (b) 80° (c) 125° (d) 45°

38. If a_1, a_2, a_3 and b_1, b_2, b_3 are in geometric progression and their common ratios are equal, then the points $A(a_1, b_1), B(a_2,b_2)$ *and* $C(a_3,b_3)$ . . .

(a) lie on the same line (b) lie on a circle (c) lie on an ellipse (d) None of these

39. The image of the point (4, –13) in the line $5x + y + 6 = 0$ is . . .

(a) (1,2) (b) (3,4) (c) (–4,13) (d) (–1, –14)

40. If the lines $x + (a - l)y + 1 = 0$ and $2x + a^2y - 1 = 0$ are perpendicular then . . .

(a) $|a| = 2$ (b) $0 < a < 1$ (c) $-1 < a < 1$ (d) $a = -1$

41. If $x + 3y - 4 = 0$ and $6x - 2y - 7 = 0$ are the lines containing the diagonals of a parallelogram *PQRS*, then parallelogram *PQRS* is . . .

(a) rectangle (b) square (c) cyclic quadrilateral (d) rhombus

42. For $a + b + c = 0$, the line $3ax + 4by + c = 0$ passes through the fixed point . . .

(a) $\left(\frac{1}{3}, -\frac{1}{4}\right)$ (b) $\left(-\frac{1}{3}, \frac{1}{4}\right)$ (c) $\left(\frac{1}{3}, \frac{1}{4}\right)$ (d) $\left(-\frac{1}{3}, -\frac{1}{4}\right)$

43. If $3l + 2m + 6n = 0$, then the family of lines $lx + my + n = 0$ passes through the fixed point . . .

(a) (2,3) (b) (3,2) (c) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (d) $\left(\frac{1}{3}, \frac{1}{2}\right)$

44. If the lines $x + y + r = 0$ and $\lambda x - 5y = 5$ are identical then $\lambda + r = . . .$,

(a) –4 (b) 4 (c) 1 (d) –1

45. If the x – *coordinate* of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is an integer, then the integer value of m is . . .

(a) 2 (b)0 (c) 4 (d) 1

46. If (4,5) is the foot of perpendicular on the line l, then the equation of the line l would be . . .

(a) $4x + 5y + 41 = 0$ (b) $4x - 5y + 9 = 0$ (c) $4x + 5y - 41 = 0$ (d) None of these

47. The $y - intercept$ of the line $y + y_1 = m(x - x_1)$ is . . .

(a) $-(y_1 + mx_1)$ (b) $y_1 - mx_1$ (c) $\frac{y_1+mx_1}{m}$ (d) None of these

48. The locus of mid points of the segment intercepted between the axes by the line $xseca + ytana = p$ is . . .

(a) $\frac{p^2}{4x^2} = 1 + \frac{p^2}{4y^2}$ (b) $\frac{x^2}{p^2} + \frac{y^2}{p^2} = 4$ (c) $\frac{p^2}{x^2} = 1 + \frac{p^2}{y^2}$ (d) $\frac{p^2}{4x^2} + \frac{p^2}{4y^2} = 1$

49. If the y – *intercept* of the perpendicular bisector of the segment obtained by joining P(l,4) and $Q(k, 3)$ is –4 then $k = . . .$

(a) 1 (b) 2 (c) –2 (d) –4

50. The y – *intercept* of the line passing through the point (2,2) and perpendicular to the line $3x + y - 3 = 0$ is . . .

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $-\frac{4}{3}$ (d) $-\frac{3}{4}$

51. The line parallel to the X – *axis* and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ where $(a, b) \neq (0,0)$ is :

(A) above the X – *axis* at a distance of $\frac{2}{3}$ from it

(B) above the X – *axis* at a distance of $\frac{3}{2}$ from it

(C) below the X – *axis* at a distance of $\frac{2}{3}$ from it

(D) below the X – *axis* at a distance of $\frac{3}{2}$ from it

52. A square of side a lies above the x – *axis* and has one vertex at the origin. The side passing through the origin makes an angle a $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x – *axis*. The eq. of its diagonal not passing through the origin is :

(A) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$ (B) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$
(C) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$ (D) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

53. If P *and* Q divides $\overline{AB}$ from A in the ratios λ *and* $-\lambda$, then A divides $\overline{PQ}$ from p in the ratio . . . . . . $(\lambda \neq 1, \lambda > 0)$.

(a) $\frac{\lambda-1}{\lambda+1}$ (b) $\frac{1-\lambda}{\lambda+1}$ (c) $\frac{\lambda-2}{\lambda+2}$ (d) $\frac{2-\lambda}{\lambda+2}$

54. The nearest point on the line $x - 3y + 25 = 0$ from the origin is . . .

(a) (–4,5) (b) (–4,3) (c) (4,3) (d) None of these

55. If the slope of a curve is constant, then the graph of a curve in the plane is . . .

(a) line (b) parabola (c) hyperbola (d) none of these

56. If $5x + 12y + 13 = 0$ is transformed into $x\cos\alpha + y\sin\alpha = p$, then $\alpha = ?$ $\alpha \in [-\pi, \pi]$

(a) $\cos^{-1}\left(-\frac{5}{13}\right)$ (b) $\sin^{-1}\left(-\frac{12}{13}\right)$ (c) $\tan^{-1}\left(\frac{12}{5}\right) - \pi$ (d) $\tan^{-1}\left(\frac{12}{5}\right)$

57. If $P(-1,0)$, $Q(0,0)$ and $R(3,3\sqrt{3})$ are given points, then the equation of the bisector of $\angle PQR$ is . . .

(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \frac{\sqrt{3}}{2}y = 0$ (c) $\sqrt{3}x + y = 0$ (d) $x + \sqrt{3}y = 0$

58. For the line $y - y_1 = m(x - x_1)$, m *and* x_1 are fixed values, if different lines are drawn according to the different value of y_1; then all such lines would be . . .

(a) all lines intersect the line $x = x_1$ (b) all lines pass through one fixed point

(c) all lines are parallel to the line $y = x_1$ (d) all lines will be the set of perpendicular lines

59. If the length of perpendicular drawn from origin to a line is 10 and $\alpha = -\frac{5\pi}{6}$ then the equation of line would be . . .

(a) $\sqrt{3}x + y = 20$ (b) $\sqrt{3}x - y = 20$ (c) $\sqrt{3}x + y + 20 = 0$ (d) $\sqrt{3}x - y + 20 = 0$

60. Find the equation of line making a triangle of area $\frac{50}{\sqrt{3}}$ units with two axes and on which a perpendicular from origin makes an angle $\frac{\pi}{6}$ with positive direction of x – *axis*.

(a) $x + \sqrt{3}y = 10$ (b) $x - y = 10$ (c) $\sqrt{3}x + y - 5 = 0$ (d) $\sqrt{3}x + y - 10 = 0$

61. If $2x + 2y - 5 = 0$ is the equation of the line containing one of the sides of an equilateral triangle and (1,2) is one vertex, then find the equations of the lines containing the other two sides.

(a) $y = (2+\sqrt{3})x - \sqrt{3}, \quad y = (2+\sqrt{3})x + \sqrt{3}$

(b) $y = (2-\sqrt{3})x - \sqrt{3}, \quad y = (2+\sqrt{3})x + \sqrt{3}$

(c) $y = (2-\sqrt{3})x + \sqrt{3}, \quad y = (2+\sqrt{3})x - \sqrt{3}$

(d) $y = (2+\sqrt{3})x + \sqrt{3}, \quad y = (2+\sqrt{3})x - \sqrt{3}$

62. Find the equation of line passing through the point $(\sqrt{3}, -1)$ and at a distance $\sqrt{2}$ units from the origin.

(a) $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 4$ or $(\sqrt{3}-1)x - (\sqrt{3}+1)y = 4$

(b) $(\sqrt{3}+1)x + (\sqrt{3}+1)y = 4$ or $(\sqrt{3}-1)x + (\sqrt{3}+1)y = 4$

(c) $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 4$ or $(\sqrt{3}-1)x + (\sqrt{3}+1)y = 4$

(d) $(\sqrt{3}-1)x + (\sqrt{3}-1)y = 4$ or $(\sqrt{3}+1)x + (\sqrt{3}+1)y = 4$

63. If (3,–2) and (–2,3) are two vertices and (6,–1) is the orthocentre of a triangle, then the third vertex would be . . .

(a) (1,6) (b) (–1,6) (c) (1, –6) (d) none of these

64. The circumcentre of the triangle formed by the lines $x + y = 0$, $x - y = 0$ and $x - 7 = 0$ is . .

(a) (7,0) (b) (3.5,0) (c) (0,7) (d) (3.5,3.5)

65. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in arithmetic sequence, then the line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ passes through the fixed point . . .

(a) (–1,–2) (b) (–1,2) (c) $\left(1,-\frac{1}{2}\right)$ (d) (1,–2)

66. Find the slope of the line passing through the point (1,2) and the point of intersection of this line with the line $x + y + 3 = 0$ is at a distance $3\sqrt{2}$ units from (1,2).

(a) $\frac{1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}-1}{2}$

67. The angle between the lines $x = 3$ and $\sqrt{3}x-y+5=0$ is . . .

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

68. The angle between the lines $y = e$ and $\sqrt{3}x-y+5=0$ is . . .

(a) $-\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

69. The angle between the lines $\{(x, 0)/x \in R\}$ and $\{(0,y)/ y \in R\}$ is . . .

(a) $\frac{\pi}{2}$ (b) $-\frac{\pi}{2}$ (c) 0 (d) π

70. If the point $\left(1+\frac{t}{\sqrt{2}},\ 2+\frac{t}{\sqrt{2}}\right)$ lies between the two parallel lines $x + 2y = 1$ and $2x + 4y = 15$, then the range of t is . . .

(a) $0<t<\frac{5}{6\sqrt{2}}$ (b) $-\frac{4\sqrt{2}}{3}<t<0$ (c) $-\frac{4\sqrt{2}}{3}<t<\frac{5\sqrt{2}}{6}$ (d) None of these

71. If $2x + 3y = 8$ is perpendicular to the line $(x + y + 1) + \lambda(2x - y - 1) = 0$, then λ = ?

(a) –5 (b) $\frac{3}{2}$ (c) 5 (d) 0

72. If the line $(a + l)x + (a^2 - a - 2)y + a = 0$ is parallel to $Y-axis$, then a = . . .

(a) –1 (b) 2 (c) 3 (d) 1

73. The equation of a straight line passing through the point (–5, 4) and which cut off an intercept of $\sqrt{2}$ unit between the lines x + y + 1 = 0 and x + y – 1 = 0 is

(a) x – 2y + 13 = 0 (b) 2x – y + 14 = 0 (c) x – y + 9 = 0 (d) x – y + 10 = 0

74. If P(1, 2), Q(4, 6), R(5, 7) and S(a, b) are the vertices of a parallelogram PQRS then

(a) a = 2, b = 4 (b) a = 3, b = 4 (c) a = 2, b = 3 (d) a = 2, b = 5

75. The sum of squares of intercepts on the axes cut off by the tangents to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ (a > 0) at $\left(\frac{a}{8}, \frac{a}{8}\right)$ is 2. Thus a has the value.

(a) 1 (b) 2 (c) 4 (d) 8

76. If two vertices of a trinangle eare (5, –1) and (–2, 3) and if its orthocentre lies at the origin then the cooridnates of the third vertex are

(a) (4, 7) (b) (–4, –7) (c) (2, –3) (d) (5, –1)

77. Line ax + by + p = 0 makes angle $\frac{\pi}{4}$ which $x\, cos\alpha + y sin\alpha = p,\ p \in R^+$. If these lines and the line $x\, sin\alpha - y\, cos\alpha = 0$ are concurrent then

(a) $a^2 + b^2 = 1$ (b) $a^2 + b^2 = 2$ (c) $2(a^2 + b^2) = 1$ (d) $a^2 - b^2 = 2$

78. A straight line passess through a point A(1, 2) and makes an angle 60^0 with the x–$axis$. This line intersect the line x + y = 6 at P. Then AP will be

(a) $3(\sqrt{3}+1)$ (b) $3(\sqrt{3}-1)$ (c) $(\sqrt{3}+1)$ (d) $3\sqrt{3}$

79. The image of origin in the line x + 4y = 1 is

(a) $\left(\frac{2}{17}, \frac{-8}{17}\right)$ (b) $\left(-\frac{2}{17}, -\frac{8}{17}\right)$ (c) $\left(-\frac{2}{17}, \frac{8}{17}\right)$ (d) $\left(\frac{2}{17}, \frac{8}{17}\right)$

80. Orthocentre of triangle with vertices (0, 0), (3, 4) and (4, 0) is

(a) $\left(3, \frac{5}{4}\right)$ (b) (3, 12) (c) $\left(3, \frac{3}{4}\right)$ (d) (3, 9)

81. The equation of three sides of triangle are x = 2, y + 1 = 0 and x + 2y = 4. The coordinates of the circumcentre of the triangle is

(a) (4, 0) (b) (2, –1) (c) (0, 4) (d) (–1, 2)

82. If a, b, c are in A.P. then ax + by + c = 0 represents

(a) a single line (b) a family of concurrent lienes

(c) a family of parallel lines (d) a family of circle

83. A(4, 0), B(0, 3), C(6, 1) be vertices of triangle ABC. Slope of bisector of angle C will be

(A) $3\sqrt{2}-7$ (b) $5\sqrt{2}-7$ (c) $6\sqrt{2}-7$ (d) none

84. The locus of the variable point whose distance from (–2, 0) is $\frac{2}{3}$ times its distance from the line $x = -\frac{9}{2}$ is

(a) ellipse (b) parabola (c) circle (d) hyperbola

85. The line $3x - 4y + 7 = 0$ is rotated through an angle $\frac{\pi}{4}$ in the clockwise direction about the point (–1, 1). The equation of the line in its new position is

(a) $7y + x - 6 = 0$ (b) $7y - x - 6 = 0$ (c) $7y + x + 6 = 0$ (d) $7y - x + 6 = 0$

86. The area of the triangle formed by the point (a, a^2), (b, b^2), (c, c^2) is ..... (a, b, c are three consecutive odd integers)

(a) $\frac{1}{2}$ *(a – b) (b – c) sq unit* (b) 8 sq unit

(c) 16 sq unit (d) $\frac{1}{2}$ *(a – b) (b – c) (a + b + c) sq unit*

87. The straight line $7x - 2y + 10 = 0$ and $7x + 2y - 10 = 0$ forms an isosceles triangle with the line $y = 2$. Area of the triangle is equal to :

(a) $\frac{15}{7}$ sq unit (b) $\frac{10}{7}$ sq unit (c) $\frac{18}{7}$ sq unit (d) $\frac{10}{13}$ sq unit

88. In triangle ABC, equation of right bisectors of the sides $\overline{AB}$ and $\overline{AC}$ are $x + y = 0$ and $y - x = 0$ respectively. If A = (5, 7) then equation of side BC is

(a) $7y = 5x$ (b) $5x = y$ (c) $5y = 7x$ (d) $5y = x$

89. The equations of the two lines each passing through (5, 6) and each making an acute angle of 45^0 with the line $2x - y + 1 = 0$ is

(a) $3x + y - 21 = 0,\ x - 3y + 13 = 0$ (b) $3x + y + 21 = 0,\ x + 3y + 13 = 0$

(c) $y = 2x,\ y = 3x$ (d) $3x + y - 21 = 0, x - 3y - 13 = 0$

90. If the equation of base of an equilateral triangle is $2x - y = 1$ and the vertex is (–1, 2), then the length of the side of the triangle is

(a) $\sqrt{\frac{20}{3}}$ (b) $\frac{2}{\sqrt{15}}$ (c) $\sqrt{\frac{8}{15}}$ (d) $\sqrt{\frac{15}{2}}$

91. Four points (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) are such that $\sum_{i=1}^{4}\left(x_i^2 + y_i^2\right) \le$ $2(x_1x_3 + x_2x_4 + y_1y_2 + y_3y_4)$. Then these points are vertices of

(a) parallellogram (b) Rectangle (c) Square (d) Rhombus

92. A variable straight line passess through a fixed point (a, b) intersecting the coordinate axes at A and B. If 'O' is the origin, then the locus of the centroid of the triangle OAB is

(a) $bx + ay = 3xy$ (b) $bx + ay = 2xy$ (c) $ax + by = 3xy$ (d) $ax + by = 2xy$

93. If the poitns (k, 2–2k), (1–k, 2k) and (–k–4, 6 – 2k) are collinear, the possible value of k are

(a) $-\frac{1}{2}, 1$ (b) $\frac{1}{2}, -1$ (c) 1, 2 (d) 1, 3

94. In a triangle ABC, coordinates of A are (1, 2) and the equations of the medians through B and C are $x + y = 5$ and $x = 4$ respectively. Then coordinates of B and C will be

(a) (–2, 7), (4, 3) (b) (7, –2), (4, 3) (c) (2, 7), (–4, 3) (d) (2, –7), (3, –4)

95. The ratio in which the join of the points (1, 2) and (–2, 3) is divided by the line $3x + 4y = 7$ *is*

(a) 4 : 1 (b) 3 : 2 (c) 3 : 1 (d) 7 : 3

96. The equation of the bisector of acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$ *is*

(a) $11x - 3y + 9 = 0$ (b) $3x + 11y - 13 = 0$ (c) $3x + 11y - 3 = 0$ (d) $11x - 3y + 2 = 0$

97. The lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$ are concurrent at the point

(a) $\left(\frac{1}{2}, \frac{3}{4}\right)$ (b) (1, 3) (c) (3, 1) (d) $\left(\frac{3}{4}, \frac{1}{2}\right)$

98. The area of parallelogram whose two sides are $y = x + 3$, $2x - y + 1 = 0$ and remaining two sides are passing through (0, 0) is

(a) 2 sq unit (b) 3 sq unit (c) $\frac{5}{2}$ sq unit (d) $\frac{7}{2}$ sq unit

99. The equation of a straight line that passes through the point (–4, 3) and is such that the portion of it between the axes is divided by the point in the ratio 5 : 3 internally is

(a) $9x - 20y + 96 = 0$ (b) $2x - y + 11 = 0$ (c) $2x + y + 5 = 0$ (d) $3x - 2y + 7 = 0$

100. Area of a quadrilateral fromed by the lines $|x| + |y| = 2$ is

(a) 8 (b) 6 (d) 3 (d) None

101. The line $x + 3y - 2 = 0$ bisect the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$. The equation of other line is

(a) $3x + 3y - 1= 0$ (b) $x - 3y + 2= 0$ (c) $5x + 5y - 3= 0$ (d) None

102. The equation of the bisector of the angle between two lines $3x - 4y + 12 = 0$ and $12x - 5y + 7 = 0$, which contain the point (–1, 4) is

(a) $21x + 27y - 121 = 0$ (b) $21x - 27y + 121 = 0$

(c) $21x + 27y + 191 = 0$ (d) $\frac{-3x+4y-12}{5} = \frac{12x-5y+7}{13}$

103. The equations of two striaght lines which are parallel to $x + 7y + 2 = 0$ and at unit distance from the point (1, –1) are

(a) $x+7y+6\pm 4\sqrt{2}=0$ (b) $x+7y+6\pm 5\sqrt{2}=0$

(c) $2x+7y+6\pm 5\sqrt{2}=0$ (d) $x+y+6\pm 5\sqrt{2}=0$

104. The points on the line x + y = 4 which lie at a unit distance from the line $4x + 3y = 10$ are

(a) (3, 1), (–7, 11) (b) (7, 11, (2, 2) (c) (7, –11), (–3, 7) (d) (1, 3), (–5, 9)

105. One side of the rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1 and (1, 1). Then the equations of other side is

(a) $7x - 4y + 25 = 0$ (b) $4x + 7y = 11$ (c) $7x - 4y - 3= 0$ (d) All of these

106. Equation of a straight line passing through the point (4, 5) and equally inclined to the lines $3x = 4y + 7$ and $5y = 12x + 6$ is (angle bisector)

(a) $9x - 7y = 1$ (b) $9x + 7y = 71$ (c) $7x - y = 73$ (d) $7x - 9y + 17 = 0$

107. The nearest point on the line $3x + 4y = 1$ from origin is

(a) $\left(\frac{7}{25}, \frac{4}{25}\right)$ (b) $\left(\frac{7}{25}, \frac{2}{25}\right)$ (c) $\left(\frac{3}{25}, \frac{4}{25}\right)$ (d) $\left(\frac{1}{25}, \frac{3}{25}\right)$

108. The locus of the mid point of the intercept of the variable line $x \cos a + y \sin a = p$ between the coordinate axes is

(a) $x^{-2}+y^{-2}=p^{-2}$ (b) $x^{-2}+y^{-2}=2p^{-2}$ (c) $x^{-2}+y^{-2}=4p^{-2}$ (d) non of these

109. Three straight lines $2x + 11y - 5 = 0$, $4x - 3y - 2 = 0$ and $24x + 7y - 20 = 0$

(a) form a triangle (b) are only concurrent

(c) are concurrent with one line bisecting the angle between the other two.

(d) none of these

110. A straight line through the point (2, 2) intersects the line $\sqrt{3}x+y=0$ and $\sqrt{3}x-y=0$ at the points A and B. The equation to the line AB so that the triangle OAB is equilateral is

(a) $x = 2$ (b) $y = 2$ (c) $x + y = 4$ (d) none

111. A triangle with vertices (4, 0), (–1, –1), (3, 5) is

(a) isosceles and right angled (b) isosceles but not right angled

(c) right angled but not isosceles (d) neither right angled nor isosceles

112. Equation of a line at a distance $\sqrt{5}$ unit from origin with intercepts 1:2 on axes is ....

(a) $2x - y \pm 5 = 0$ (b) $2x + y \pm 5 = 0$ (c) $x - 2y \pm 5 = 0$ (d) $x + 2y \pm 5 = 0$

113. The equation of the lines with slope –2 and intersecting x–axis at points distance 3 unit from the origin is ....

(a) $2x + y \pm 6 = 0$ (b) $x + 2y \pm 6 = 0$ (c) $2x + y \pm 3 = 0$ (d) $x + 2y \pm 3 = 0$

114. The equation of a line containing a side of an equilateral triangle is $\sqrt{3}x + 4 = 0$. If (0, –1) is one of the vertices then the length of its side is.....

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

115. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1, b_2)y + c = 0$, then the value of C will be

(a) $\frac{1}{2}\left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right)$ (b) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(c) $\frac{1}{2}\left(a_1^2 + a_2^2 + b_1^2 + b_2^2\right)$ (d) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

116. Locus of the centroid of the triangle whose vertices are (a cos t, a sin t), (b sin t, – b cost) and (1, 0), where t is a parameter is

(a) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ (b) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

(c) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$ (d) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

117. A square of side 'a' lies above the *x–axis* and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of *x–axis*. The equation of the diagonal not passing through the origin is

(a) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = 0$ (b) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = 0$

(c) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha + \cos\alpha) = 0$ (d) $y(\cos\alpha - \sin\alpha) - x(\cos\alpha - \sin\alpha) = 0$

118. If x_1, x_2, x_3 and y_1, y_2, y_3 both are in GP with the same common ratio, then the points (x_1, y_1) (x_2, y_2) and (x_3, y_3)

(a) lie on a striaght line (b) lie on a ellipse

(c) lie on a circle (d) are vertices of a triangle

119. The length of a side of a square OPQR is a, O is the origin $\overline{OP}$ and $\overline{OR}$ are along positive direction of the X and Y axes respectively. If A and B are mid points of $\overline{PQ}$ and $\overline{QR}$ respectively then measure of angle between $\overline{OA}$ and $\overline{OB}$ is....

(a) $\cos^{-1}\frac{3}{5}$ (b) $\tan^{-1}\frac{4}{3}$ (c) $\cot^{-1}\frac{3}{4}$ (d) $\sin^{-1}\frac{3}{5}$

120. The incentre of a triangle whose vertices A(2, 4), B(2, 6) and $C(2+\sqrt{3}, 5)$ is....

(a) $\left(2+\frac{1}{\sqrt{3}}, 5\right)$ (b) $\left(1+\frac{1}{2\sqrt{3}}, \frac{5}{2}\right)$ (c) (2, 5) (d) None of these

121. If a line $3x + 4y = 24$ intersects the axes at A and B, then inradius of ΔOAB is .....

(a) 1 (b) 2 (c) 3 (d) 4

122. The equation of straight line passing through (1, 2) and having intercept of length 3 between the straight lines $3x + 4y = 24$ and $3x + 4y = 12$ is

(a) $7x - 24y + 41 = 0$ (b) $7x + 24y - 55 = 0$ (c) $24x - 7y - 10 = 0$ (d) $24x + 7y - 38 = 0$

123. Let A(2, –3) and B(–2, 1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then locus of the vertex C is the line

(a) $2x + 3y = 0$ (b) $2x - 3y = 7$ (c) $3x + 2y = 5$ (d) $3x - 2y = 3$

124. The line parallel to the x–axis and passing through the intersection of lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ where $(a, b) \neq (0, 0)$ is

(a) abover the x–axis at a distance of $\frac{2}{3}$ from it

(b) above the x–axis at a distance $\frac{3}{2}$ from it.

(c) below that x–axis at a distance $\frac{2}{3}$ from it.

(d) below the x–axis at a distance $\frac{3}{2}$ from it.

125. If non–zero numbers a, b, c are in HP, then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point that point is

(a) $\left(1, -\frac{1}{2}\right)$ (b) (1, –2) (c) (–1, –2) (d) (–1, 2)

126. If a vertex of a triangle is (1, 1) and the mid–points of two sides through this vertex are (–1, 2) and (3, 2), then centroid of the triangle is

(a) $\left(\frac{1}{3}, \frac{7}{3}\right)$ (b) $\left(1, \frac{7}{3}\right)$ (c) $\left(-\frac{1}{3}, \frac{7}{3}\right)$ (d) $\left(-1, \frac{7}{3}\right)$

127. The reflection of the point (4, –13) in the line $5x + y + 6 = 0$ is.....

(a) (1, 2) (b) (3, 4) (c) (–4, 13) (d) (–1, –14)

128. If P_1 and P_2 denote the lengths of the perpendiculars from the origin on the lines $x \sec \alpha + y \operatorname{cosec} \alpha = 2a$ and $x \cos \alpha + y \sin \alpha = a \cos 2a$ respectively then $\left(\frac{p_1}{p_2}+\frac{p_2}{p_1}\right)^2$ is equal to .....

(a) $4 \sin^2 4\alpha$ (b) $4 \cos^2 4\alpha$ (c) $4 \operatorname{cosec}^2 4\alpha$ (d) $4 \sec^2 4\alpha$

129. Locus of mid point of rod having length 2c begins to slide on two perpendicular lines is...
(a) $x^2 - y^2 = c^2$ (b) $x^2 + y^2 = c^2$ (c) $x^2 + y^2 = 2c^2$ (d) $x^2 - y^2 = 2c^2$

130. $A(3t^2, 6t)$, $B\left(\frac{3}{t^2}, -\frac{6}{t}\right)$ and S(3, 0). Then value of $\frac{1}{SA}+\frac{1}{SB}$ is
(a) 1 (b) 3 (c) $\frac{1}{3}$ (d) 6

131. A(6, 7), B(–2, 3) and C(9, 1) are vertices of ΔABC, then coordinates of point of intersection of bisector of ∠B and side $\overline{AC}$ is
(a) $\left(-\frac{22}{3}, \frac{13}{3}\right)$ (b) $\left(\frac{22}{3}, \frac{13}{3}\right)$ (c) $\left(\frac{22}{3}, -\frac{13}{3}\right)$ (d) $\left(-\frac{22}{3}, -\frac{13}{3}\right)$

132. A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. It's equation is
(a) $3x - 4y + 7 = 0$ (b) $4x + 3y = 24$ (c) $3x + 4y = 25$ (d) $x + y = 7$

133. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}, x > 0$ and $y = 3x$, $x > 0$ the 'a' belongs to
(a) $(3, \infty)$ (b) $\left(\frac{1}{2}, 3\right)$ (c) $\left(-3, -\frac{1}{2}\right)$ (d) $\left(0, \frac{1}{2}\right)$

134. Let A(h, k), B(1, 1) and C(2, 1) be the vertices of right angled triangle with $\overline{AC}$ as its hypotenuse. If the area of a triangle is 1, then the set of vvalues which 'k' can take is given by
(a) (1, 3) (b) (0, 2) (c) (–1, 3) (d) (–3, –2)

135. Let P(–1, 0), Q(0, 0) and $R(3, 3\sqrt{3})$ be three points. The equation of the bisector of the ∠PQR is
(a) $\sqrt{3}x + y = 0$ (b) $x + \frac{\sqrt{3}}{2}y = 0$ (c) $\frac{\sqrt{3}}{2}x + y = 0$ (d) $x + \sqrt{3}y = 0$

136. The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y–intercept –4. Then a possible value of k is
(a) –4 (b) 1 (c) 2 (d) –2

137. If A(1, 2) and B(6, 2), 3AB = 2BC and A – B – C athe value of C can be
(a) $\left(-\frac{3}{2}, \frac{3}{3}\right)$ (b) $\left(\frac{27}{2}, 2\right)$ (c) $\left(-\frac{27}{2}, 2\right)$ (d) $\left(\frac{27}{2}, -2\right)$

138. The equation of a striaght line passing through the point (4, 3) and making intercepts on the coordinate axes whose sum is –1 is given by
(a) $3x - 2y = 6$ and $x - 2y = -2$ (b) $3x - 2y = -6$ and $x - 2y = 2$
(c) $3x - 2y = 6$ and $x + 2y = 2$ (d) $3x - 2y = -6$ and $x - 2y = -2$

139. The obtuse angle bisector of the lines $x - 2y + 4 = 0$ and $4x - 3y + 2 = 0$ is
(a) $x(4-\sqrt{5}) + y(2\sqrt{5}-3) + (2-4\sqrt{5}) = 0$ (b) $x(4-\sqrt{5}) + y(2\sqrt{5}-3) + (2+4\sqrt{5}) = 0$
(c) $x(4+\sqrt{5}) + y(2\sqrt{5}-3) + (2+4\sqrt{5}) = 0$ (d) $x(4+\sqrt{5}) + y(2\sqrt{5}+3) + (2+4\sqrt{5}) = 0$

140. Equation of line which is equally inclined to the axis and passes through a common points of family of lines $4acx + y(ab + bc + ca - abc) + abc = 0$

(a) $y - x = \frac{7}{4}$ (b) $y + x = \frac{7}{4}$ (c) $y - x = \frac{1}{4}$ (d) $y + x = \frac{1}{4}$

141. The equation of a line passing through the point of intersection of $3x - 2y = 0$ and $5x + y - 2 = 0$ and making an angle of measure $\tan^{-1}(-5)$ with positive direction of x–axis is

(a) $3x - 2y = 0$ (b) $5x + y - 2 = 0$ (c) $5x + y = 0$ (d) $3x + 2y + 1 = 0$

142. The straight line perpendicular to the straight line $\sqrt{3}x + y = 1$ makes which of the following angles with the positive direction of y–axis

(a) 30^0 (b) 60^0 (c) 45^0 (d) none

143. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line in 2D geometry for

(a) exactly one value of p (b) exactly two value of p
(c) more than two value of p (d) no value of p

144. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1.$ Then distance between L and K is

(a) $\sqrt{17}$ (b) $\frac{17}{\sqrt{15}}$ (c) $\frac{23}{\sqrt{17}}$ (d) $\frac{23}{\sqrt{15}}$

145. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of 'a' is the interval

(a) $(0, \infty)$ (b) $[1, \infty)$ (c) $(-1, \infty)$ (d) $(-1, 1]$

146. Consider three points $P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$ where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$ then

(a) P lies on the $\overline{RQ}$ (b) Q lie on the $\overline{PR}$ (c) R lie on the $\overline{QP}$ (d) P, Q, R are non collinear

147. Triangle is formed by the coordinates (0, 0), (0, 21) and (21, 0). Find the number of intergral co–ordinates strictly inside the triangle (intergral coordinates has both x and y)

(a) 190 (b) 105 (c) 231 (d) 205

148. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively, then the point O divide the segment PQ in the ratio

(a) 1:2 (b) 3:4 (c) 2:1 (d) 4:3

149. A triangle is formed by the tangents to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinate axes, lies in the first Quadrant. If the area is 2, then value of b is :

(a) –1 (b) 3 (c) –3 (d) 1

150. Area of the parallelogram formed by the lines y = mx, y = mx + 1, y = nx and y = nx + 1 equals

(a) $\frac{|m+n|}{(m-n)^2}$ (b) $\frac{2}{|m+n|}$ (c) $\frac{1}{(m-n)}$ (d) $\frac{1}{|m+n|}$

151. Let PS be the median of the triangle with vertices P(2, 2), Q(6, –1) and R(7, 3). The equation of the line passing through (1, –1) and parallel to PS is

(a) $2x - 9y - 7 = 0$ (b) $2x - 9y - 11 = 0$ (c) $2x + 9y - 11 = 0$ (d) $2x + 9y + 7 = 0$

152. P(3, 1) and Q(6, 5) and R(x, y) are three points such that the angle PRQ is a right angle and the area of ΔRQP = 7, then the number of such points R is

(a) 0 (b) 1 (c) 2 (d) 4

153. If one of the diagonal of a square is along the line x = 2y and one of its vertices is (3, 0) then its sides through this vertex are given by the equaions.

(a) $y - 3x + 9 = 0, 3y + x - 3 = 0$ (b) $y + 3x + 9 = 0, 3y + x - 3 = 0$

(c) $y - 3x + 9 = 0, 3y - x + 3 = 0$ (d) $y - 3x + 3 = 0, 3y + x + 9 = 0$

154. The orthocentre of the triangle with vertices $\left[2, \frac{\sqrt{3}-1}{2}\right]$, $\left[\frac{1}{2}, -\frac{1}{2}\right]$ and $\left[2, -\frac{1}{2}\right]$ is

(a) $\left(\frac{3}{2}, \frac{\sqrt{3}-3}{6}\right)$ (b) $\left(2, -\frac{1}{2}\right)$ (c) $\left(\frac{3}{4}, \frac{\sqrt{3}-2}{4}\right)$ (d) $\left(\frac{1}{2}, -\frac{1}{2}\right)$

155. If the extremities of the base of an isoscelese triangle are the points (2a, 0) and (0, a) and the equation of one of the sides is x = 2a, then area of the triangle is

(a) 5 sq. unit (b) $\frac{5}{2}$ sq unit (c) $\frac{25}{2}$ sq unit (d) none of these

156. The equation of the line on which the perpendiculars from the origin makes 30^0 angle with x–axis and which form a triangle of area $\frac{50}{\sqrt{3}}$ with axes are

(a) $x+\sqrt{3}y \pm 10 = 0$ (b) $\sqrt{3}x + y \pm 10 = 0$ (c) $x + \sqrt{3}y - 10 = 0$ (d) none of these

157. If the lines $x + ay + a = 0$, bx + y + b = 0 and cx + cy + 1 = 0 (a, b, c being distinct ≠ 1) are concurrent, then the value of $\frac{a}{a-1} + \frac{b}{b-1} + \frac{c}{c-1}$ is

(a) –1 (b) 0 (c) 1 (d) none

158. The equations of perpendicular bisectors of the sides AB and AC of a triangle ABC are $x - y + 5 = 0$ and $x + 2y = 0$ respectively. If the point A is (1, –2) then equation of line BC is

(a) $23x + 144y - 40 = 0$ (b) $14x + 23y - 40 = 0$

(c) $23x + 14y + 40 = 0$ (d) $14x + 23y + 40 = 0$

159. Comrehensive type : A straight line L with negative slope passes through the point (9, 4) and cuts the positive corodinate axes at the points P and Q respectively. Now answer the following

(A) Minimum value of OP + OQ, as L varies, where O is the origin is

(a) 18 (b) 25 (c) 36 (d) 49

(B) Area of ΔOPQ, when OP + OQ becomes minimum is _____ sq units

(a) 75 (b) 225 (c) 125 (d) 200

(C) Let R be a moving point on the x – y plane such that OPRQ becomes a ractangle then locu of R as L varies is

(a) $\frac{x}{9}+\frac{4}{y}=\frac{1}{2}$ (b) $\frac{x}{9}+\frac{4}{y}=1$ (c) $\frac{9}{x}+\frac{4}{y}=1$ (d) $\frac{4}{x}+\frac{1}{y}=1$

160. If the lines $x = a + m$, $y = -2$ and $y = mx$ are concurrent, the least value of $|a|$ is.....

(a) 0 (b) $\sqrt{2}$ (c) $2\sqrt{2}$ (d) none

161. A(–3, 4), B(5, 4) and C and D form a rectangle, $x - 4y + 7 = 0$ is a diameter of the circumcircle of rectangle ABCD, the area of ABCD is

(a) 8 (b) 16 (c) 32 (d) 64

162. The line $3x + 2y = 24$ meets y–axis at A and x–axis at B. The perpendicular bisector of AB meets the x–axis at c, then area of ΔABC is

(a) 78 (b) 92 (c) $\frac{93}{2}$ (d) None

163. The lines $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ are concurrent then

(a) a, b, c are in A.P. (b) a, b, c are in G.P. (c) a, b, c are in H.P. (d) None of these

UNIT- 11 line-lines

Hints

1. ANS : B

$y - y_1 = m(x - x_1)$ where m = 1

equidistant from (1, -2) and (3, 4)

$$\left|\frac{-2-1-a}{\sqrt{2}}\right| = \left|\frac{4-3-a}{\sqrt{2}}\right| \Rightarrow a = -1$$

$\therefore$ RL y-x+1=0

2. ANS : C

$\perp$ distance between ℓ_1 and $\ell_2 = \frac{2}{\sqrt{5}}$

$\therefore \ell_3$ is $\perp$ to both ℓ_1 and ℓ_2 $\quad \therefore \ell_3 : 2x - y + 14 = 0$

also A(-5, 4) $\in \ell_3 \Rightarrow$ k=14

3. ANS : C

eq^n of angle bisector

$$\frac{3x-4y-2}{\sqrt{9+16}} = \pm\frac{5x-12y+2}{\sqrt{25+144}}$$

$\therefore 7x + 4y - 18 = 0$ or $4x - 7y - 1 = 0$

4. ANS : B

point of intersection of the lines is $\left(\frac{4}{13}, \frac{6}{13}\right)$

m=-5 ´Ü³ÜÜ $y - y_1 = m(x - x_1)$

$\therefore$ RL : 5x+y-2=0

5. ANS : D

lines are concurrent $\therefore \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$\Rightarrow (a+b+c)(ab+bc+ca-a^2-b^2-c^2) = 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0 \quad [\because a+b+c \neq 0]$

$\Rightarrow a = b = c$

6. ANS : A

$y - y_1 = m(x - x_1)$ and eq^n of line passes through (1, 2)

mx-y+2-m=0—(A)

This line intersect to the given line at point $A\left(\frac{4+4m}{3+4m},\frac{6+9m}{3+4m}\right)$ and $B\left(\frac{16+4m}{3+4m},\frac{6+21m}{3+4m}\right)$

also $AB=3\Rightarrow m=\frac{7}{24}$

$\therefore$ RL : 7x-24y+41=0

7. ANS : C

there for $x>0 \therefore a>0\ (a,a^2)$

$$\left.\begin{aligned} y=\tfrac{x}{2}\Rightarrow a^2-\tfrac{a}{2}>0\Rightarrow a>\tfrac{1}{2}\_\_\_\_(1)\\ y=3x\Rightarrow a^2-3a<0\Rightarrow a<3\_\_(2)\end{aligned}\right\}\frac{1}{2}<a<3$$

8. ANS : C **Figure**

slop of $\overleftrightarrow{QR}=\tan\theta=\sqrt{3}\Rightarrow\theta=\frac{\pi}{3}$

$\overrightarrow{QS}$ is bisector of $\angle PQR$ $\therefore m=\sqrt{3}$

which passes through (0, 0)

from, $y-y_1=m(x-x_1)$, $y-0=-\sqrt{3}(x-0)$

$\therefore \sqrt{3}x+y=0$

9. ANS : A $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ H.P and $\left.\begin{aligned}\therefore \tfrac{1}{a}-\tfrac{2}{b}+\tfrac{1}{c}=0\\ \tfrac{x}{4}+\tfrac{y}{b}+\tfrac{1}{c}=0\end{aligned}\right\}$

by comparing x = 1, y=-2

$\therefore$ line passes thorugh (1, -2)

10. ANS : B

perpendicular distance between (0, 0) and 2x+y+6=0 = $OQ=\frac{6}{\sqrt{5}}$

perpendicular distance between (0, 0) and 4x+2y-9=0 = OP = $\frac{9}{2\sqrt{5}}$

$\lambda=\frac{OP}{OQ}=\frac{3}{4}$ required ratio

11. ANS : B The point lies on the line 3x-2y-2=0

X - co-ordinate : a then y - co-ordinate : $\frac{3a-2}{2}$

then the perpendicular distance formula : $|9a-12|=15\ \therefore a=3,\ -\frac{1}{3}$

$\therefore a=3\Rightarrow x=3, y={}^{7}\!/\!_{2}$ or $a=-\frac{1}{3}\Rightarrow x=-\frac{1}{3},\ y=-\frac{3}{2}$

$\therefore$ required points are $\left(3, \frac{7}{2}\right), \left(-\frac{1}{2}, -\frac{3}{2}\right)$

12. ANS : C $\quad AP = \frac{7}{3}AB \quad \therefore AP > AB \quad \therefore P \notin \overline{AB}$

$\therefore$ A - P - B is not possible

13. ANS : B $\quad \therefore \lambda = \frac{-AP}{PB} < 0$, also $\frac{PA}{AB} = \frac{4}{1} \therefore \frac{PA}{PB} = \frac{4}{5} \therefore \lambda = -4:5$

14. ANS : A $\quad p = \frac{|5k + 0 - 20|}{\sqrt{k^2 + 16}} = 1 \Rightarrow (3k - 16)(k - 3) = 0$

$\therefore k = \frac{16}{3}$, or $k = 3$

15. ANS : D $\quad p = \sin 2\alpha,\ q = \cos 2\alpha$

$\therefore p^2 + q^2 = \sin^2 2\alpha + \cos^2 2\alpha = 1$

16. ANS : C $\quad$ (0, b) be the point on the y-axis then $\Rightarrow \frac{|4b - 12|}{\sqrt{17}} = 4$

$\therefore = \sqrt{17} + 4$ or $b = -\sqrt{17} + 3$

$\therefore p(0,\ 3 + \sqrt{17})$ or $p(0,\ -\sqrt{17} + 3)$

17. ANS : B $\quad \Delta = \frac{1}{2} \times a.b$

$\therefore \frac{1}{2}(2b)(p - b) = b^2$ **†Ü'äÝ´Ü**

$\therefore p = 0$ or $p = 2b$

$\therefore$ vertex of triangle lies on line x=0

18. ANS : A $\quad$ using $x = x^1 + h,\ \ y = y^1 + k$

given eq^n : $(x - 2)^2 + (y - 4)^2 = 105$

$\therefore$ shifting the origin at (h, k) = (2, 4)

So $x^2 + y^2 = 105$

19. ANS : B $\quad AQ^2 = (\pm 1 + BQ)^2$, $Q(x, y)$ ´ÜÜå

$\therefore (4x + 1)^2 = 4[(x + 1)^2 + y^2]$ **†Ü'äÝ´Ü**

$\therefore 12x^2 - 4y^2 = 3$

20. ANS : B $\quad (h,\ k) = \left(\frac{a}{2}, \frac{b}{2}\right)$

$OA^2 + OB^2 = AB^2 \Rightarrow a^2 + b^2 = 4c^2$

$\Rightarrow h^2 + k^2 = c^2$

$\therefore$ locus of the mid point : $x^2 + y^2 = c^2$

21. ANS : D slope of $\overline{OA} = \frac{1}{2}$, slope of $\overline{OB} = 2$

$\theta = \tan^{-1}\left|\frac{\frac{1}{2} - 2}{1 + \frac{1}{2}.2}\right| = \tan^{-1} \frac{3}{4}$

$= \sin^{-1} \frac{3}{5}$ **†Ü'äÝ´Ü**

22. ANS : A $\overline{AM} \perp \overline{BC}$ $AM = \frac{3}{2}$

from right ΔAMB, $AM^2 = a^2 - \left(\frac{a}{2}\right)^2$

$\Rightarrow a = \sqrt{3}$ **†Ü'äÝ´Ü**

23. ANS : C

if A lies on the line x+2y=1 then, $t = \frac{-4\sqrt{2}}{3}$

if A lies on the line 2x +4y=15 then, $t = \frac{5\sqrt{2}}{6}$

$\therefore \frac{-4\sqrt{2}}{3} < t < \frac{5\sqrt{2}}{6}$

24. ANS : C $A(r_1 \cos\theta, r_1 \sin\theta)$, $B(-r_2 \sin\theta, r_2 \cos\theta)$ are on line

$\therefore \frac{r_2 \sin\theta}{a} + \frac{r_2 \cos\theta}{b} = 1$ and $\therefore \frac{r_1 \cos\theta}{a} + \frac{r_2 \sin\theta}{b} = 1$

Now $\frac{1}{OA^2} + \frac{1}{OB^2} = \frac{1}{r_1^2} + \frac{1}{r_2^2}$

$\therefore \frac{1}{OA^2} + \frac{1}{OB^2} = \frac{a^2 + b^2}{a^2 b^2}$

25. ANS : B

take $a = \frac{b}{2}$ in $\frac{x}{a} + \frac{y}{b} = 1$

also take distance between 2x+y-b=0 and (0, 0) is $\sqrt{5}$

$\therefore 2x + y \pm 5 = 0$ which is RL.

26. ANS : D

x+y-1=0 and 2x-3y+1=0 (sloving the eq^n) $\left(\frac{2}{5}, \frac{3}{5}\right)$

27. ANS : C

$t<0 \quad \therefore \frac{t}{1-t}<0 \qquad \therefore \lambda=\frac{t}{1-t}$

28. ANS : B x=4t+1, y=5t+2 $\therefore y-x-1=t>0$

$\therefore y-x-1$ Positive

29. ANS : A x=2t+2, y=4t+3 $\therefore 3x+y=10t+9$

$(x, y) \in \overline{AB} \Rightarrow 0 \le t \le 1 \Rightarrow 9 \le 10t+9 \le 19$

$\therefore 3x+y$ maximum value $=19$

30. ANS : C x=8t-2, y=5-3t and $t \in R-[0,1]$

$\therefore \overleftrightarrow{AB}-\overline{AB}=\{(8t-2, 5-3t) / t \in R-[0,1]\}$

31. ANS : B $\cos\alpha=\frac{a}{\sqrt{a^2+b^2}}=\frac{1}{2}, \ \sin\alpha=\frac{b}{\sqrt{a^2+b^2}}=\frac{\sqrt{3}}{2} \quad \therefore \alpha=\frac{-\pi}{3}$

$p=\frac{|-4|}{\sqrt{1+3}}=2$, Now from $x\cos\alpha+y\sin\alpha=p$

$\therefore x\cos \pi/3 + y\sin \pi/3 = 2$

32. ANS : B circumcentre = controid

$AD = AB\sin 60^\circ$ [from ΔABD]

$r=\frac{a}{2\sqrt{3}} \qquad \left[\because 2=\frac{1}{3}AD\right]$ **†Ü‘äÝ´Ü**

one side of PQRS = x $\therefore x^2+x^2=(2r)^2=\frac{a^2}{6}$,

$\therefore$ area of square $=\frac{a^2}{6}$

33. ANS : B $|\ |=0$ [concurrent] $\Rightarrow 2ac=ab+bc \Rightarrow \frac{2}{b}=\frac{1}{c}+\frac{1}{a}$

$\therefore$ a, b, c H.P

34. ANS : A 5x+4y+k=0 which passes thorugh (2, 3)

$\therefore$ required vertex : (6, -2) “å.

35. ANS : D take b=-a-1 in $\frac{x}{a}+\frac{y}{b}=1$, also (4, 3) on given line

$\therefore a=\pm 2$, RL , $\frac{x}{2}-\frac{y}{3}=1$, and $x/{-2}+y/1=1$

36. ANS : A $12^2+9^2=15^2$ **†Ü‘äÝ´Ü**

$\therefore \frac{x}{\pm 9}+\frac{y}{\pm 12}=1$

$\therefore \pm 3x \pm 4y = 36$ **or** $\pm 4x \pm 3y = 36$

37. ANS : D $m_1 = -\cot 85^\circ,\ m_2 = -\cot 40^\circ \quad \tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|,$

$\therefore \tan(85^\circ - 40^\circ) = \tan 45^\circ,$

$\therefore \theta = 45^\circ$

38. ANS : A equal common ratio = r $\quad a_2 = a_1 r,\ a_3 = a_1 r^2$

$b_2 = b_1 r,\ b_3 = b_1 r^2$

slope of $\overleftrightarrow{AB} = \dfrac{b_1}{a_1} =$ Slope of $\overleftrightarrow{BC}$

$\therefore$ A, B, C are on one line

39. ANS : D C is mid point of $\overline{AB}$ which is lie on 5x+y+6=0

$m_1 m_2 = -1$

sloving both eq^n: $x_1 = -1, y_1 = -14$

$\therefore$ image point B(-1, -14) †Ü‘äÝ´Ü

40. ANS : D $\ell_1 \perp \ell_2 \ \therefore m_1.m_2 = -1$ then $(a+1)(a^2 - 2a + 2) = 0$

$a^2 - 2a + 2 = 0$ not possible $\therefore a = -1$

41. ANS : D $m_1 = -\dfrac{1}{3},\ m_2 = 3,\ m_1 m_2 = -1,$ diagonals bisect at right angle

$\therefore \square$ PQRS rhombus

42. ANS : C take c=-a-b in $3a\left(x - \dfrac{1}{3}\right) + 4b\left(y - \dfrac{1}{4}\right) = 0$

$\therefore$ line passes thorugh fixed point $\left(\frac{1}{3}, \frac{1}{4}\right)$

43. ANS : C take $n = -\dfrac{\ell}{2} - \dfrac{m}{3}$ in $\ell\left(x - \dfrac{1}{2}\right) + m\left(y - \dfrac{1}{3}\right) = 0$

$\therefore$ fixed point is $\left(\frac{1}{2}, \frac{1}{3}\right)$

44. ANS : A indentical : $\dfrac{1}{\lambda} = \dfrac{1}{-5} = \dfrac{\mu}{-5} \Rightarrow \lambda = -5,\ \mu = 1$

$\therefore \lambda + \mu = -4$

45. ANS : A by sloving $x = \dfrac{5}{3 + 4m},$

$\therefore 3 + 4m = \pm 1, \pm 5. \quad \therefore m = \dfrac{1}{2}$, -1 $, \dfrac{-1}{2}$, -2

$\therefore$ integer no. of m = 2

46. ANS : C

M(4, 5) is foot of perpendicular from 0(0, 0) slope of $\overleftrightarrow{OM} = 5/4$ $\therefore$ slope of ℓ

=- $-4/5$, which passes thorugh (4, 5)

$\therefore 4x + 5y - 41 = 0$

47. ANS : A

$mx - y - (mx_1 + y_1) = 0$ Y intercept $= -\frac{c}{b}$ (formula)

$\therefore$ y - intercept $= -(mx_1 + y_1)$

48. ANS : A

vertex P of $\overline{AB}$: (x, y)=$\left(\frac{p\cos\alpha}{2}, \frac{p\cot\alpha}{2}\right)$

$\therefore \sec\alpha = \frac{p}{2x}, \tan\alpha = \frac{p}{2y}$

also, $\sec^2\alpha - \tan^2\alpha = 1$

$\therefore \frac{p^2}{4x^2} = 1 + \frac{p^2}{4y^2}$

49. ANS : D mid point of $\overline{PQ}$ $\left(\frac{k+1}{2}, \frac{7}{2}\right)$

eq^n of perpendicular bisector : $y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$ whose y-intercept = - 4

$\therefore k = \pm 4$

50. ANS : B the eq^n of line perpendicular to the given line and passing through (2, 2) is : x-3y+4=0

$\therefore$ y - intercept $= 4/3$

51. ANS : B the y co-ordinate of point of intersection $= -3/2$

$\therefore$ required eq^n of line parallel to X - axis : $y = -3/2$

52. ANS : B required eq^n of diagonal: $\frac{x - a\cos\alpha}{-a\sin\alpha - a\cos\alpha} = \frac{y - a\sin\alpha}{a\cos\alpha - a\sin\alpha}$

$\therefore y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$ **†Ü‘äÝ´Ü**

53. ANS : B P divides $\overline{AB}$ from A in the ratio λ.

Q divides $\overline{AB}$ from A in the ratio -λ.

$\therefore P\left(\frac{\lambda b}{\lambda+1}, 0\right), Q\left(\frac{-\lambda b}{-\lambda+1}, 0\right)$

suppose A divides $\overline{PQ}$ from p in the ratio k.

$\therefore k = \frac{x - x_1}{x_2 - x} = \frac{1-\lambda}{1+\lambda}$

54. ANS : D none of the point outof A, B, C is not on the line or point of intersection $\left(-5/2, 15/2\right)$

55. ANS : A slope of the curve is constant $\therefore \frac{dy}{dx} = m$.

$\therefore y = mx + c$

56. ANS : C $\frac{-5}{13}x - \frac{12}{13}y = 1 \quad \therefore \cos\alpha = \frac{-5}{13}, \sin\alpha = \frac{-12}{13}$

$\alpha = \tan^{-1}\frac{12}{5} - \pi$ [α is in the third quadrant]

57. ANS : C slope $= \tan\theta = \sqrt{3} \quad m\angle PQS = 60^\circ$

slope of $\overline{QS} = -\sqrt{3}$

using $y - y_1 = m(x - x_1)$ †Ü‘äÝ´Ü

$\sqrt{3}x + y = 0$

58. ANS : A y is not fixed so all the lines are not parallel to $x = x_1$

$\therefore$ they intersect to the line $x = x_1$

59. ANS : C from the eq^n $x\cos\alpha + y\sin\alpha = p$

we get $-x\cos\frac{\pi}{6} - y\sin\frac{\pi}{6} = 10$

$\therefore \sqrt{3}x + y + 20 = 0$

60. ANS : D $x\cos\alpha + y\sin\alpha = P$, where $\alpha = 30^\cdot$ †Ü‘äÝ´Ü

$A\left(\frac{2p}{\sqrt{3}}, 0\right)$, $B(0, 2p)$

$\therefore BOA = \frac{50}{\sqrt{3}} \Rightarrow \frac{1}{2}(OA)(OB) = \frac{50}{\sqrt{3}} \Rightarrow p^2 = 25, \therefore p = 5$

$\therefore \sqrt{3}x + y = 10$

61. ANS : C

slope of $\overleftrightarrow{BC} = m_1 = -1$, slope of $\overleftrightarrow{AB} = m_2$

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \left.\begin{matrix} m_2 = 2 + \sqrt{3} \text{ OR} \\ m_2 = 2 - \sqrt{3} \end{matrix}\right\}$, which passes thorugh $\left(\sqrt{3}, -1\right)$.

$y = (2 - \sqrt{3})x + \sqrt{3}$

or $y = (2 + \sqrt{3})x - \sqrt{3}$

62. ANS : A

$x\cos\alpha + y\sin\alpha = p$ where $p = \sqrt{2}$ which passes through $\left(\sqrt{3}, -1\right)$

$4\sin^2\alpha + 2\sqrt{2}\sin\alpha - 1 = 0$

$\therefore \sin\alpha = \dfrac{\sqrt{3}-1}{2\sqrt{2}}, \cos\alpha = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore (\sqrt{3}+1)x - (\sqrt{3}-1)y = 4$

OR $\sin\alpha = \dfrac{-\left(\sqrt{3}+1\right)}{2\sqrt{2}}, \cos\alpha \dfrac{\sqrt{3}-1}{2\sqrt{2}},$

$\therefore (\sqrt{3}-1)x + (\sqrt{3}+1)y = 4$

63. ANS : B slope of $\overline{BC}$ $\times$ slope of $\overline{AM}$ = -1

$\Rightarrow 3a - b + 9 = 0$ — (1)

slope of $\overline{AC}$ $\times$ slope of $\overline{BH}$ = -1

$\Rightarrow 2a + b - 4 = 0$ — (2) solve (1) and (2)

$\therefore c(a, b) = c(-1, 6)$

64. ANS : A x+y=0 and x-y=0 are perpendicular

the circumcenter of Δ is on the line x-7=0

$\therefore$ circumcenter is (7, 0)

65. ANS : D from $\dfrac{1}{c} = \dfrac{2}{b} - \dfrac{1}{b}$ we get $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{2}{b} - \dfrac{1}{a} = 0$

$\therefore \dfrac{1}{a}(x-1) + \dfrac{1}{b}(y-(-2)) = 0$

$\therefore$ which passes through (1, -2)

66. ANS : B slope of line x+y+3=0 =-1 †Ü‘äÝ´Ü

$\therefore$ slope of the line perpendicular to it =1

67. ANS : A X=3 is a vertical line and slope of other line $= \tan\theta = \sqrt{3}$ $\therefore \theta = \dfrac{\pi}{3}$

$\alpha = \left|\dfrac{\pi}{2} - \theta\right| = \left|\pi/2 - \pi/3\right| = \pi/6$

68. ANS : C slope of y=e is $m_1 = 0$ slope of other line $m_2 = -\dfrac{1}{\sqrt{3}}$

$\tan\alpha = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\dfrac{0 + \dfrac{1}{\sqrt{3}}}{1+0}\right| = \dfrac{1}{\sqrt{3}}, \therefore \alpha = \pi/6$

69. ANS : A X- axis and Y- axis are perpendicular to each other.

$\therefore \ell_1 \perp \ell_2$

70. ANS : C If point A is on the line x+2y=1 then $t = -\frac{-4\sqrt{2}}{3}$

or on the line 2x+4y=15 then $t = \frac{5\sqrt{2}}{6}$

$\therefore -\frac{4\sqrt{2}}{3} < t < \frac{5\sqrt{2}}{6}$

71. ANS : A $(1+2\lambda)x + (1-\lambda)y + (1-\lambda) = 0$

slope $= -\left(\frac{1+2\lambda}{1-\lambda}\right) = 3/2$

$\therefore \lambda = -5$

72. ANS : B line parallel to Y-axis (vertical line)

$\therefore$ co-officient of Y = 0 and $a+1 \neq 0$

$\therefore a^2 - a + 2 = 0 \Rightarrow a = 2$

73. (C) $x - y + 9 = 0 \perp$ distance between two liners is $\sqrt{2}$. eq of RL passes through (–5, 4) any line $\perp$ to given line is x – y + k = 0 $\therefore$ –5 –4 + K = 0

$\therefore$ K = 9

74. (C) a = 2 b = 3

diagonals bisect each other choose the 4th vertecx as (a, b) $\left(\frac{1+5}{2}, \frac{7+2}{2}\right) = \left(\frac{a+4}{2}, \frac{b+6}{2}\right)$

$\Rightarrow$ a = 2 and b = 3

75 (C) $x^{2/3} + y^{2/3} = a^{2/3}$

differentiating

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$\therefore \quad \frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}} \text{ at } \left(\frac{a}{8}, \frac{a}{8}\right), \frac{dy}{dx} = -1$$

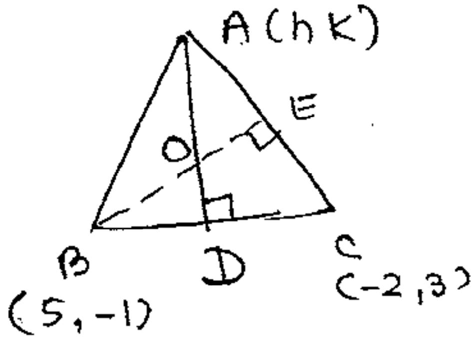
$\therefore$ eq of tanget at $\left(a/8, a/8\right)$ is

$$y - a/8 = -\left(x - a/8\right) \Rightarrow x + y - \frac{a}{4} = 0$$

$\therefore$ sum of intercepts $= \frac{a}{4} + \frac{a}{4} = \frac{a}{2} = 2$

$\therefore$ $a = 4$

76. (B) $(-4, -7)$



$\overline{AD} \perp \overline{BC} \Rightarrow \overline{OA} \perp \overline{BC}$

$$\left(\frac{k-0}{h-0}\right)\left(\frac{4}{-7}\right) = -1 \Rightarrow 2h = 4k$$

$\overline{OB} \perp \overline{AC} \Rightarrow$

$$\left(\frac{k-3}{h+2}\right)\left(-\frac{1}{5}\right) = -1 \Rightarrow 5h - k + 13 = 0$$

$\therefore$ $h = -4 \quad k = -7$

77. (B) $a^2 + b^2 = 2$ $\quad$ $ax + by + P = 0$ is angle bisector of given two lines

$\therefore$ $ax + by + p = 0$ and

$$\frac{x\cos\alpha + y\sin\alpha - p}{1} = \pm\frac{(x\sin\alpha - y\cos\alpha)}{1}$$

$x(\cos\alpha + \sin\alpha) + y(\sin\alpha + \cos\alpha) - p = 0$

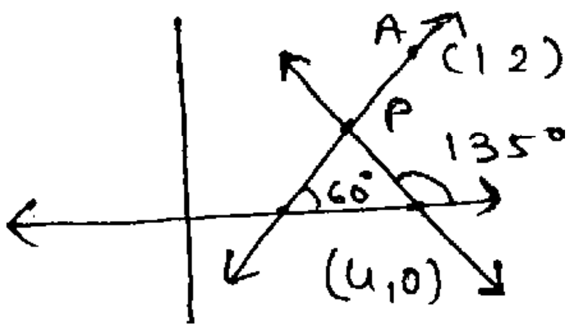
$x(\cos\alpha + \sin\alpha) + y(\sin\alpha - \cos\alpha) - p = 0$

$\therefore$ $\cos + \sin = -a$

$\sin - \cos = -b$

$\Rightarrow a^2 + b^2 = 2$

78. (B) $3\left(\sqrt{3} - 1\right)$ (Parametric from)



$$\frac{x-1}{\frac{1}{2}} = \frac{y-2}{\sqrt{3/2}} = r$$

$\Rightarrow$ $x = \frac{r}{2} + 1$ $\qquad \Rightarrow x + y = 6$

$y = \frac{\sqrt{3}}{2}r + 2 \qquad \Rightarrow \left(\frac{r}{2} + 1\right) + \left(\frac{\sqrt{3}}{2}r + 2\right) = 6$

$$\therefore \quad r = \frac{6}{\sqrt{3}+1} = 3\left(\sqrt{3}-1\right) = AP$$

79. (D) $\left(\frac{2}{17}, \frac{8}{17}\right)$

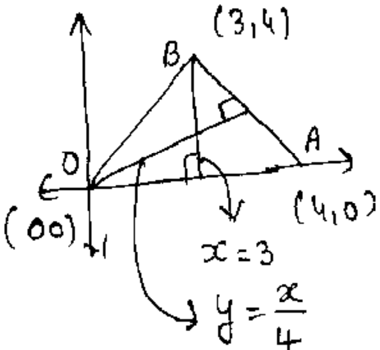
image of $(x_1\ y_1)$ is $(x_2\ y_2)$ in line $ax + by + c = 0$ then

$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = -2\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$$

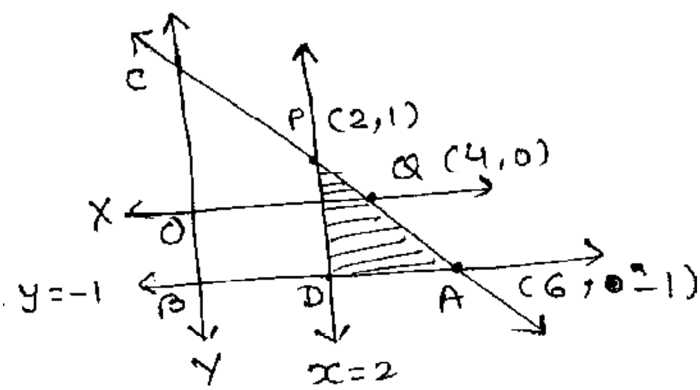
$$\frac{x_2 - 0}{1} = \frac{y_2 - 0}{4} = \frac{-2(0 + 0 - 1)}{17}$$

$$x_2 = \frac{2}{17} \qquad y_2 = \frac{8}{17}$$

80. (C) $\left(3, \frac{3}{4}\right)$



81. (A) (4, 0)



For ΔPDA mid pt of $\overline{PA}$ is cir cumecenter

82. (B) family of concurrent lines

$2b = a + c$

$a - 2b + c = 0$

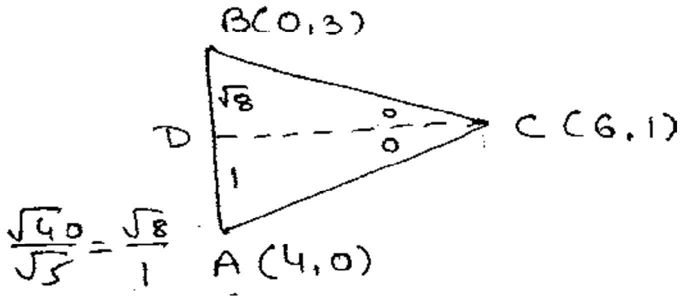
$\Rightarrow$ $ax + by + c = 0$ passesthrough $(1, -2)$

83. (B) $5\sqrt{2} - 7$

BC = $\sqrt{40}$

AC = $\sqrt{5}$

$\therefore \quad \dfrac{BC}{AC} = \dfrac{BD}{AD} = \dfrac{\sqrt{40}}{\sqrt{5}} = \dfrac{\sqrt{8}}{1}$



$\therefore$ coordinate of D = $\left(\dfrac{\sqrt{8}.4 + 1.0}{1 + \sqrt{8}}, \dfrac{\sqrt{8}.0 + 1.3}{\sqrt{8} + 1}\right)$ $\therefore$ slope of $\overleftrightarrow{CD}$ = $5\sqrt{2} - 7$

84. (A) ellipse : P(h, k) Q(–2, 0)

$$PQ = \frac{2}{3}\left|\frac{h + \frac{9}{2}}{\sqrt{1^2 + 0^2}}\right| = \frac{2}{3}\left|\frac{2h + 9}{2}\right| \qquad \therefore \sqrt{(h+2)^2 + k^2} = \left|\frac{2h+9}{3}\right|$$

$\Rightarrow$ $5x^2 + 9y^2 = 45$

$\Rightarrow$ $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$

85. (A) $7y + x - 6 = 0$, $(-1, 1) \in 3x - 4y + 7 = 0$

$\therefore$ Slope of line in new position

$$= \frac{\frac{3}{4} - 1}{1 + \frac{3}{4}} = -\frac{1}{7}$$

$\therefore$ Req eq of line

$y - 1 = -\dfrac{1}{7}(x + 1)$

$\Rightarrow$ $7y + x - 6 = 0$

86. Area of $\Delta = \frac{1}{2}\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

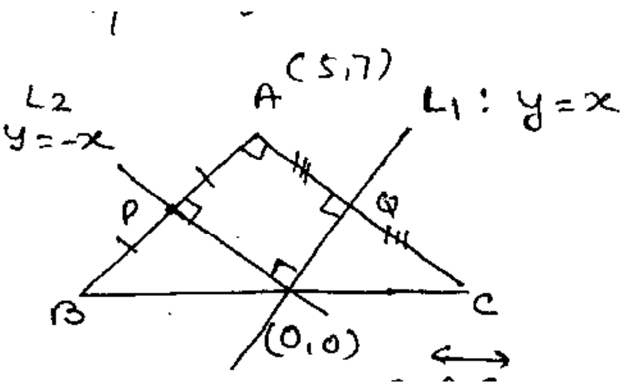
$= \frac{1}{2}$ (a – b) (b – c) (c – a)

$= \frac{1}{2}$ (–2) (–2) (4) = 8 sq unit

87. (B) Vertices of Δ are $\left(-\frac{6}{7}, 2\right)\left(\frac{6}{7}, 2\right), (0,5)$

∴ its area = $\frac{18}{7}$ sq unit

88. (A) 7y = 5x



eq of $\overleftrightarrow{AB}$

$y - 7 = 1\,(x - 5)$

∴ $y - x = 2$ ......(1)

Also $y + x = 0$ ......(2)

∴ P ↔ (–1,1)

P is midpoint of $\overline{AB}$

∴ B = (–7, –5)

eq of $\overleftrightarrow{AC}$

$y - 7 = -(x - 5)$

$x + y = 12$ .....(3)

$x - y = 0$ .....(4)

∴ Q ↔ (6, 6)

Q is midpoint AC

∴ C = (7, 5)

eq of $\overleftrightarrow{BC}$ $= \dfrac{x+7}{-7-7} = \dfrac{y+5}{-5-5}$ 10x + 70 = 14y + 70

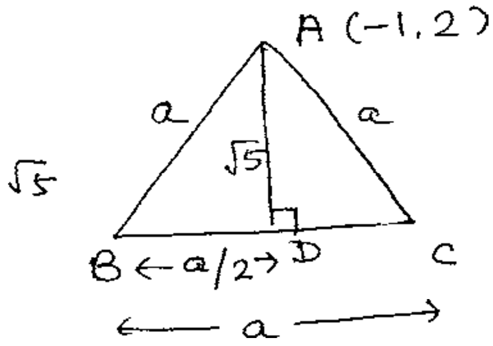
\ $\dfrac{x+7}{-14} = \dfrac{y+5}{-10}$ 5x = 7y

89. (A) $1 = \left|\dfrac{m-2}{1+2m}\right|$

$\Rightarrow$ m = –3 and m = $\dfrac{1}{3}$

90. (A) $\sqrt{\dfrac{20}{3}}$

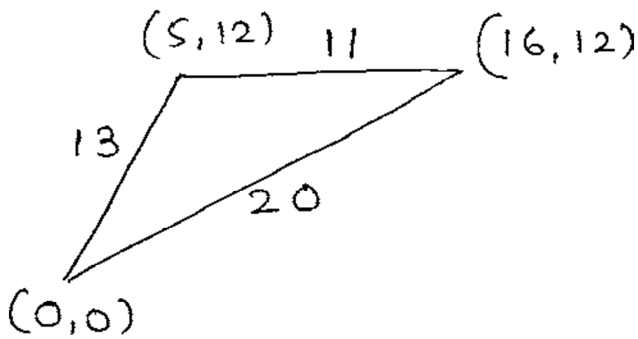
AD = $\left|\dfrac{2(-1)-2-1}{\sqrt{2^2+(-1)^2}}\right| = \sqrt{5}$



tan 60° = $\dfrac{\sqrt{5}}{a/2}$

$\therefore$ a = $\sqrt{\dfrac{20}{3}}$

91. (B) Rectangle



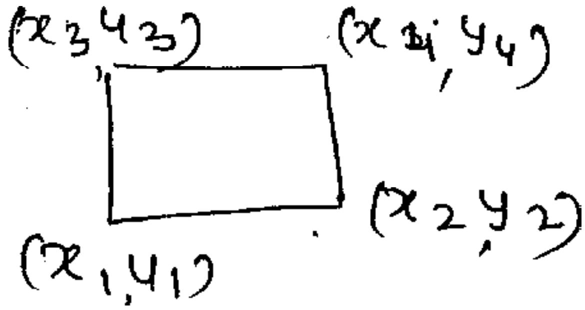
$$x_1^2 + y_1^2 + x_2^2 + y_2^2 + x_3^2 + y_3^2 + x_4^2 + y_4^2 \le (x_1 + x_3 + x_2x_4 + y_1y_2 + y_3y_4)$$

$\Rightarrow$ $(x_1 - x_3)^2 + (x_2 - x_4)^2 + (y_1 - y_2)^2 + (y_3 - y_4)^2 \le 0$

$\Rightarrow$ $x_1 = x_3$ $y_1 = y_2$

$x_2 = x_4$ $y_3 = y_4$

92. (A) $bx + ay = 3xy$



eq of $\overleftrightarrow{AB} = y - b = m(x - a)$

$$G = \left(\frac{a - \frac{b}{m}}{3}, \frac{b - am}{3}\right)$$

$3h = a - \frac{b}{m}$, $\quad 3k = b - am$

eliminating 'm' we will get bh + ak – 3hk = 0 ie bx + ay – 3xy = 0

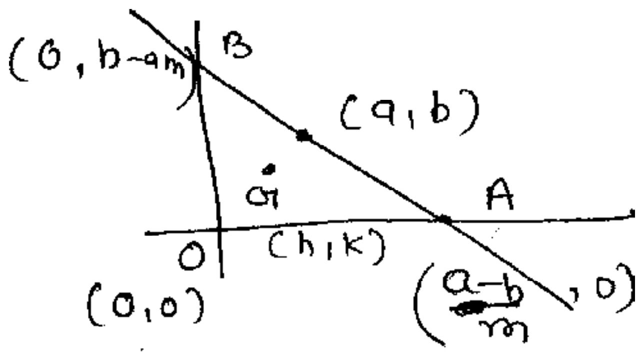
93. (B) $\frac{1}{2}, -1$ Slope of $\overleftrightarrow{AB}$ = Slope of $\overleftrightarrow{BC}$

$$\frac{2 - 2k - 2k}{k - 1 + k} = \frac{2k - 6 - 2k}{1 - k + k + 4}$$

$\Rightarrow$ $(4k - 6)(2k - 1) + 10(2k - 1) = 0$

$\therefore$ $k = \frac{1}{2}$ or $k = -1$

94. (B) (7, –2) (4, 3)



$x_1 + y_1 = 5$

$x_2 = 4$

$\therefore$ $G = (4, 1)$

$$\frac{1 + x_1 + x_2}{3} = 4 \; \& \; \frac{y_1 + y_2 + 2}{3} = 1$$

$\therefore$ $x_1 + x_2 = 11 \qquad y_1 + y_2 = 1$

$\therefore$ $x_1 = 7 \qquad x_2 = 4$

$y_2 = 3 \qquad y_1 = -2$

95. (A) 4 : 1

$$P = \left(\frac{-2\lambda + 1}{\lambda + 1}, \frac{3\lambda + 2}{\lambda + 1}\right)$$

$\therefore$ $3\left(\frac{-2\lambda + 1}{\lambda + 1}\right) + 4\left(\frac{3\lambda + 2}{\lambda + 1}\right) - 7 = 0$

$\Rightarrow$ $\lambda = 4$

96. (A) $11x - 3y + 9 = 0$

eq of lines

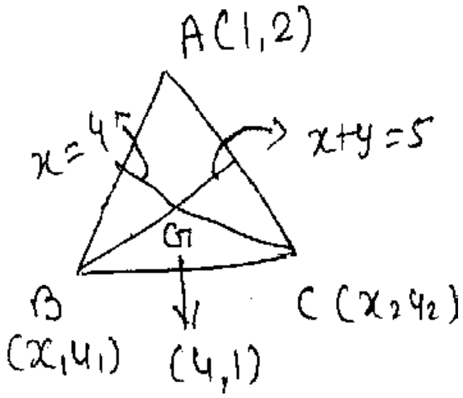
$3x - 4y + 7 = 0$

$-12x - 5y + 2 = 0$

$a_1a_2 + b_1b_2 = -36 + 20 < 0 \quad \therefore$ eq of acute angle bisector is

$$\frac{3x-4y+7}{5} = +\frac{-12x-5y+2}{13}$$

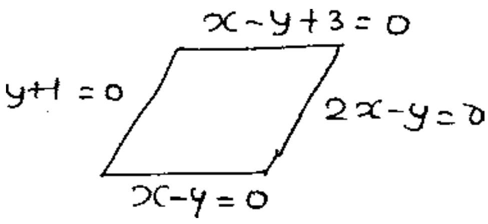
$11x - 3y + 9 = 0$



97. (D) $\left(\frac{3}{4}, \frac{1}{2}\right)$

$ax + by + c = 0 \Rightarrow \left(\frac{3}{4}, \frac{1}{2}\right)$

98. (B) 3 sq unit Area of // gm



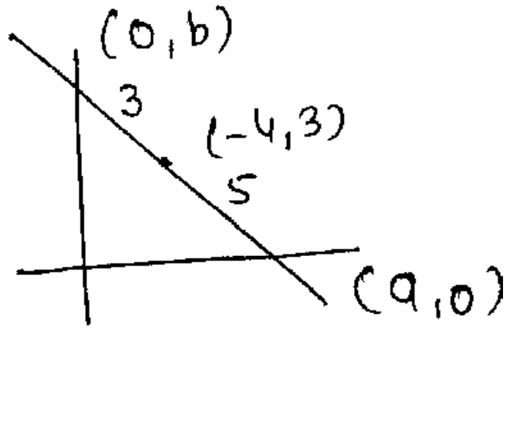
$$= \left| \frac{(3-0)(1-0)}{\begin{vmatrix} 2-1 \\ 1-1 \end{vmatrix}} \right| = 3$$

99. (A) $9x - 20y + 96 = 0$

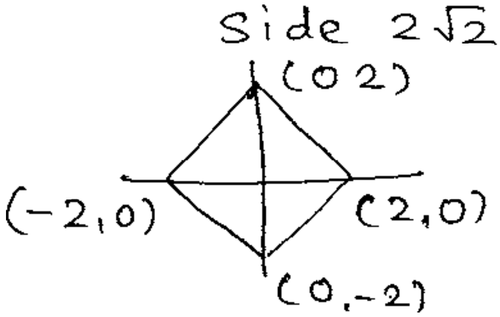
$\frac{3a}{8} = -4 \quad \Rightarrow \quad a = -\frac{32}{3}$

$\frac{5b}{8} = 3 \quad \Rightarrow \quad b = \frac{24}{5}$

$\therefore$ REOL $\frac{3x}{-32} + \frac{5y}{24} = 1 \Rightarrow 9x - 20y + 96 = 0$



100. (A) 8



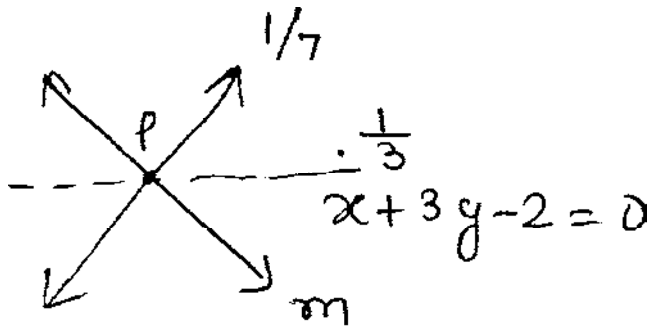
$|x| + |y| = 2$ represent square of side $2\sqrt{2}$

$\therefore$ its area is 8

101. (C) 5x + 5y = 3

POI of given two lines is

$P = \left(-\frac{1}{10}, \frac{7}{10}\right)$



$\therefore \left[\frac{-\frac{1}{3} - m}{1 - \frac{m}{3}}\right] = \left[\frac{-\frac{1}{3} - \frac{1}{7}}{1 - \frac{1}{21}}\right] \Rightarrow m = -1$

$\therefore$ REOL $y - \frac{7}{10} = -1\left(x + \frac{1}{10}\right) \Rightarrow 5x + 5y = 3$

102. (A) 21x + 27y – 121 = 0 ; at (–1, 4), $\frac{3x - 4y + 12}{12x - 5y + 12} > 0$

$\therefore$ we have to take +ve sign

$\frac{3x - 4y + 12}{5} = \frac{12x - 5y + 7}{13}$

$\Rightarrow$ 21x + 27y – 121 = 0

103. (B) x + 7y + 6 $\pm$ $5\sqrt{2} = 0$

iet line is x + 7y + λ = 0 distance of this line from (1, –1) is

$\left|\frac{1 - 7 + \lambda}{\sqrt{50}}\right|$ But as per Que $\left|\frac{1 - 7 + \lambda}{\sqrt{50}}\right| = 0$

$\Rightarrow \lambda = 6 \pm 5\sqrt{2}$

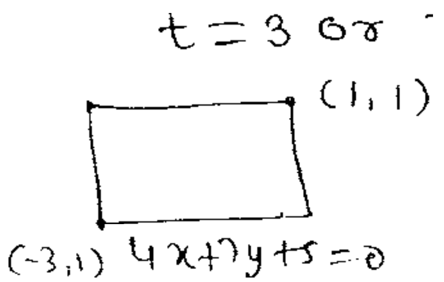
104. (A) (3, 1) and (–7, 11), any pt on line x + y = 4 can be taken as (t, 4 – t) the ⊥ distance of this pt from the line 4x + 3y – 10 = 0 is 1

$$\therefore \quad \left|\frac{4t+3(4-t)-10}{5}\right|=1$$

$$\Rightarrow \quad \left|\frac{t+2}{5}\right|=1$$

$\therefore$ t = 3 or t = –7

105. (D) All 4x + 7y – 11 = 0 , 7x – 4y + 25
7x – 4y – 3
7x – 4y + λ = 0



$\therefore$ λ = 25 or λ = – 3

106. (A) 9x – 7y = 1

$$\frac{3x-4y-7}{5}=\pm\frac{12x-5y+6}{13}$$

ie 21x + 27y + 121 = 0 &
99x – 77y – 61 = 0

there slopes = $-\frac{7}{9}$ and $\frac{9}{7}$

eq of lines passing through (4, 5)

$y - 5 = -\frac{7}{9}(x-4) \quad \Rightarrow \quad 7x + 9y = 73$

$y - 5 = \frac{9}{7}(x-4) \quad \Rightarrow \quad 9x - 7y = 1$

107. (C)

We know that foot of ⊥ from $(x_1\ y_1)$ on the line ax + by + c = 0 is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\frac{(ax_1+by_1+c)}{a^2+b^2}$$

ie $\frac{\alpha-0}{3} = \frac{\beta-0}{4} = \frac{-(-1)}{25}$ \ $\alpha = \frac{3}{25},\ \beta = \frac{4}{25}$

108. (C) $x^{-2} + y^{-2} = 4p^{-2}$

109. (C) $\frac{24x+7y-20}{25} = \pm\frac{4x-3y-2}{5}$

$\Rightarrow$ $27x + 7y - 20 = 20x - 15y - 10$ (by +ve sign)

$\Rightarrow$ $4x + 22y - 10 = 0$

$\Rightarrow$ $2x + 11y - 5 = 0$

110. (B) $y = 2$, $\sqrt{3}x + y = 0$ makes an angle of 120° with OX,

$\sqrt{3}x - y = 0$ makes angle of 60° with OX

$\therefore$ Rap line is $y - 2 = 0$

111. (A) Isoclese and rt L Δ

AB = $\sqrt{26}$, BC = $\sqrt{52}$, CA = $\sqrt{26}$

112. (B) $2x + y \pm 5 = 0$, $\frac{x}{a} + \frac{y}{b} = 1$, $\frac{a}{b} = \frac{1}{2}$

$\therefore$ $\frac{2x}{b} + \frac{y}{b} = 1$ $\therefore$ $2x + y - b = 0$

$\Rightarrow$ $\sqrt{5} = \frac{|-b|}{\sqrt{5}}$ $\therefore$ $b = \pm 5$ $a = \pm\frac{5}{2}$

$\therefore$ REOL $\frac{2x}{\pm 5} + \frac{y}{\pm 5} = 1$

113. (A) $2x + y \pm 6 = 0$

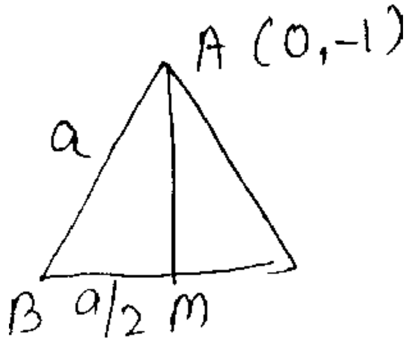
Line intersect x axis at pt (3, 0), (–3, 0) with slope – 2

$\therefore$ $y - 0 = -2(x - 3)$ $y - 0 = -2(x + 3)$

$y + 2x - 6 = 0$ $y + 2x + 6 = 0$

114. (A) $\sqrt{3}$

$$AM = \frac{|0-1-2|}{\sqrt{3+1}} = \frac{3}{2}$$



$$\therefore \quad a^2 - \frac{a^2}{4} = \frac{9}{4}$$

$\Rightarrow \quad a^2 = 3$

$\Rightarrow \quad a = \sqrt{3}$

115. (A) Let P(x, y) is any pt A $(a_1\ b_1)$, B(a_2, b_2)

$PA^2 = PB^2$

\ $(x - a_1)^2 + (y - b_1)^2 = (x - a_2)^2 + (y - b_2)^2$

– $2(a_1 - a_2)\ x + 2\ (b_1 - b_2)y + a_2^2 + b_2^2 - a_1^2 - b_1^2 = 0$

– $(a_1 - a_2)\ x + (b_1 - b_2)y + \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2) = 0$

on compaining

$(a_1 - a_2)x + (b_1 - b_2)\ y + c = 0$

$c = \frac{1}{2}\ (a_2^2 + b_2^2 - a_1^2 - b_1^2)$

116. (B) $\alpha = \frac{a\cos t + b\sin t + 1}{3}$ $(\alpha, \beta) = \text{centriod}$

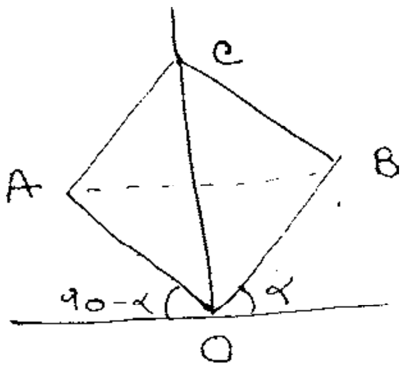
$$\beta = \frac{a\sin t - b\cos t}{3}$$

$a\cos t + b\sin t = (3\alpha - 1)$

$a\sin t - b\cos t = 3\beta$

sq sadd $a^2 + b^2 = (3\alpha - 1)^2 + (3\beta)^2$

117. (D) eq of $\overleftrightarrow{AB}$:



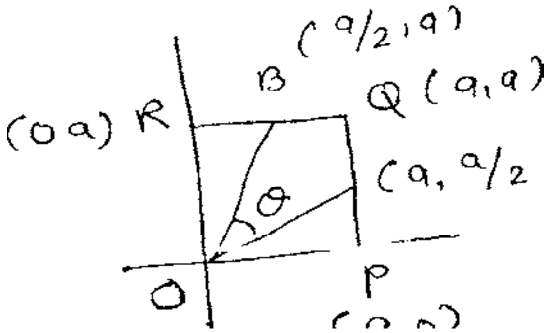
$$y - a\sin\alpha = \frac{a\cos\alpha - a\sin\alpha}{-a\sin\alpha - a\cos\alpha}(x - a\cos\alpha)$$

$$y - a\sin\alpha = -\frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha}(x - a\cos\alpha)$$

$\Rightarrow \quad y(\cos\alpha + \sin\alpha) + (\cos\alpha - \sin\alpha) = 0$

118. (A) Lie on a straight line

119. (D) $\sin^{-1}\frac{3}{5}$



slope of $\overline{OA} = \frac{1}{2} = m_1$

slope of $\overline{OB} = 2 = m_2$

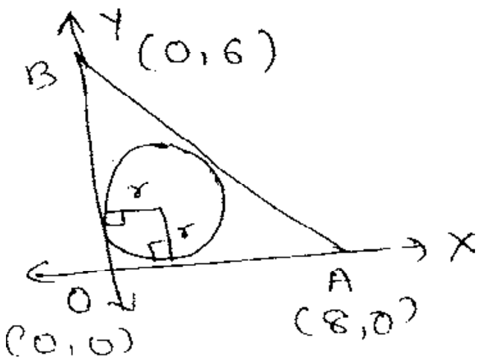
$$\left(\overline{OA}\,\hat{}\,\overline{OB}\right) = \theta = \tan^{-1}\left|\frac{\frac{1}{2}-2}{1+\frac{1}{2}\cdot 2}\right| = \tan^{-1}\frac{3}{4} = \sin^{-1}\frac{3}{5}$$

120. (A) $\left(2+\frac{1}{\sqrt{3}}, 5\right)$

Incentre = centroid

∴ AB = BC = CA = 2

121. (B) 2



Inradius = $\frac{\Delta}{S} = \dfrac{\frac{1}{2}8\times 6}{\frac{1}{2}(8+6+10)} = 2$

122. (A) $7x - 24y + 41 = 0$

Let eq of Rap line is $y - 2 = m(x - 1)$

∴ this line meats tha lines $3x + 4y - 12 = 0$ and $3x + 4y - 24 = 0$ at A & B

∴ $A = \left(\frac{4+4m}{3+4m}, \frac{6+9m}{3+4m}\right)$ $B = \left(\frac{16+4m}{3+4m}, \frac{6+21m}{3+4m}\right)$

But AB = 3 ∴ $AB^2 = 9$

∴ $\left(\frac{12}{3+4m}\right)^2 + \left(\frac{12}{3+4m}\right)^2 = 9 \quad \Rightarrow \quad m = \frac{7}{24}$

∴ REOL $7x - 24y + 41 = 0$

123. (A) 2x + 3y = 9

Let C is (α, β)

$\therefore$ controdi is $\left(\frac{\alpha}{3}, \frac{\beta-2}{3}\right)$

$\therefore$ $2\left(\frac{\alpha}{3}\right)+3\left(\frac{\beta-2}{3}\right)=1$

$\Rightarrow$ $2\alpha+3\beta=9$

124. (D)

125. (B) (1, –2)

a, b, c are in H.P.

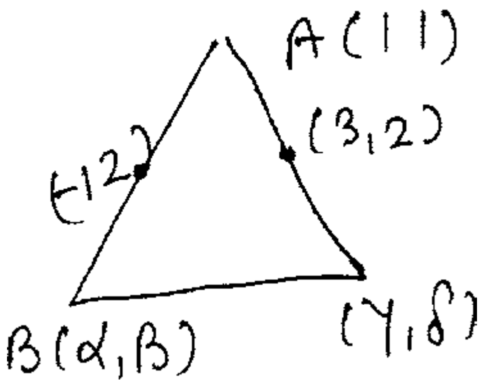
$\Rightarrow$ $\frac{2}{b}=\frac{1}{a}+\frac{1}{c}$

$\Rightarrow$ $\frac{1}{a}-\frac{2}{b}+\frac{1}{c}=0$

$\Rightarrow$ $\frac{x}{a}+\frac{y}{b}=-\frac{1}{c}$

$\therefore$ it passes through point (1, –2)

126. (B) $\left(1, \frac{7}{3}\right)$



$y+1=6$ $\qquad$ $\delta+1=4$

$y=5$ $\qquad$ $\delta=3$

$\alpha+1=-2$ $\qquad$ $\beta+1=4$

$\alpha=-3$ $\qquad$ $\beta=3$

$\therefore$ centroid $\left(\frac{1-3+5}{3}, \frac{1+3+3}{3}\right)$

$=\left(1, \frac{7}{3}\right)$

127. (D) (–1, –14)

Let B(x_1, y_1) is reflection of A(4, –12)

$\therefore$ $c\left(\frac{x_1+4}{2}, \frac{y_1-13}{2}\right)$ it lie on line 5x + y + 6 = 0

$\therefore$ $5\left(\frac{x_1+y}{2}\right)+\frac{y_1-13}{2}+6=0$ $\therefore$ $5x_1+y_1+7=0$

Slope of $\overline{AB}\times$ Slope of (5x + y + 6) = –1

$\left(\frac{y_1+13}{x_1-4}\right)\times(-5)=-1$ $\Rightarrow$ $-5y_1+x_1-69=0$

$\therefore$ $x_1 = -1$ $y_1 = -14$

128. (C) 2cosec 4α

$$p_1^{\ 2}+p_2^{\ 2}=\frac{4a^2}{\sec^2\alpha+\text{cosec}^2\alpha}+\frac{a^2\cos^2 2\alpha}{\cos^2\alpha+\sin^2\alpha}$$

$$=\frac{a^2 4\tan^2\alpha}{(1+\tan^2\alpha)^2}+\frac{a^2\cos^2 2\alpha}{\cos^2\alpha+\sin^2\alpha}$$

$$=a^2\left(\frac{2\tan\alpha}{1+\tan^2\alpha}\right)^2+a^2\cos^2 2\alpha$$

$$=a^2\left(\sin^2 2\alpha+\cos^2 2\alpha\right)=a^2$$

$$p_1^{\ 2}p_2^{\ 2}=\frac{1}{4}a^4\sin^2 4\alpha$$

$$\therefore \quad \frac{p_1}{p_2}+\frac{p_2}{p_1}=\frac{p_1^{\ 2}+p_2^{\ 2}}{p_1p_2}=\frac{2}{\sin 4\alpha}=2\,\text{cosec}\,4\alpha$$

129. (B) $x^2 + y^2 = c^2$

a = 2h

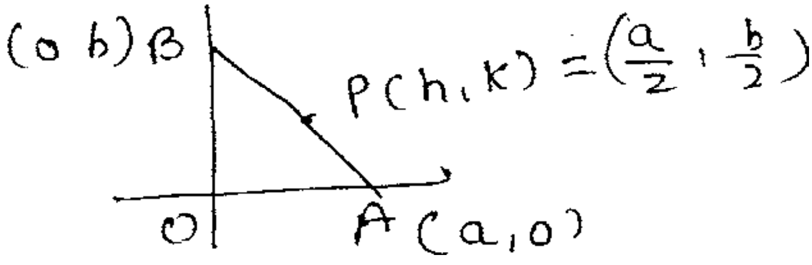
b = 2k

$OA^2 + OB^2 = AB^2$

$a^2 + b^2 = 4c^2$

$4h^2 + 4k^2 = 4c^2$

$h^2 + k^2 = c^2$



130. (C) $\frac{1}{3}$

$SA^2 = (3 - 3t^2)^2 + (6t)^2$

$= 9[1 - 2t^2 + t^4 + 4t^2]$

$= 9(1 + t^2)^2$

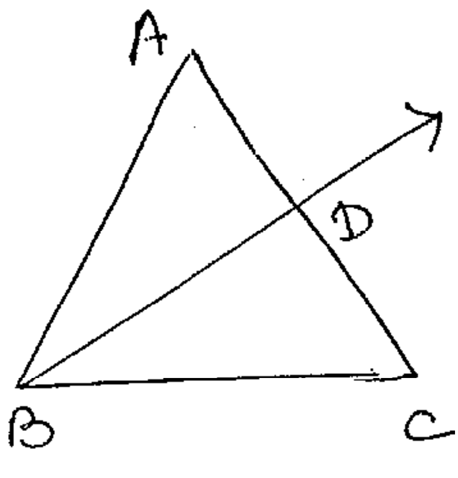
$SB^2 = \left(3 - \frac{3}{t^2}\right)^2 + \left(0 + \frac{6}{t}\right)^2$

$= 9\left[1 - \frac{2}{t^2} + \frac{1}{t^4} + \frac{4}{t^2}\right]$

$= 9\left(1 + \frac{1}{t^2}\right)^2$

$\therefore \quad \frac{1}{SA} + \frac{1}{SB} = \frac{1}{3}$

131. (B) $\left(\frac{22}{3}, \frac{13}{3}\right)$



$AB = \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$

$BC = \sqrt{12 + 4} = \sqrt{125} = 5\sqrt{5}$

$\frac{BC}{BA} = \frac{5}{4}$

coordinate of $D = \left[\frac{\frac{5}{4} \cdot 6 + 9}{\frac{5}{4} + 1}, \frac{\frac{5}{4} \cdot 7 + 1}{\frac{5}{4} + 1}\right]$

$= \left(\frac{22}{3}, \frac{13}{3}\right)$

132. (B) $4x + 3y = 24$

$\frac{x}{a} + \frac{y}{b} = 1$

$\left(\frac{3}{a}, 4\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$ $\therefore$ a= 6, b = 8

$\Rightarrow$ 6y + 8x = 24

33. (B) $\left(\frac{1}{2}, 3\right)$

$y = \frac{x}{2}, x > 0$

$y = 3x, \quad x > 0$

$a^2 - 3a < 0, \quad a^2 - \frac{a}{2} > 0$

$-\frac{1}{2} < a < 3$

134. (C) (–1, 3)

135. (A) $\sqrt{3}x + y = 0$

y – 0 = tan120° (x – 0)

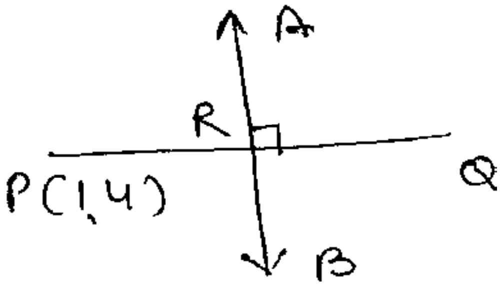
Slope of QR = $\sqrt{3}$

136. (A) (–4)

Slope of PQ = $-\frac{1}{k-1}$

Slope of AB = k –1

R is mid point of PQ $\therefore$ $\left(\frac{k+1}{2}, \frac{7}{2}\right) = R$



eq of $\overleftrightarrow{AB}$ $y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$

$\Rightarrow$ $k^2 = 16$ $\therefore$ $k = \pm 4$

137. (B) $\left(\frac{27}{2}, 2\right)$

$\frac{BC}{AB} = \frac{3}{2}$ and $A - B - C$

$\therefore$ B divide $\overline{AC}$ from C in ratio 3 : 2

$$(6, 2) = \left(\frac{\frac{3}{2}(1) + x}{\frac{3}{2}+1}, \frac{\frac{3}{2}(2) + y}{\frac{3}{2}+1}\right)$$

$\therefore$ $x = \frac{27}{2}$ & $y = 2$

138. (A) $\frac{x}{a} + \frac{y}{b} = 1$ $\therefore$ $\frac{4}{a} + \frac{3}{b} = 1$ Also a + b = -1

$\therefore$ $\frac{4}{a} + \frac{3}{-1-a} = 1$ $\therefore$ $a = \pm 2$

$\therefore$ $a = 2 \Rightarrow b = -3$

$a = -2 \Rightarrow b = 1$

139. (B) $x - 2y + 4 = 0$ $c_1 > 0$ $a_1a_2 + b_1b_2 > 0$

$4x - 3y + 2 = 0$ $c_2 < 0$

$\therefore$ Obtuse angle bisector is $\frac{x - 2y + 4}{\sqrt{5}} = \frac{4x - 3y + 2}{5}$

$\Rightarrow$ $x\left(4-\sqrt{5}\right) + y\left(2\sqrt{5}-3\right) + \left(2-4\sqrt{5}\right) = 0$

140. Lines van be written as (a, b, c > 0 and are in H P)

$\frac{4}{b}x + y\frac{3}{b} + 1 - y = 0$

$\frac{1}{b}(4x + 3y) + 1 - y = 0$ $\therefore$ lines are concurrent at $\left(-\frac{3}{4}, 1\right)$ and

Rep line is $y - 1 = \pm 1\left(x + \frac{3}{4}\right)$

$y + x = \frac{1}{4}$, $y - x = \frac{7}{4}$, (A) and (D)

141. (B) $5x + y - 2 = 0$

POI of 3x – 2y = 0 and 5x + y – 2 = 0 is $\left(\frac{4}{13}, \frac{6}{13}\right)$

The line makes an angle of measure $\tan^{-1}(-5)$ with x-axis

$\therefore \quad \theta = \tan^{-1}(-5) \quad \Rightarrow \quad \tan\theta = -5$

$\therefore$ REOL $\quad y - \frac{6}{13} = -5\left(x - \frac{4}{13}\right)$

$\Rightarrow \quad 5x + y - 2 = 0$

$\therefore \quad \sqrt{3}x + y + 20 = 0$

142. (B) 60° any line $\perp$ to $\sqrt{3}x + y = 1$ is

$x - \sqrt{3}y + k = 0 \quad \therefore \quad m_1 = \frac{1}{\sqrt{3}}$

$\tan\alpha = \frac{\left|\frac{1}{\sqrt{3}} - 0\right|}{|1-0|} = \frac{1}{\sqrt{3}} \quad \therefore \quad \alpha = \frac{\pi}{6}$

$\therefore$ angle with the +ve direction of y-axis is $\left|\frac{\pi}{2} - \alpha\right| = \left|\frac{\pi}{2} - \frac{\pi}{6}\right| = \frac{\pi}{3}$

143. (A) exactly one value of p

given lines are 11 $\quad \therefore \quad m_1 = m_2$

$\Rightarrow \quad p\left(p^2+1\right) = -\frac{\left(p^2+1\right)^2}{p^2+1}$

$\Rightarrow \quad p\left(p^2+1\right)^2 = -\left(p^2+1\right)^2$

$\Rightarrow \quad p = -1$

144. (C) $\frac{23}{\sqrt{17}}$

since L : $\frac{x}{5} + \frac{y}{b} = 1$ Passess through (13, 22)

$\therefore \quad \frac{13}{5} + \frac{32}{b} = 1 \quad \Rightarrow \quad b = -20$

$\therefore$ line L becomes

$\frac{x}{5} + \frac{y}{-20} = 1 \quad \Rightarrow \quad 4x - y - 20 = 0$ .....(1)

L is // to K : $\frac{x}{c} + \frac{y}{3} = 1$

$\therefore \quad \frac{4}{1} = -\frac{1/c}{1/3} \quad \Rightarrow \quad c = -\frac{3}{4}$

$\therefore$ K becomes $4x - y + 3 = 0$ ....(2)

$\therefore$ distance between K & L = $23/\sqrt{17}$

145. (2) $[1, \alpha)$

146. (D) P, Q and R are non colinear

P = $(-\sin(\beta - \alpha), -\cos\beta) = (x_1, y_1)$

Q = $(\cos(\beta - \alpha), \sin\beta) = (x_2\ y_2)$

R = $(x_2 \cos\theta + x_1 \sin\theta, y_2 \cos\theta + y_1 \sin\theta)$

$\therefore \quad T \equiv \left(\frac{x_2\cos\theta + x_1\sin\theta}{\cos\theta + \sin\theta}, \frac{y_2\cos\theta + y_1\sin\theta}{\cos\theta + \sin\theta}\right)$

$\therefore$ P, Q, T are collinear

$\Rightarrow$ P, Q, R are non colliner

147. (A) 190

consider the line x = 1,

which cuts the line.

Joining points (0, 21) and (21,0)

at (1,20), so there ar 19 integral points

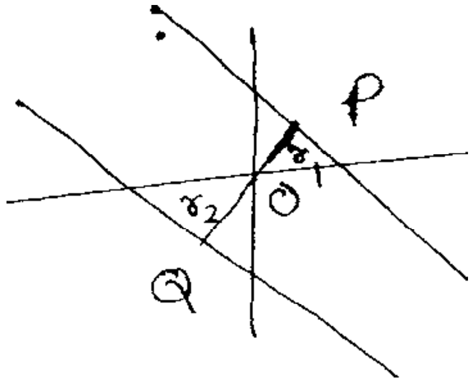
on this line inside the Δ.

lly x = 2, x = 3, ... x = 20

contain respectively 18, 17,...... o integral points.

$\therefore$ Total points = 19 + 18 + 17 + ... + 1

148. (B) 3 : 4 Let $\left(r_1\cos\theta, r_1\sin\theta\right)$ is on



$4x + 2y = 9 \qquad \therefore \qquad r_1 = \dfrac{9}{4\cos\theta + 2\sin\theta}$

Let $\left(-r_2\cos\theta, -r_2\sin\theta\right)$ lie on

$2x + y + 6 = 0$

$\therefore \quad r_2 = \dfrac{6}{2\cos\theta + \sin\theta}$

$\therefore \quad \dfrac{OP}{OQ} = \dfrac{r_1}{r_2} = \dfrac{3}{4}$

149. (C) –3 $f(x) = x^2 + bx - b$

$f'(x) = 2x + b$

$f'(1) = 2 + b$

eq of tangent at (1, 1) will be

$y - 1 = (2 + b)(x - 1)$

– $\quad \dfrac{y}{2+b} - \dfrac{1}{2+b} = x - 1$

\ $\quad \dfrac{x}{(1+b)/(2+b)} - \dfrac{y}{(1+b)} = 1$

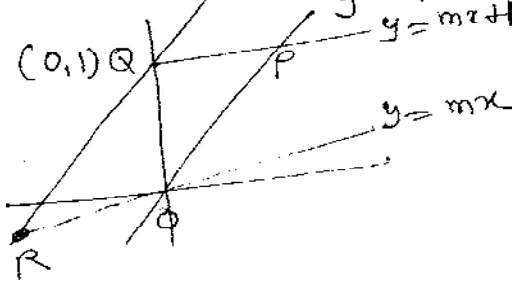
Ininter sept form $OA = \dfrac{1+b}{2+b}$ and $OB = -(1+b)$

Area of $OAB = \dfrac{1}{2}(OA \cdot OB) = 2$ given

$\Rightarrow \quad (1+b)^2 = -4(2+b)$

$\Rightarrow \quad b = -3$

150. (D) $\dfrac{1}{|m-n|}$



coordinate of pare $\left(\dfrac{1}{n-m}, \dfrac{n}{n-m}\right)$

Area of // gm OPQR = 2 × area of Δ OPQ

$\therefore$ Desired area = $2\times\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ \frac{1}{n-m} & \frac{n}{n-m} & 1 \end{vmatrix}$

$= \frac{1}{|n-m|}$

151. (D) 2x + 9y + 7 = 0

Mid point of Q (6, –1) and R(7, 3) is $\left(\frac{6+7}{2}, \frac{-1+3}{2}\right) \equiv \left(\frac{13}{2}, 1\right)$

Slope of median through $P = \frac{1-2}{\frac{13}{2}-2} = \frac{-2}{9}$

Equation of the required line is

$y + 1 = -\frac{2}{9}(x - 1)$ or 2x + 9y + 7 = 0

152. (C) 2 points

$\angle PRQ = \pi/2$

$\therefore$ Slope of RPX slope of RQ = –1

$\therefore$ $\frac{y-1}{x-3} \times \frac{5-1}{6-3} = -1$ – 3x + 4y = 13 ......(1)

Aare of $\Delta RPQ = 7$

$\Rightarrow$ $\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 6 & 5 & 1 \end{vmatrix} = \pm 7$

$\therefore$ 3y – 4x = 5 or 3y – 4x = –23 .....(2)

\ Solving (1) and (2) we get tow points

153. (A) y – 3x + 9 = 0 and 3y + x – 3 = 0

Point (3, 0) does not lie on the diagonal x = 2y,

let m be the slope of a side passing through (3, 0)

then eq is y – 0 = m (x – 3) an ther side is x = 2, Now $\tan\frac{\pi}{4} = \pm\frac{m - \frac{1}{2}}{1 + \frac{m}{2}}$ $\Rightarrow$ m = 3, $\frac{-1}{3}$

154. (B) $\left(2, -\frac{1}{2}\right)$ Given Δ is right angled Δ at vertex $\left(2, -\frac{1}{2}\right)$

$\Rightarrow$ AC = BC = t

$= \sqrt{4a^2 + (a - t)^2}$

$\Rightarrow$ $t = \frac{5a}{2}$

$\therefore$ coordinats of third vertex C = $\left(2a, \frac{5a}{2}\right)$

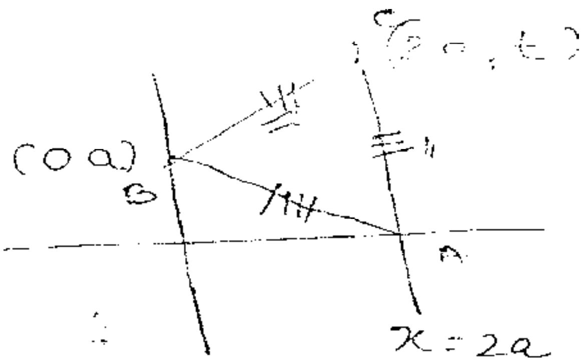
155. (B) $\sqrt{3}x + y \pm 10 = 0$

Let p is length of ⊥ from the original on the given line. Then its equation in normal from x cos 30° + y sin 30° = p or

$\sqrt{3}x + y = 2p$

This meets the coordinats arces at

$A\left(\frac{2p}{\sqrt{3}}, 0\right)$ and $B(0, 2p)$



$\therefore$ Area of ΔOAP = $\frac{1}{2}\left(\frac{2p}{\sqrt{3}}\right) \cdot 2p = \frac{2p^2}{\sqrt{3}} = \frac{50}{\sqrt{3}}$.

– p = ± 5, REq $\sqrt{3}x + y = \pm 10$

156. (C) 1

lines are concurrent

$\therefore \begin{vmatrix} 1 & a & a \\ b & 1 & b \\ c & c & 1 \end{vmatrix} = 0$

$$\Rightarrow \quad abc\begin{vmatrix} 1/a & 1 & 1 \\ 1 & 1/b & 1 \\ 1 & 1 & 1/c \end{vmatrix} = 0$$

$$\Rightarrow \quad abc\begin{vmatrix} \frac{1}{a} & 1 & 1 \\ 1-\frac{1}{a} & \frac{1}{b}-1 & 0 \\ 1-\frac{1}{a} & 0 & \frac{1}{c}-1 \end{vmatrix} = 0$$

$R_3 \to R_3 - R_1$

$R_2 \to R_2 - R_1$

$$\Rightarrow \quad abc\left[\frac{1}{a}\left(\frac{1}{b}-1\right)\left(\frac{1}{c}-1\right)+\left(\frac{1}{a}-1\right)\left(\frac{1}{b}-1\right)-\left(\frac{1}{c}-1\right)\left(1-\frac{1}{a}\right)\right]=0$$

$\Rightarrow$ (1 – b) (1 – c) + c(1 – a) (1 – b) + b(1 – c) (1 – c) (1 – a) = 0

$$\Rightarrow \quad \frac{1}{1-a}+\frac{c}{1-c}+\frac{b}{1-b}=0$$

$$\Rightarrow \quad 1+\frac{a}{1-a}+\frac{b}{1-a}+\frac{c}{1-c}=0$$

$$\Rightarrow \quad \frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$$

157. (C) 14x + 25y – 40 = 0

Line AB $\perp$ to x – y + 5 = 0 is x + y + c_1 = 0 it passess throuh A(1, –2)

$\therefore$ c_1 = 1

AB : x + y + 1 = 0

Let B $\leftrightarrow$ (h, k), M . P of $\overline{AB}$ is $\left(\frac{h+1}{2}, \frac{k-2}{2}\right)$

le on $\overline{AB}$ as well as its bisector

$$\therefore \quad \frac{h+1}{2}+\frac{k-2}{2}+1=0 \quad \& \quad \frac{h+1}{2}-\frac{k-2}{2}+5=0$$

$\Rightarrow$ B = (–7, 6) with line $\overline{AC}$ we get C = $\left(\frac{11}{5}, \frac{2}{5}\right)$

$\therefore$ eq of $\overline{BC}$: $y - 6 = \dfrac{\frac{2}{5} - 6}{\frac{11}{5} + 7}(x + 7)$

ie $14x + 23y - 40 = 0$

158. (A) (b) 25 (B) (a) 75 C (c) $\dfrac{9}{x} + \dfrac{4}{y} = 1$

(A) Let eq of line is $y - 4 = m(x - 9)$

$P = \left(\dfrac{9m-4}{m}, 0\right)$ $Q = (0, 4 - 9m)$

$OP + OQ = 9 - \dfrac{4}{m} + 4 - 9m \geq 13 + 2\sqrt{\left(-\dfrac{4}{m}\right)(-9m)} = 25$

(B) OP + OQ is minimum when

$\dfrac{4}{m} = 9m \Rightarrow m^2 = \dfrac{4}{9} \Rightarrow m = -\dfrac{2}{3}$

$P = (15, 0)$ & $Q = (0, 10)$

Area of $\Delta OPQ = \dfrac{1}{2} \times 15 \times 10 = 75$

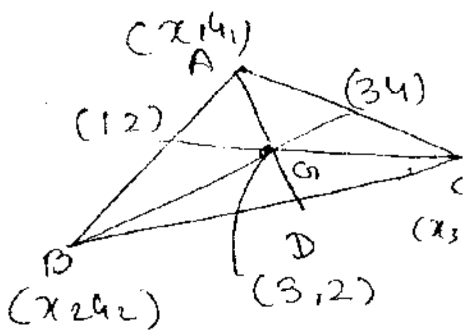
(C) $h = \dfrac{9m-4}{m} \Rightarrow \dfrac{9}{h} + \dfrac{4}{k} = 1$

$k = 4 - 9m$

$\therefore \dfrac{9}{x} + \dfrac{4}{y} = 1$

159. (A) (a) $(2x + y = 4)$ (B) (b) $(5, 0)$ C(c) $6\sqrt{2}$

Let $D = (\alpha, \beta)$



$\therefore \quad \frac{\alpha+1+3}{3}=3 \quad \Rightarrow \quad \alpha = 5, \ \frac{\beta+2+4}{3}=2 \quad \Rightarrow \quad \beta = 0$

$\therefore \quad D = (5, 0)$

$\frac{x_1+x_2}{2}=1 \quad \frac{x_2+x_3}{2}=5 \ \& \ \frac{x_3+x_1}{2}=3$

$\frac{y_1+y_2}{2}=2 \quad \frac{y_2+y_3}{2}=0 \ \& \ \frac{y_1+y_3}{2}=4$

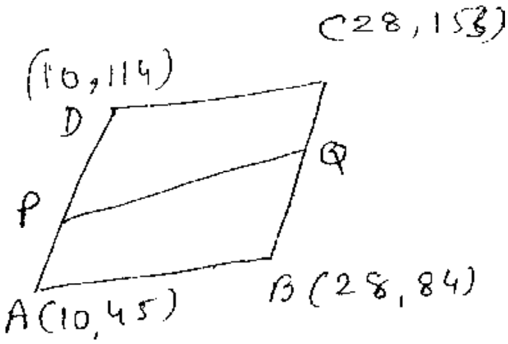
$\therefore \quad A = (-1, 6) \qquad B = (3, -2) \qquad C = (7, 2)$

eq of AB = $2x + y = 4$

Height of altitude from A = $\frac{2 \times \text{Ar } \Delta ABC}{BC} = 6\sqrt{2}$

160. (B) $\frac{99}{19}$

AP = CQ = x



– $\frac{45+x}{10} = \frac{153-x}{28}$

– $x = \frac{135}{19}$

slope of PQ = $\frac{45+\frac{135}{19}}{10} - = \frac{99}{19}$

161. (C) $2\sqrt{2}$

$\therefore$ lines are concurrent

$\therefore \quad \begin{vmatrix} 1 & 0 & -a-m \\ 0 & 1 & 2 \\ m & -1 & 0 \end{vmatrix} = 0 \quad \Rightarrow \quad m^2 + am + 2 = 0$

$\therefore$ m is real $\qquad \therefore \quad a^2 \geq 8 \qquad \Rightarrow \qquad |a| \geq 2\sqrt{2}$

162. (A) a, b, c are in A.P

Answer Key

1	**B**	**35**	**D**	**69**	**A**		
2	**C**	**36**	**A**	**70**	**C**		
3	**C**	**37**	**D**	**71**	**A**		
4	**B**	**38**	**A**	**72**	**B**		
5	**D**	**39**	**D**				
6	**A**	**40**	**D**				
7	**C**	**41**	**D**				
8	**C**	**42**	**C**				
9	**A**	**43**	**C**				
10	**B**	**44**	**A**				
11	**B**	**45**	**A**				
12	**C**	**46**	**C**				
13	**B**	**47**	**A**				
14	**A**	**48**	**A**				
15	**D**	**49**	**D**				
16	**C**	**50**	**B**				
17	**B**	**51**	**B**				
18	**A**	**52**	**C**				
19	**B**	**53**	**B**				
20	**B**	**54**	**D**				
21	**D**	**55**	**A**				
22	**A**	**56**	**C**				
23	**C**	**57**	**C**				
24	**C**	**58**	**A**				
25	**B**	**59**	**C**				
26	**D**	**60**	**D**				
27	**C**	**61**	**C**				
28	**B**	**62**	**A**				
29	**A**	**63**	**B**				
30	**C**	**64**	**A**				
31	**B**	**65**	**D**				
32	**B**	**66**	**B**				
33	**B**	**67**	**A**				
34	**A**	**68**	**C**				
73	c	101	c	129	b		
74	c	102	a	130	c		
75	c	103	b	131	b		
76	b	104	a	132	b		
77	b	105	d	133	b		
78	b	106	a	134	c		
79	d	107	c	135	a		
80	c	108	c	136	a		
81	a	109	c	137	b	157	c
82	b	110	b	138	a	158	c

83	b	111	a	139	b	159	a&b&c
84	a	112	b	140	a&d	160	b
85	a	113	a	141	b	161	c
86	b	114	a	142	b	162	a
87	c	115	a	143	b		
88	a	116	b	144	a		
89	a	117	d	145	c		
90	a	118	a	146	b		
91	b	119	d	147	d		
92	a	120	a	148	b		
93	b	121	b	149	c		
94	d	122	a	150	b		
95	a	123	a	151	d		
96	a	124	b	152	c		
97	d	125	b	153	a		
98	b	126	b	154	b		
99	a	127	d	155	b		
100	a	128	c	156	b		

Unit – 11 – Circle and Conic Section

MCQ

(1) The number of integral values of m for which $x^2 + y^2 \ (1 - m)x + my + 5 = 0$ is the equation of a circle whose radius cannot exceed 5, is

(a) 20 (b) 18 (c) 8 (d) 24

(2) The circle $x^2 + y^2 - 6x - 10y + \quad = 0$ does not touch or intersect the coordinate axes and point (1, 4) is inside the circle, then the range of the values of   is

(a) (0, 25) (b) (5, 29] (c) (25, 29) (d) (9, 25)

(3) Equation of smallest circle touching these four circle $(x \quad 1)^2 + (y \quad 1)^2 = 1$ is ________

(a) $x^2 + y^2 = 3 - \sqrt{2}$

(b) $x^2 + y^2 = 5 - 2\sqrt{2}$

(c) $x^2 + y^2 = 6 - 2\sqrt{2}$

(d) $x^2 + y^2 = 3 - 2\sqrt{2}$

(4) If two circle $(x - 1)^2 + (y - 3)^2 = a^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(a) $2 < a < 8$ (b) $a > 2$ (c) $a < 2$ (d) $a = 2$

(5) If the tangents are drawn to the circle $x^2 + y^2 = 12$ at the point where it meets the circle $x^2 + y^2 - 5x + 3y - 2 = 0$, then the point of intersection of these tangent is

(a) (6, – 6) (b) $\left(6, \frac{18}{5}\right)$ (c) $\left(6, \frac{18}{5}\right)$ (d) $\left(\ 6, \frac{18}{5}\right)$

(6) Two tangents to the circle $x^2 + y^2 = 4$ at the points A and B meet at P(–4, 0). The area of the quadrilateral PAOB, where O is the origin is

(a) $4\sqrt{3}$ (b) 4 (c) $6\sqrt{2}$ (d) $2\sqrt{3}$

(7) The radius of the circle passing through the points (5, 2), (5, –2) and (1, 2) is

(a) $2\sqrt{5}$ (b) $3\sqrt{2}$ (c) $5\sqrt{2}$ (d) $2\sqrt{2}$

(8) The line $x\sin \quad - y\cos \quad = k$ touches the circle $x^2 + y^2 = k^2$ then

(a) $\left(\frac{\ }{2}, \frac{\ }{2}\right)$ (b) [0,] (c) [– ,] (d) is any angle

(9) One of the diameters of the circle circumscribing the rectangle ABCD is $x - 4y + 7 = 0$. If A and B are points (–3, 4) and (5, 4) respectively, then the area of the rectangle is ________

(a) 32 sq. units (b) 16 sq. units (c) 64 sq. units (d) 8 sq. units

(10) Let C be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. If the tangents at the point A(1, 7)

and B(4, –2) on the circle meet at piont D. Then area of the quadrilateral ABCD is ________

(a) 150 sq. units (b) 100 sq. units (c) 75 sq. units (d) 50 sq. units

(11) The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co–ordinates axes. The locus of the circumcentre of the triangle is $x + y - xy + k \sqrt{x^2 \quad y^2} \quad 0$ then k = ________

(a) 0 (b) 1 (c) 2 (d) 3

(12) A square is inscribed in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$. Its sides are parallel to the co–ordinate axes. Then one vertex of the square is

(a) $(1 \quad \sqrt{2}, \quad 2)$ (b) $(1 \quad \sqrt{2}, \quad 2)$ (c) $(1, -2+\sqrt{2})$ (d) $(1 \quad \sqrt{2}, \quad 2 \quad \sqrt{2})$

(13) If the equation $\frac{m(x \quad 1)^2}{3} \quad \frac{(y \quad 2)^2}{4} \quad 1$ represents a circle then m = ________

(a) 0 (b) $\frac{3}{4}$ (c) $\frac{3}{4}$ (d) 1

(14) The circle whose equation is $x^2 + y^2 - 2 \quad x - \quad y + \quad ^2 = 0$

(a) passes through origin (b) touches only X–axis

(c) touches only Y–axis (d) touches both the axes

(15) The line $(x + g) \cos \quad + (y + f) \sin \quad = k$ touches the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ only its

(a) $g^2 + f^2 = c + k^2$ (b) $g^2 + f^2 = c^2 + k^2$ (c) $g^2 + f^2 = c - k^2$ (d) $g^2 + f^2 = c^2 - k^2$

(16) The centre of the circle passing throug (0, 0) and (1, 0) and touching the circle $x^2 + y^2 = 9$ is ________

(a) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \quad \sqrt{2}\right)$

(17) The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y - 24 = 0$ is ________

(a) 0 (b) 1 (c) 2 (d) None of these

(18) The equation of the set of complex number $z = x + iy$, So that $|z - z_1| = 5$, where $z_1 = 1 + 2i$

(a) $x^2 + y^2 - 2x - 4y - 20 = 0$ (b) $x^2 + y^2 + 2x - 4y - 20 = 0$

(c) $x^2 + y^2 - 2x + 4y - 20 = 0$ (d) $x^2 + y^2 + 2x + 4y + 20 = 0$

(19) A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x–axis, then the locus of its centre is ______

(a) $\{(x, y) : x^2 = 4y\} \quad \{(x, y) : y \quad 0\}$ (b) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \quad \{(x, y) : y \quad 0\}$

(c) $\{(x, y) : x^2 = y\} \quad \{(0, y) : y \quad 0\}$ (d) $\{(x, y) : x^2 = 4y\} \quad \{(0, y) : y \quad 0\}$

(20) Tangent to the circle $x^2 + y^2 = 5$ at the point (1, –2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ then point of contact is ______

(a) (3, 1) (b) (3, –1) (c) (–3, –1) (d) (–3, 1)

(21) Four distinct points (1, 0), (0, 1), (0, 0) and (2a, 3a) lie on a circle for

(a) only one value of a (0, 1) (b) $a > 2$

(c) $a < 0$ (d) a (1, 2)

(22) The length of the chord joining the points (2cos , 2sin) and (2cos(+ 60°), 2sin(+ 60°)) of the circle $x^2 + y^2 = 4$ is

(a) 2 (b) 4 (c) 8 (d) 16

(23) A square is formed by the two points of straight lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$. A circle is inscribed in it. The centre of the circle is

(a) (6, 5) (B) (5, 6) (c) (7, 4) (d) (4, 7)

(24) If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is ______

(a) 3 (b) $\sqrt{3}$ (c) 2 (d) $\sqrt{2}$

(25) The lines $2x - 3y - 5 = 0$ and $3x - 4y - 7 = 0$ are diameters of a circle of area 154 square units then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y - 62 = 0$ (b) $x^2 + y^2 + 2x - 2y - 47 = 0$

(c) $x^2 + y^2 - 2x + 2y - 47 = 0$ (d) $x^2 + y^2 - 2x + 2y - 62 = 0$

(26) The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

(a) $9x - 3y + 2 = 0$ (b) $2x - y + 1 = 0$ (c) $x - 2y + 8 = 0$ (d) $x - y + 2 = 0$

(27) The lengthof the common chord of the parabolas $y^2 = x$ and $x^2 = y$ is

(a) 1 (b) $\sqrt{2}$ (c) $4\sqrt{2}$ (d) $2\sqrt{2}$

(28) The straight line $y = a - x$ touches the parabola $x^2 = x - y$ if $a =$ ______

(a) –1 (b) 0 (c) 1 (d) 2

(29) If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$ then one of the values of k is

(a) 4 (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) 8

(30) If M is the foot of the perpendicular from *a* point P on a parabola to its directrix and SPM is an equilateral triangle, where S is the focus, then SP is equal to

(a) $8a$ (b) $2a$ (c) $3a$ (d) $4a$

(31) The chord AB of the parabola $y^2 = 4ax$ cuts the axis of the parabola at C. If A = $(at_1^2, 2at_1)$, B = $(at_2^2, 2at_2)$ and AB : AC = 3 : 1 then

(a) $t_2 = 2t_1$ (b) $t_1 + 2t_2 = 0$ (c) $t_2 + 2t_1 = 0$ (d) $t_1 - 2t_2 = 0$

(32) Equation of common tangents of $y^2 = 4bx$ and $x^2 = 4by$ is

(a) $x + y + b = 0$ (b) $x - y + b = 0$ (c) $x - y - b = 0$ (d) $x + y - b = 0$

(33) Angle between the tangents drawn to $y^2 = 4x$, where it is intersected by the line $x - y - 1 = 0$ is equal to

(a) $\frac{}{2}$ (b) $\frac{}{3}$ (c) $\frac{}{4}$ (d) $\frac{}{6}$

(34) The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

(a) $\frac{}{2}$ (b) $\frac{}{3}$ (c) $\frac{}{4}$ (d) $\frac{}{6}$

(35) The shortest distance between the line $x - y + 1 = 0$ and the curve $x = y^2$ is ________

(a) $\frac{3\sqrt{2}}{5}$ (b) $\frac{2\sqrt{3}}{8}$ (c) $\frac{3\sqrt{2}}{8}$ (d) $\frac{2\sqrt{2}}{5}$

(36) Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid–point of $\overline{PQ}$ is

(a) $y^2 + 4x + 2 = 0$ (b) $y^2 - 4x + 2 = 0$ (c) $x^2 - 4y + 2 = 0$ (d) $x^2 + 4y + 2 = 0$

(37) If tangents to the parabola $y^2 = 4ax$ at the points $(at_1, 2at_1)$ and $(at_2^2, 2at_2)$ intersect on the axis of the parabola, then

(a) $t_1t_2 = -1$ (b) $t_1t_2 = 1$ (c) $t_1 = t_2$ (d) $t_1 + t_2 = 0$

(38) The focus of the parabola $x^2 - 8x + 2y + 7 = 0$ is ________

(a) $\left(4, \frac{9}{2}\right)$ (b) $\left(0, \frac{1}{2}\right)$ (c) $\left(4, \frac{9}{2}\right)$ (d) (4, 4)

(39) The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is ________

(a) (–1, 0) (b) (1, 0) (c) (0, 0) (d) (0, 1)

(40) If the line $y = 1 - x$ touches the curve $y^2 - y + x = 0$, then the point of contact is

(a) (0, 1) (b) (1, 0) (c) (1, 1) (d) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(41) The line $y = c$ is a tangent to the parabola $y^2 = 4ax$ if c is equal to

(a) a (b) 0 (c) $2a$ (d) None of these

(42) The vertex of the parabola $(x - b)^2 = 4b\,(y - b)$ is ________

(a) $(b, 0)$ (b) $(0, b)$ (c) (0, 0) (d) (b, b)

(43) The axis of the parabola $9y^2 - 16x - 12y - 57 = 0$ is

(a) $y = 0$ (b) $16x + 61= 0$ (c) $3y - 2 = 0$ (d) $3y - 61 = 0$

(44) If P$(at^2, 2at)$ be one end of a focal chord of the parabola $y^2 = 4ax$, then the length of the chord is ________

(a) $a\left(t \quad \frac{1}{t}\right)$ (b) $a\left(t \quad \frac{1}{t}\right)$ (c) $a\left(t \quad \frac{1}{t}\right)^2$ (d) $a\left(t \quad \frac{1}{t}\right)^2$

(45) The latus rectum of a parabola is a line

(a) through the focus (b) parallel to the directrix

(c) perpendicular to the axis (d) all of these

(46) A tangent to the parabola $y^2 = 9x$ passes through the point (4, 10). Its slope is

(a) $\frac{3}{4}$ (b) $\frac{9}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

(47) The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$ if $m =$ ________

(a) 4 (b) 3 (c) 2 (d) 1

(48) If a chord of the parabola $y^2 = 4ax$, passing through its focus F meets it in P and Q, then

$\frac{1}{|FP|} \quad \frac{1}{|FQ|} =$ ________

(a) $\frac{1}{a}$ (b) $\frac{2}{a}$ (c) $\frac{4}{a}$ (d) $\frac{1}{2a}$

(49) The equation of the chord of parabola $y^2 = 8x$. Which is bisected at the point (2, –3) is

(a) $3x + 4y - 1 = 0$ (b) $4x + 3y + 1 = 0$ (c) $3x - 4y + 1 = 0$ (d) $4x - 3y - 1 = 0$

(50) If $x + y + 1 = 0$ touches the parabola $y^2 = ax$ then $a =$ ________

(a) 8 (b) 6 (c) 4 (d) 2

(51) If y_1, y_2 and y_3 are the ordinates of the vertices of a triangle inscribed in the parabola $y^2 = 4ax$, then its area is

(a) $\left|\frac{1}{8a}(y_1 \quad y_2)(y_2 \quad y_3)(y_3 \quad y_1)\right|$ (b) $\left|\frac{1}{4a}(y_1 \quad y_2)(y_2 \quad y_3)(y_3 \quad y_1)\right|$

(c) $\left|\frac{1}{2a}(y_1 \quad y_2)(y_2 \quad y_3)(y_3 \quad y_1)\right|$ (d) $\left|\frac{1}{a}(y_1 \quad y_2)(y_2 \quad y_3)(y_3 \quad y_1)\right|$

(52) The centre of the ellipse $\frac{(x \quad y \quad 2)^2}{9} \quad \frac{(x \quad y)^2}{16} = 1$ is ________

(a) (1, 1) (b) (0, 0) (c) (0, 1) (d) (1, 0)

(53) Let E be the ellipse $\frac{x^2}{9} \quad \frac{y^2}{4} \quad 1$ and C be the circle $x^2 + y^2 = 9$. Let P and Q be the piont (1, 2) and (2, 1) respe. Then

(a) P lies inside C but outside E (b) P lies inside both C and E

(c) Q lies outside both C and E (d) Q lies inside C but outside E

(54) The ellipse $x^2 + 4y^2 = 4$ is incribed in a rectangle aligned with the co–ordinate axes. Which in turn is inscribed in an other ellipse that passes through the point (4, 0). Then the equation of the ellipse is

(a) $4x^2 + 48y^2 = 48$ (b) $x^2 + 16y^2 = 12$ (c) $x^2 + 16y^2 = 16$ (d) $x^2 + 12y^2 = 16$

(55) Chords of an ellipse are drawn through the positive end of the minor axis. Then their mid point lies on

(a) a circle (b) a parabola (c) an ellipse (d) a hyperbola

(56) The distance from the foci of P(x_1, y_1) on the ellipse $\frac{x^2}{9} \quad \frac{y^2}{25} \quad 1$ is ________

(a) $4 \quad \frac{5}{4}y_1$ (b) $5 \quad \frac{4}{5}y_1$ (c) $5 \quad \frac{4}{5}x_1$ (d) $4 \quad \frac{4}{5}y_1$

(57) If S and S' are two foci of an ellipse $16x^2 + 25y^2 = 400$ and PSQ is a focal chord such that SP = 16 then S'Q = ________

(a) $\frac{74}{9}$ (b) $\frac{54}{9}$ (c) $\frac{64}{9}$ (d) $\frac{44}{9}$

(58) Tangents are drawn to the ellipse $\frac{x^2}{9} \quad \frac{y^2}{5} \quad 1$ at ends of latus recturm line. The area of quadrilateral so formed is ________

(a) $\frac{27}{4}$ (b) $\frac{27}{55}$ (c) 27 (d) $\frac{27}{2}$

(59) Let P be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e. If A, A' are the vertices and S, S' are the foci of an ellipse, then area of APA' : area of PSS' = ________

(a) e (b) e^2 (c) e^3 (d) $\frac{1}{e}$

(60) A focus of an ellipse is at the origin. The directrix is the line $x - 4 = 0$ and eccentricity is $\frac{1}{2}$, then the length of semi–major axis is

(a) $\frac{5}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{2}{3}$

(61) The equation $\frac{x^2}{1-r}-\frac{y^2}{1+r}=1\,;\ r>1$ represents.

(a) a parabola (b) an ellipse (c) a circle (d) None of these

(62) If P(m, n) is a point on an ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ with foci S and S' and eccentricty e, then area of SPS' is ________

(a) $ae\sqrt{a^2 \quad m^2}$ (b) $ae\sqrt{b^2 \quad m^2}$ (c) $be\sqrt{b^2 \quad m^2}$ (d) $be\sqrt{a^2 \quad m^2}$

(63) If P(x_1, y_1) is a point on an ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ and it's one focus is S(ae, 0) then PS is equal to ________

(a) $a + ex_1$ (b) $a - ex_1$ (c) $ae + x_1$ (d) $ae - x_1$

(64) If $\sqrt{3}bx + ay = 2ab$ touches the ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ then eccentric angle θ of point of contact = ________

(a) $\frac{}{2}$ (b) $\frac{}{3}$ (c) $\frac{}{4}$ (d) $\frac{}{6}$

(65) If P is a point on an ellipse $5x^2 + 4y^2 = 80$ whose foci are S and S'. Then PS + PS' = ________

(a) $4\sqrt{5}$ (b) 4 (c) 8 (d) 10

(66) If $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ is an ellipse, then length of it's latus–rectum is ________

(a) $\frac{2b^2}{a}$ (b) $\frac{2a^2}{b}$

(c) depends on whether $a > b$ or $b > a$ (d) $\frac{2a}{b^2}$

(67) The curve represented *by* $x = 3(\cos t + \sin t)$; $y = 4(\cos t - \sin t)$ is

(a) circle (b) parabola (c) ellipse (d) hyperbola

(68) The length of the common chord of the ellipse $\frac{(x \quad 1)^2}{9} \quad \frac{(y \quad 2)^2}{4} \quad 1$ and the circle $(x - 1)^2 + (y - 2)^2 = 1$

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 4 (d) None of these

(69) S and T are the foci of an ellipse and B is an end of the minor axis. If   STB is an equilateral, then e = ________

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

(70) If the line $lx + my + n = 0$ cuts an ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ in points whose eccentric angles differ by $\frac{\ }{2}$, then $\frac{a^2l^2 \quad b^2m^2}{n^2}$ = ________

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$

(71) Area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ is

(a) ab (b) $2ab$ (c) $\frac{a}{b}$ (d) $\sqrt{ab}$

(72) The equation $2x^2 + 3y^2 - 8x - 18y + 35 = k$ represents

(a) parabola if $k > 0$ (b) circle if $k > 0$ (c) a point if $k = 0$ (d) a hyperbola if $k > 0$

(73) If $\frac{x}{a} \quad \frac{y}{b} \quad \sqrt{2}$ touches the ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$, then its eccentric angle θ of the contact piont is ________

(a) 0^o (b) 45^o (c) 60^o (d) 90^o

(74) The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x = 4$, then equation of an ellipse is

(a) $3x^2 + 4y^2 = 1$ (b) $3x^2 + 4y^2 = 12$ (c) $4x^2 + 3y^2 = 12$ (d) $4x^2 + 3y^2 = 1$

(75) The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} \quad \frac{y^2}{9} \quad 1$ and having its centre (0, 3) is ________

(a) 4 (b) 3 (c) $\sqrt{12}$ (d) $\frac{7}{2}$

(76) The equations of the common tangents to the parabola $y = x^2$ and $y = -(x - 2)^2$ is

(a) $y = 4(x - 1)$ (b) $y = 2$ (c) $y = -4(x - 1)$ (d) $y = -30x - 50$

(77) If e_1 and e_2 be the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e_1^2} \quad \frac{1}{e_2^2} =$ ________

(a) 2 (b) 1 (c) 0 (d) 3

(78) A hyperbola, having the transverse axis of length 2 sin is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

(a) $x^2\text{cosec}^2 \quad - y^2\sec^2 \quad = 1$ (b) $x^2\sec^2 \quad - y^2\text{cosec}^2 \quad = 1$

(c) $x^2\sin^2 \quad - y^2\cos^2 \quad = 1$ (d) $x^2\cos^2 \quad - y^2\sin^2 \quad = 1$

(79) The locus of a point P(,) moving under the condition that the line $y = \quad x + \quad$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

(a) a circle (b) a parabola (c) an ellipse (d) a hyperbola

(80) If ($a$sec , $b$tan) and ($a$sec , $b$tan) are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ then =

$\tan\frac{\alpha}{2} \cdot \tan\frac{\beta}{2} =$ ________

(a) $\frac{e \quad 1}{e \quad 1}$ (b) $\frac{1 \quad e}{1 \quad e}$ (c) $\frac{1 \quad e}{1 \quad e}$ (d) $\frac{e \quad 1}{e \quad 1}$

(81) If AB is a double ordinates of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OAB is an equilateral triangle, O being the centre of the hyperbola, then the eccentricity e of the hyperbola satisfies.

(a) $1 \quad e \quad \frac{2}{\sqrt{3}}$ (b) $e \quad \frac{1}{\sqrt{3}}$ (c) $e \quad \frac{\sqrt{3}}{2}$ (d) $e \quad \frac{2}{\sqrt{3}}$

(82) The value of m for which $y = mx + 6$ is a tangent to the hyperbola $\frac{x^2}{100} \quad \frac{y^2}{49} \quad 1$ is

(a) $\sqrt{\frac{17}{20}}$ (b) $\sqrt{\frac{20}{3}}$ (c) $\sqrt{\frac{20}{17}}$ (d) $\sqrt{\frac{3}{20}}$

(83) The vertices of the hyperbola $9x^2 - 16y^2 - 36x + 96y - 252 = 0$ are

(a) (6, 3), (–6, 3) (b) (–6, 3), (–6, –3) (c) (6, –3), (2, –3) (d) (6, 3),(–2, 3)

(84) Which of the following in independent of in the hyperbola $0 \quad \frac{}{2} \quad \frac{x^2}{\cos^2} \quad \frac{y^2}{\sin^2} \quad 1$?

(a) Vertex (b) Eccentricity (c) Abscissa of foci (d) Directrix

(85) The equation of the tangent to the curve $4x^2 - 9y^2 = 1$. Which is parallel to $5x - 4y + 7 = 0$ is

(a) $30x - 24y + 17 = 0$ (b) $24x - 30y \quad \sqrt{161} = 0$

(c) $3x - 24y \quad \sqrt{161} = 0$ (d) $24x + 30y \quad \sqrt{161} = 0$

(86) Two straight lines pass through the fixed points (a, 0) and have slopes whose products is $p > 0$. Then, the locus of the points of intersection of the lines is

(a) a circle (b) a parabola (c) an ellispe (d) a hyperbola

(87) The equations to the common tangents to the two hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and are $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

(a) $y \quad x \quad \sqrt{a^2 \quad b^2}$ (b) $y \quad x \quad \sqrt{b^2 \quad a^2}$

(c) $y \quad x \quad \sqrt{a^2 \quad b^2}$ (d) $y = \pm x \pm (a^2 - b^2)$

(88) If the line $2x \quad \sqrt{6}y \quad 2$ touches the hyperbola $x^2 - 2y^2 = 4$ then the point of contact is

(a) $(4, \quad \sqrt{6})$ (b) $(-5, 2\sqrt{6})$ (c) $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ (d) $(\quad 2, \sqrt{6})$

(89) A common tangent to $9x^2 - 16y^2 = 144$ and $x^2 + y^2 = 9$ is

(a) $y \quad 3\sqrt{\frac{2}{7}}x \quad \frac{15}{\sqrt{7}}$ (b) $y \quad 2\sqrt{\frac{3}{7}}x \quad 15\sqrt{7}$

(c) $y \quad \frac{3}{\sqrt{7}}x \quad \frac{15}{\sqrt{7}}$ (d) $y \quad 2\sqrt{\frac{3}{7}}x \quad 15\sqrt{7}$

(90) The coordinates of a point on the hyperbola $\frac{x^2}{24} \quad \frac{y^2}{18} \quad 1$ which is nearest to the line $3x + 2y + 1 = 0$ are

(a) (6, –3) (b) (6, 3) (c) (–6, 3) (d) (–6, –3)

(91) The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ is

(a) $3x \quad \sqrt{3}y \quad 1 \quad 0$ (b) $x \quad \sqrt{3}y \quad 3 \quad 0$ (c) $x \quad \sqrt{3}y \quad 3 \quad 0$ (d) $3x \quad \sqrt{3}y \quad 1 \quad 0$

(92) If $a > 2b > 0$ and $y = mx - b\sqrt{1 \quad m^2}$ $(m > 0)$ is a tangent to circles $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ then $m =$ ________

(a) $\frac{2b}{\sqrt{a^2 \quad 4b^2}}$ (b) $\frac{2b}{a \quad 2b}$ (c) $\frac{b}{a \quad 2b}$ (d) $\frac{\sqrt{a^2 \quad 4b^2}}{2b}$

(93) If $x = 9$ is the chord of the hyperbola $x^2 - y^2 = 9$ then the equation of the corresponding pair of tangents at the end points of the chord is ________

(a) $9x^2 - 8y^2 + 18x - 9 = 0$ (b) $9x^2 - 8y^2 - 18x + 9 = 0$

(c) $9x^2 - 8y^2 - 18x - 9 = 0$ (d) $9x^2 - 8y^2 + 18x + 9 = 0$

(94) The latus rectum of the hyperbola $9x^2 - 16y^2 - 18x - 32y - 151 = 0$ is

(a) $\frac{9}{2}$ (b) $\frac{3}{2}$ (c) 9 (d) $\frac{9}{4}$

(95) The locus of the vertices of the family of parabolas $y \quad \frac{a^3x^2}{3} \quad \frac{a^2x}{2} \quad 2a$ is

(a) $xy \quad \frac{105}{64}$ (b) $xy \quad \frac{3}{4}$ (c) $xy \quad \frac{35}{16}$ (d) $xy \quad \frac{64}{105}$

(96) The area bounded by the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 4$ and the pair of lines $\sqrt{3}(x^2 + y^2) = 4xy$ is equal to ________

(a) $\frac{}{4}$ (b) $\frac{}{2}$ (c) $\frac{5}{2}$ (d) 3

(97) The equation of the tangent to the circle $x^2 + y^2 + 4x - 4y + 4 = 0$. Which makes equal intercepts on the positive coordinate axes is ________

(a) $x + y = 8$ (b) $x + y = 4$ (c) $x + y = 2\sqrt{2}$ (d) $x + y = 2$

(98) Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point (1, 1) is

(a) $x^2 + y^2 - 6x + 4 = 0$ (b) $x^2 + y^2 - 3x + 1 = 0$

(c) $x^2 + y^2 - 4y + 2 = 0$ (d) None of these

(99) If the circle $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q, then the line $5x + by - a = 0$ passes through P and Q fore

(a) no value of a (b) exactly one value of a

(c) exactly two values of a (d) infinitely many value of a

(100) The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have coordinates (3, 4) and (–4, 3) respectively, then QPR is equal to

(a) $\frac{\;}{2}$ (b) $\frac{\;}{3}$ (c) $\frac{\;}{4}$ (d) $\frac{\;}{6}$

(101) If PN is the perpendicular from a point on a rectangular hyperbola to its asymptotes, the locus, the midpoint of PN is

(a) A circle (b) a hyperbola (c) a parabola (d) An ellipse

(102) The equation $\left| \sqrt{x^2 \quad (y \quad 1)^2} \quad \sqrt{x^2 \quad (y \quad 1)^2} \right| \quad k$ will represent a hyperbola for

(a) k (0,) (b) k (2,) (c) k (–3, 0) (d) k (0, 2)

(103) The asymptote of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ from with any tangent to the hyperbola a triangle whose area is a^2 tan in magnitude then its eccentricity is

(a) cosec (b) sec (c) cosec^2 (d) sec^2

(104) The area of the triangle formed by any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with its asymptotes is

(a) ab (b) $4ab$ (c) a^2b^2 (d) $4a^2b^2$

(105) The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is

(a) $\frac{x}{y_1 \quad y_2} \quad \frac{y}{x_1 \quad x_2} \quad 1$ (b) $\frac{x}{x_1 \quad x_2} \quad \frac{y}{y_1 \quad y_2} \quad 1$

(c) $\frac{x}{y_1 \quad y_2} \quad \frac{y}{x_1 \quad x_2} \quad 1$ (d) $\frac{x}{x_1 \quad x_2} \quad \frac{y}{y_1 \quad y_2} \quad 1$

(106) The product of the lengths of perpendiculars drawn from any point on the hyperbola $x^2 - 2y^2 = 2$ to its asymptotes is

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$

SOLUTION

(1) Answer : (c) 8

Here, radius $\sqrt{\left(\frac{1 \quad m}{2}\right)^2 \quad \left(\frac{m}{2}\right)^2 \quad 5} \quad 5$

$2m^2 - 2m - 119 \quad 0$

$\frac{1 \quad \sqrt{239}}{2} \quad m \quad \frac{1 \quad \sqrt{239}}{2}$

$-7.2 \quad m < 8.2$ (apporixemetaly)

$m = -7, -6, ..., 5, 6, 7, 8$

(2) Answer : (c) (25, 29)

The equation of the circleis $x^2 + y^2 - 6x - 10y + \quad = 0$ ... ... (1)

Whose centre is C(3, 5) and radius $r \quad \sqrt{34}$

If the circle does not touch or intersect the x–axis, then radius $r < y$ coordinate of centre C

or $\sqrt{34} \quad 5$

$34 - \quad < 25$

$34 - 25 < \quad > 9$ ... ... (2)

Also, circle doesnot touch

or intersect the y–axis, then the radius $r < x$ – coordinate or centre C

or $\sqrt{34} \quad 3 \quad > 25$

$34 - \quad < 9$ ... ... (3)

If the point (1, 4)) is inside the circle, then its distance from centre C < r (radius)

or $\sqrt{(3 \quad 1)^2 \quad (5 \quad 4)^2} \quad \sqrt{34}$

$5 < 34 - \quad < 29$ ... . (4)

From (2), (3) and (4) are satisfied if 25 < < 29

(3) Answer : (d) $x^2 \quad y^2 \quad 3 \quad 2\sqrt{2}$

$A_1B_1 \quad \sqrt{4 \quad 4} \quad 2\sqrt{2}$

AB $\quad 2\sqrt{2} \quad 2 \quad 2(\sqrt{2} \quad 1)$ Diameter

Thus, equation of the required circle is

$$x^2 + y^2 = (\sqrt{2} \quad 1)^2$$

$$3 \quad 2\sqrt{2}$$

$$x^2 \quad y^2 \quad 3 \quad 2\sqrt{2}$$

(4) Answer : (a) $2 < a < 8$

If d is the distance between the centre of two circles of radii r_1 and r_2, then they intersect in two distinct points, iff $| r_1 - r_2 | < d < r_1 + r_2$

Here, radii of two circles are a and 3 and distance between the centre is 5.

Thus $| a - 3 | < 5 < a + 3 \qquad -2 < a < 8$ and $a > 2$

$$2 < a < 8$$

(5) Answer : (c) $\left(6, \frac{18}{5}\right)$

Let (h, k) be the point of intersection of the tangents. Then the chord of contact of tangents is the common chord of the circle $x^2 + y^2 = 12$ and $x^2 + y^2 - 5x + 3y - 2 = 0$

i.e. $5x - 3y - 10 = 0$ ... ... (1)

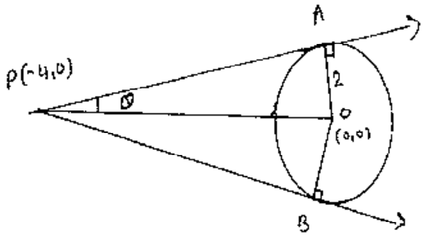
Also, the equation of the chord of contact is $hx + ky - 12 = 0$ ... ... (2)

Equation (1) & (2) represent the same line

$$\frac{h}{5} \quad \frac{k}{\ \ 3} \quad \frac{12}{10} \qquad h \quad 6 \qquad k \quad \frac{18}{5}$$

Hence, the required point is $\left(6, \frac{18}{5}\right)$

(6) Answer : (a) $4\sqrt{3}$



sin $\frac{2}{4}$ $\frac{1}{2}$

So, area of POA $\frac{1}{2}$ 2 4 sin60^{o}

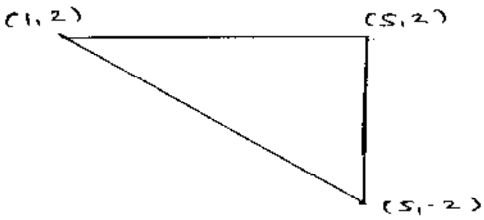
$4\frac{\sqrt{3}}{2}$ $2\sqrt{3}$

area (quadrilatural PAOB) = 2 area of POA

2 $2\sqrt{3}$

$4\sqrt{3}$

(7) Answer : (d) $2\sqrt{2}$



Triangle is right angled triangle

Di*am*eter = length of h*y*potenuse

$\sqrt{16 \quad 16}$

$4\sqrt{2}$

Radius $2\sqrt{2}$

(8) Answer : (d) is any angle

$y = mx + C$ touches the circle, if $C^2 = a^2 (1 + m^2)$

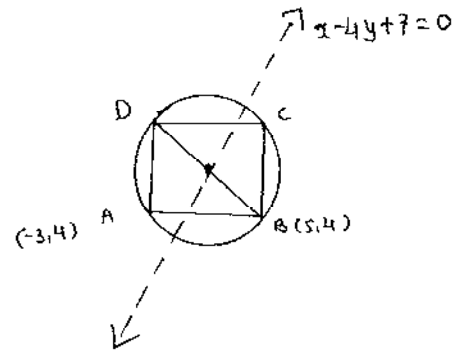
Now, *y*cos = *x*sin – *k*

$y = x\tan \quad - k\sec$

$k^2\sec^2 \quad = k^2 (1 + \tan^2 \ )$

True for all value of

(9) **Answer :** (a) 32 sq. units



First, we note that none of the point A(–3, 4), B(5, 4) lie on the diameter $x - 4y + 7 = 0$

Let E(,) be the centre of the circle, them 4 = + 7 ... ... (i)

Since ABCD is a rectangle

| EA | = | EB | $\quad EA^2 = EB^2$

$(\quad + 3)^2 + (\quad - 4)^2 = (\quad - 5)^2 + (\quad - 4)^2$

6 + 9 = –10 + 25

= 1 = 2 (Putting in Equation (1))

Now | AB | $\sqrt{(5 \quad 3)^2 \quad (4 \quad 4)^2} \quad 8$

and | BD | = 2| EB |

$2\sqrt{(5 \quad 1)^2 \quad (4 \quad 2)^2} \quad 4\sqrt{5}$

From right angle ABD

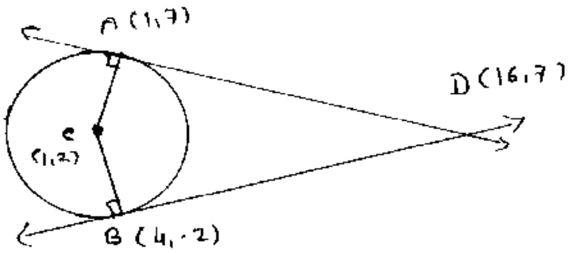
$AD^2 = BD^2 - AB^2 = 80 - 64 = 16$ | AD | = 4

Area of teh rectangle ABCD

= | AB | | AD |

= 8(4) = 32 sq. units

(10) **Answer :** (c) 75 sq. units



The centre of the circle C is (1, 2).

The equations of the tangents to the given circle at the points A and B are

$x(1) + y(7) - (x + 1) - 2\,(y + 7) - 20 = 0$ and

$4x - 2y - (x + 4) - 2(y - 2) - 20 = 0$

$y = 7$ ... ... (i)

and $3x - 4y - 20 = 0$ ... ... (ii)

Solving (i) and (ii)

The point D(16, 7)

Now area of quadrilateral ABCD

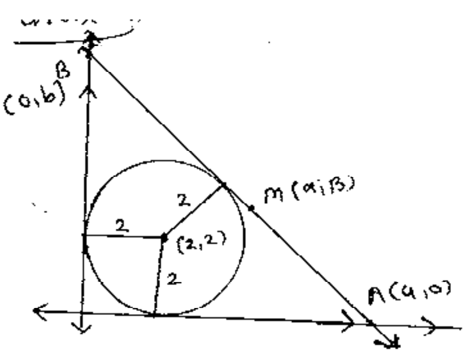
= 2 Area of ACD

$$2\left(\frac{1}{2}\right) \text{ modulus of } \begin{vmatrix} 1 & 2 & 1 \\ 1 & 7 & 1 \\ 16 & 7 & 1 \end{vmatrix}$$

$= |1(7 - 7) - 2\,(1 - 16) + 1\,(7 - 112)|$

$= |-75| = 75$ sq. units

(11) Answer : (b) 1



Given circle is

$x^2 + y^2 - 4x - 4y + 4 = 0$ ... ... (i)

centre (2, 2) and radius = 2

From figure in AOB

Let the equation of AB be $\frac{x}{a} \quad \frac{y}{b} \quad 1$

So that A(a, 0) & B(0, b)

Since AOB = 90^{o}

[AB] is diameter of the circum circle of AOB,

Hence its centre, say M(,), is mid point of

[AB], we have $\frac{a \quad 0}{2}$ and $\frac{0 \quad b}{2}$

$a = 2$ and $b = 2$

Equation of AB becomes $\frac{x}{2} \quad \frac{y}{2} \quad 1$

$x + \quad y - 2 \quad = 0$ ... ... (ii)

As AB touches the circle, (i) we have $\frac{|2 \quad 2 \quad 2 \quad |}{\sqrt{\quad ^2 \quad ^2}} \quad 2$

$| \qquad | \quad \sqrt{\quad ^2 \quad ^2}$

$\sqrt{\quad ^2 \quad ^2}$

locus of M(,) is $x + y - xy \quad \sqrt{x^2 \quad y^2} \quad 0$

$k = \quad 1$

(12) Answer : (d) None of these

Centre of the circle is (1, –2) and radius $\sqrt{1^2 \quad 2^2 \quad 3} \quad \sqrt{2}$. So the sides of the square are $x \quad 1 \quad \sqrt{2}$ and $y \quad 2 \quad \sqrt{2}$. Hence the four corners of the square are $(1 \quad \sqrt{2}, \quad 2 \quad \sqrt{2})$

(13) Answer : (b) $\frac{3}{4}$

Given equation a circle   coefficient of x^2 = coefficient of y^2

$$\frac{\ }{3} \quad \frac{1}{4} \qquad\qquad \frac{3}{4}$$

(14) Answer : (b) Touches only x–axis

Center $\left(\ , \frac{\ }{2} \right)$ and radius $\sqrt{\ ^{2} \ \frac{^{2}}{4} \ \ ^{2}} \quad \frac{\ }{2}$

radius = y co–ordinate of the centre

radius = distance of the centre from the x–axis

circle touches x–axis

Moreover, x co–ordinate of the centre is not (numerically) equal to the radius, therefore, y–axis does not touch the circle

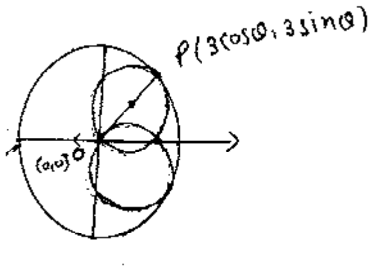
(15) Answer : (a) $g^2 + f^2 = c + k^2$

The given line touches the circle iff the length or perpendicular from $(-g, -f)$ upon the line equals radius of the circle

$$\frac{1 \quad k1}{\sqrt{\cos^2 \quad \sin^2}} \quad \sqrt{g^2 \quad f^2 \quad c}$$

$k^2 = g^2 + f^2 - c \qquad\qquad g^2 + f^2 = c + k^2$

(16) Answer : (d) $\left(\frac{1}{2}, \quad \sqrt{2} \right)$



The centre of the circle passing through the points (0, 0) and (1, 0) has coordinate $\left(\frac{1}{2}, a \right)$ for some real value of a

Also, circle touching $x^2 + y^2 = 9$ must have its centre on a line passing through the origin.

Let P(x, y) be the point of contact of two circles.

$\overline{OP}$ is the diameter of the smallest circle and hence midpoint of OP = centre of the circle

$$\left(\frac{0 \quad x}{2}, \frac{0 \quad y}{2}\right) \quad \left(\frac{1}{2}, a\right)$$

$x = 1$ and $y = 2a$

But (1, 2a) must lies on the circle $x^2 + y^2 = 9$

$1 + 4a^2 = 9 \qquad a^2 = 2 \qquad a = \quad \sqrt{2}$

The required centre are $\left(\frac{1}{2}, \quad \sqrt{2}\right)$

(17) Answer : (b) 1

$x^2 + y^2 = 4$ given c_1 (0, 0) and $r_1 = 2$

Also for circle $x^2 + y^2 - 6x - 8y - 24 = 0$, then $c_2 = (3, 4)$ wad $r_2 = 7$

$c_1c_2 \quad \sqrt{3^2 \quad 4^2} \quad 5$

$r_2 - r_1 = 7 - 2 = 5$

$c_1c_2 = r_2 - r_1$

Given circles touch internally such that they can have just one common tangent at the point of contact.

(18) Answer : (a) $x^2 + y^2 - 2x - 4y - 20 = 0$

We have $| z - z_1 | = 5$

$| z - z_1 |^2 = 25$

$| (x + iy) - (1 + 2i) |^2 = 25$

$| (x - 1) + i(y - 2) |^2 = 25$

$(x - 1)^2 + (y - 2)^2 = 25$

$x^2 + y^2 - 2x - 4y - 20 = 0$

(19) Answer : (d) $\{(x, y) : x^2 = 4y\} \quad \{(0, y) \mid y \quad 0\}$

Let the centre of the circle C be (h, k)

Circle touches X axis radius = $| k |$

Also it touches the given circle $x^2 + (y - 1)^2 = 1$,

centre (0, 1) radius 1, externally

Distance between centres = sum of radii

$\sqrt{(h \quad 0)^2 \quad (k \quad 1)^2} \quad 1 \quad |k|$

$h^2 + k^2 - 2k + 1 = 1 + 2|k| + k^2$

$h^2 = 2k + 2|k|$

locus of (h, k) is $x^2 = 2y + 2|y|$

Now if $y > 0$, it becomes $x^2 = 4y$ and if $y \quad 0$, it becomes $x = 0$

Combining the two, the required locus is $\{(x, y) : x^2 = 4y\} \quad \{(0, y) \mid y \quad 0\}$

(20) Answer : (b) (3, –1)

The equation of the tangent to the circle $x^2 + y^2 = 5$ at the point (1, –2) is

$(1)x + (-2)y = 5 \qquad x - 2y = 5$ ... ... (i)

other circle is $x^2 + y^2 - 8x + 6y + 20 = 0$ ... ... (ii)

Solving (i) and (ii), we get

$(2y + 5)^2 + y^2 - 8(2y + 5) + 6y + 20 = 0$

$5y^2 + 10y + 5 = 0$

$y^2 + 2y + 1 = 0$

$(y + 1)^2 = 0$

$y = -1$

$x = 3$

Hence, the line (i) meet the circle (ii) in two coincident points

Touches the circle (ii) and point of contact is (3, –1)

(21) Answer : (a) only one value of a : $a \quad (0, 1)$

The equation of the circle through (0, 0), (1, 0) and (0, 1) is $x^2 + y^2 - x - y = 0$

Point $(2a, 3a)$ lies on this circle if $(2a)^2 + (3a)^2 - 2a - 3a = 0$

$13a^2 - 5a = 0 \qquad a \quad \frac{5}{13} \quad \because a \quad 0$

(22) Answer : (a) 2

Hint : Equil*a*ter*a*l Tri*a*ngle

(23) **Answer :** (d) (4, 7)

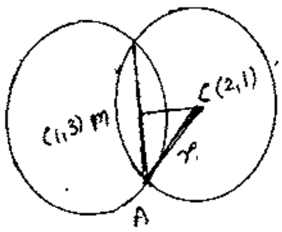
Centre of circle

= mid point of $\overline{AC}$

$\left(\frac{2 \quad 6}{2}, \frac{5 \quad 9}{2}\right)$

= (4, 7)

(24) Answer : (a) 3



Centre of the given circle is (1, 3) and its radius

$\sqrt{1^2 \quad 3^2 \quad 6} \quad 2$

If r is the radius of the other circle, then

$r^2 = AM^2 + MC^2 = 2^2 + 5 = 9$

r = 3

(25) Answer : (c) $x^2 + y^2 - 2x + 2y - 47 = 0$

Centre of the circle is the point of intersection of given line i.e. (1, –1)

Area of a circle = $\quad r^2$

$154 = \frac{22}{7} \quad r^2 \qquad\qquad r^2 \quad \frac{7 \quad 7}{22}$

$r = 7$

centre (1, –1)

Equation circle is $(x - 1)^2 + (y + 1)^2 = 7^2$

$x^2 + y^2 - 2x + 2y - 47 = 0$

(26) Answer : (d) $y = x + 2$

Parabola $y^2 = 8x$

$= 2(4)x \qquad\qquad a = 2$

Any tangent to this parabola is $y = mx + \frac{2}{m} : m \quad 0$ ... ... (1)

This intersect $xy = -1$ where

$x\left(mx \quad \frac{2}{m}\right) \quad 1$

$mx^2 + 2x + m = 0$ ... ... (2)

$A = m^2 : B = 2 : C = m$ line (1) touch $xy = -1$

$= B^2 - 4AC$

$0 = 4 - 4\,m^2 m$

$= 1 - m^3$

$m = 1$

Hence, the required common tangent is $x - y + 2 = 0$

(27) Answer : (b) $\sqrt{2}$

Two parabolas meet in the points (0, 0) and (1, 1). Hence, the length of the common chord

$\sqrt{(1 \quad 0)^2 \quad (1 \quad 0)^2} \quad \sqrt{2}$

(28) Answer : (c) 1

Line $y = a - x$ and parabola $y = x - x^2$

$a - x = x - x^2$

$x^2 - 2x + a = 0$

Since the line touches the parabola, we must have equal roots

$= B^2 - 4AC = (-2)^2 - 4(1)a = 0$

$a = 1$

(29) Answer : (a) 4

Parabola is $y^2 = k\left(x \quad \frac{8}{k}\right)$ OR

$y^2 = 4AX$

Where 4A = k, Y = y, X = $x \quad \frac{8}{k}$

Its directirx is X = –A or $x - \frac{8}{k} \quad \frac{k}{4}$ or $x \quad \frac{8}{k} \quad \frac{k}{4}$

Comparing with $x = 1$, we get $1 \quad \frac{32 \quad k^2}{4k}$

$k^2 + 4k - 32 = 0$

$(k + 8)(k - 4) = 0$

$k = 4$ or $k = -8$

(30) Answer : (d) $4a$

Let point on parabola is $P(at^2, 2at)$

From the definition of the parabola,

We ahve SP = PM = $a + at^2$

From the question point M is $(-a, 2at)$

SPM is an equilateral triangle

$SP = PM = SM$

$SP^2 = PM^2$

$4a^2 + 4a^2t^2 = (a + at^2)^2$

$4a^2 + 4a^2t^2 = a^2 + 2a^2t^2 + a^2t^4$

$4 + 4t^2 = 1 + 2t^2 + t^4$

$t^4 - 2t^2 - 3 = 0$

$(t^2 - 3)(t^2 + 1) = 0$

$t^2 = 3 \qquad t \qquad \sqrt{3}$

$SP = a + 3a$

$SP = 4a$

(31) Answer : (c) $t_2 + 2t_1 = 0$

$$C\left(\frac{2at_1^2 \quad at_2^2}{3}, \frac{4at_1 \quad 2at_2}{3}\right)$$

It lies on $y = 0$

$$\frac{4at_1 \quad 2at_2}{3} \quad 0$$

$t_2 + 2t_1 = 0$

(32) Answer : (c) $x + y + b = 0$

Equation of tangent to $y^2 = 4by$ having slope m is $y = mx + \frac{b}{m}$

It will touch $x^2 = 4by$

$x^2 = 4b\left(mx \quad \frac{b}{m}\right)$ has equal roots.

$m = -1$

Thus, common tangent is $x + y + b = 0$

(33) Answer : (a) $\frac{\ }{2}$

The line $y = x - 1$ passes through (1, 0), hence, it is focal chord

Angle between tangent is $\frac{\ }{2}$

(34) Answer : (b) $\frac{\quad}{3}$

Tangent to parabola $y^2 = 4x$ having slope m is $y = mx + \frac{1}{m}$

above tangent passes through (1, 4)

$$4 = m + \frac{1}{m}$$

$$m^2 - 4m + 1 = 0$$

Now, angle between the lines is given by

tan $\left|\frac{m_1 \quad m_1}{1 \quad m_1 m_2}\right|$

$$\frac{\sqrt{(m_1 \quad m_2)^2 \quad 4m_1 m_2}}{1 \quad m_1 m_2}$$

$$= \frac{\sqrt{16-4}}{1+1} = \sqrt{3} \qquad \therefore\ \theta = \frac{\pi}{3}$$

(35) Answer : (c) $\frac{3\sqrt{2}}{8}$

$$1 \quad 2y\frac{dy}{dx} \quad \frac{dy}{dx} \quad \frac{1}{2y} = \text{slope of given line } x - y + 1 = 0$$

$$\frac{1}{2y} \quad 1 \quad y \quad \frac{1}{2} \qquad x \quad \left(\frac{1}{2}\right)^2 \quad \frac{1}{4}$$

$$(x, y) \quad \left(\frac{1}{4}, \frac{1}{2}\right)$$

Shortest distance is $\frac{\left|\frac{1}{4} \quad \frac{1}{2} \quad 1\right|}{\sqrt{1 \quad 1}} \quad \frac{3}{4\sqrt{2}} \quad \frac{3\sqrt{2}}{8}$

(36) Answer : (c) $y^2 - 4x + 2 = 0$

Let R(h, k) be the mid point or PQ

Q($2h - 1$, $2k$)

Since Q lies on $y^2 = 8x$

$(2k)^2 = 8(2h - 1)$

$4k^2 = 16h - 8$

Hence, locus of Q(h, k) is $y^2 = 2(2x - 1)$

or $y^2 = 4x - 2 \qquad y^2 - 4x + 2 = 0$

(37) Answer : (d) $t_1 + t_2 = 0$

tangent at $(at_1^2, 2at_1)$ is $x - t_1y + at_1^2 = 0$ ... ... (1)

tangent at $(at_2^2, 2at_2)$ is $x - t_2y + at_2^2 = 0$ ... ... (2)

intersection point $(at_1t_2, a(t_1 + t_2))$ $\quad x$–$ax$is

$t_1 + t_2 = 0$

(38) Answer : (c) $\left(4, \frac{9}{2}\right)$

Given parabola is $x^2 - 8x + 2y + 7 = 0$

$(x - 4)^2 = -2y - 7 + 16$

$(x - 4)^2 = -2\left(y \quad \frac{9}{2}\right)$

$X^2 = -4aY$

$4a = 2 \qquad a \quad \frac{1}{2}$

$X = x - 4, \ Y = y - \frac{9}{2}$

Its focus is given by $X = 0$ $Y = -a$

i.e. $x - 4 = 0$

$y = \frac{9}{2}$

$\left(4, \frac{9}{2}\right)$

(39) Answer : (a) (–1, 0)

Given parabola is $y^2 = 4x$, here $a = 1$,

End points of latus rectum are L(1, 2) and L'(1, –2)

Equation of tangents to the given parabola at L and L' are

$2y = 2(x + 1)$ and $y(-2) = 2(x + 1)$

i.e. $x - y + 1 = 0$ and $x + y + 1 = 0$

Point of intersection of these points is (–1, 0)

(40) Answer : (a) (0, 1)

Given curve is $y^2 - y + x = 0$ ... ... (1)

Given line is $y = 1 - x$ ... ... (2)

Eliminating y between (1) and (2), we get $(1 - x)^2 - (1 - x) + x = 0$

or $x^2 = 0 \quad x = 0$

Substituting $x = 0$ in (2) we get $y = 1 - 0 = 1$

Required point of contact is (0, 1)

(41) Answer : (d)

A line parallel to the axis of the parabola cannot be a tangent to the parabola

(42) Answer : (d) None of these

The vertex of the given parabola is at (b, b)

(43) Answer : (c) $3y - 2 = 0$

$9y^2 - 16x - 12y - 57 = 0$

$$9\left(y^2 \quad \frac{12}{9}y\right) \quad 16x \quad 27$$

$$9\left(y \quad \frac{2}{3}\right)^2 \quad 16x \quad 27$$

$$\left(y \quad \frac{2}{3}\right)^2 \quad \frac{16}{9}\left(x \quad \frac{61}{16}\right)$$

Its axis is giveny by $y \quad \frac{2}{3} \quad 0$ (Right hand parabola)

(44) Answer : (d) $a\left(t \quad \frac{1}{t}\right)^2$

If the other and of the chord is $Q(at_1^2, 2at_1)$ then $tt_1 = -1 \quad t_1 = \quad \frac{1}{t}$

Length of chord = | PQ |

$$\sqrt{(at_1^2 \quad at^2)^2 \quad (2at_1 \quad 2at)^2}$$

$$\sqrt{a^2\left(\frac{1}{t^2} \quad t^2\right)^2 \quad 4a^2\left(\frac{1}{t} \quad t\right)^2}$$

$$a\sqrt{\left(t \quad \frac{1}{t}\right)^2\left(t \quad \frac{1}{t}\right)^2 \quad 4\left(t \quad \frac{1}{t}\right)^2}$$

$$a\left(t \quad \frac{1}{t}\right)\sqrt{\left(t \quad \frac{1}{t}\right)^2 \quad 4}$$

$$a\left(t \quad \frac{1}{t}\right)^2$$

(45) Answer : (d) *A*ll of these

(46) Answer : (b) $\frac{9}{4}$ & (c) $\frac{1}{4}$

$$y^2 \quad 9x \quad 4\left(\frac{9}{4}\right)x \qquad a \quad \frac{9}{4}$$

Equation of tangent is $y = mx = \dfrac{\frac{9}{4}}{m}$ p*a*sses through (4, 10)

$$10 = 4m + \frac{9}{4m}$$

$$16m^2 - 40m + 9 = 0$$

$$16m^2 - 36m - 4m + 9 = 0$$

$$4m(4m - 9) - 1\,(4m - 9) = 0$$

$$(4m - 9)\,(4m - 1) = 0$$

$$m \quad \frac{9}{4} \quad \text{or} \quad m \quad \frac{1}{4}$$

(47) Answer : (d) 1

Given line is $y = mx + 1$ ... ... (1)

Given parabola is $y^2 = 4x$ ... ... (2)

Equation of tangents to this parabola with slope m is

$$y = mx + \frac{1}{m} \quad \dots\dots\dots (2) \quad y \quad mx \quad \frac{a}{m}$$

$$1 \quad \frac{1}{m} \quad m \quad 1$$

(48) Answer : (a) $\frac{1}{a}$

Focus of the parabola is F(a, 0).

Let P(at_1^2, $2at_1$) and Q(at_2^2, $2at_2$)

The equation of the chord PQ is

$$y - 2at_2 \quad \frac{2a(t_2 \quad t_1)}{a(t_2^2 \quad t_1^2)} (x \quad at_1^2)$$

$$\frac{2}{t_1 \quad t_2} (x \quad at_1^2)$$

Since F(a, 0) lies on it,

$$0 - 2at_1 = \frac{2}{t_1 \quad t_2} (a - at_1^2)$$

$$t_1 t_2 = -1$$

Hence $\frac{1}{|FP|} \quad \frac{1}{|FQ|} \quad \frac{1}{a(1 \quad t_1^2)} \quad \frac{1}{a(1 \quad t_2)^2}$

$$\frac{1}{a} \frac{(1 \quad t_2^2 \quad 1 \quad t_1^2)}{(1 \quad t_1^2 \quad t_2^2 \quad t_1^2 \quad t_2^2)} \quad \frac{1}{a}$$

(49) Answer : (b) $4x + 3y + 1 = 0$

Required equation is

$(-3)y - 4(x + 2) = (-3)^2 - 8(2)$

$-3y - 4x - 8 = 9 - 16$

$4x + 3y + 1 = 0$

(50) Answer : (c) 4

$$y^2 \quad ax \quad 4\left(\frac{a}{4}\right)x$$

$y = mx + \frac{\frac{a}{4}}{m}$ $\qquad$ $y = mx + \frac{a}{4m}$ $\qquad$ ... ... (1)

$x + y + 1 = 0$ ... ... (2) are same line

$y = -x - 1$

$m \quad 1 \; ; \; \frac{a}{4m} \quad 1 \quad a \quad 4$

(51) Answer : (a) $\left| \frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1) \right|$

Let x_1, x_2, x_3 be the abscissae of the points on the parabola whose ordinates are y_1, y_2 and y_3 respe.

Then $y_1^2 = 4ax_1$, $y_2^2 = 4ax_2$ and $y_3^2 = 4ax_3$.

Area of the triangle whose vertices are (x_1, y_1), (x_2, y_2) & (x_3, y_3) is

$$\Delta = \frac{1}{2}|D|$$

$$D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} \frac{y_1^2}{4a} & y_1 & 1 \\ \frac{y_2^2}{4a} & y_2 & 1 \\ \frac{y_3^2}{4a} & y_3 & 1 \end{vmatrix} = \frac{1}{4a}\begin{vmatrix} y_1^2 & y_1 & 1 \\ y_2^2 & y_2 & 1 \\ y_3^2 & y_3 & 1 \end{vmatrix}$$

$$\Delta = \left| \frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1) \right|$$

(52) Answer : (a) (1, 1)

Centre is given by point of intersection of lines

$x - y - 2 = 0$ and $x - y = 0$ which is (1, 1)

(53) Answer : (a) P lies inside C but outside E

Since $1^2 + 2^2 - 9 < 0$ and $2^2 + 1^2 - 9 < 0$, both P and Q lie inside C.

Also $\frac{1^2}{9} \quad \frac{2^2}{4} \quad 1 \quad 0$ and $\frac{2^3}{9} \quad \frac{1}{4} \qquad 0$,

P lies outside E and Q lies inside E.

Thus, P lies inside C but outside E

(54) Answer : (d) $x^2 + 12y^2 = 16$

$x^2 + 4y^2 = 4$

$$\frac{x^2}{4} \quad \frac{y^2}{1} \quad 1$$

$a = 2,\ b = 1 \qquad P(2,\ 1)$

Required ellipse is $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1 \qquad \frac{x^2}{4} \quad \frac{y^2}{b^2} \quad 1$

(2, 1) lies on it.

the point

$$\frac{4}{16} + \frac{1}{b^2} = 1 \Rightarrow \frac{1}{b^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$b^2 \quad \frac{4}{3} \qquad \frac{x^2}{16} \quad \frac{y^2}{\left(\frac{4}{3}\right)} \quad 1 \qquad x^2 \quad 12y^2 \quad 16$$

(55) **Answer :** (c) An ellipse

Equation of chord of ellipse whose mid point is $(h,\ k)$ is

$$\frac{hx}{a^2} \quad \frac{ky}{b^2} \quad 1 \quad \frac{h^2}{a^2} \quad \frac{k^2}{b^2} \quad 1 \text{ (using } T = S_1\text{)}$$

This passes through $(0,\ b)$ $\qquad \frac{k}{b} \quad \frac{h^2}{a^2} \quad \frac{k^2}{b^2}$

Hence, the locus of $(h,\ k)$ is $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad \frac{4}{5}$ which is an ellipse

(56) **Answer :** (b) $5 \quad \frac{4}{5}y_1$

Comparing to given ellipse to $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$, we have $a = 3$ and $b = 5$. Thus $a < b$, So the major axis is y–axis and two foci lie on y–axis and their co–ordinates are $(0, \quad be)$

Now $e \quad \sqrt{1 \quad \frac{a^2}{b^2}} \quad \sqrt{1 \quad \frac{9}{25}} \quad \frac{4}{5}$.

The focal distance of a point (x_1, y_1) are given $b \quad ey_1 = 5 \quad \frac{4}{5} y_1$

(57) Answer : (a) $\frac{74}{9}$

We known that $\frac{1}{SP} \quad \frac{1}{SQ} \quad \frac{2a}{b^2}$

$$\frac{1}{16} + \frac{1}{SQ} = 2\left(\frac{5}{16}\right) \quad \therefore \quad \frac{1}{SQ} = \frac{5}{8} - \frac{1}{16} = \frac{9}{16}$$

SQ $\frac{16}{9}$ Now SQ + SQ' = $2a$ = 10

SQ' 10 $\frac{16}{9} \quad \frac{74}{9}$

(58) Answer : (c) 27

$$\frac{x^2}{9} \quad \frac{y^2}{5} \quad 1 \qquad e^2 \quad 1 \quad \frac{5}{9} \quad \frac{4}{9}$$

$$e \quad \frac{2}{3}$$

Equation of tangent at $\left(2, \frac{5}{3}\right)$ is $\frac{2x}{9} \quad \frac{y}{3} \quad 1$

F and F' be foci

Area of CPQ = $\frac{1}{2} \quad \frac{9}{2} \quad 3 \quad \frac{27}{4}$

□ Area of quadrilateral PQRS 4 $\frac{27}{4}$ 27

(59) Answer : (d) $\frac{1}{e}$

$$\frac{\text{Area of} \quad \text{APA'}}{\text{Area of} \quad \text{PSS'}} \quad \frac{\frac{1}{2}(\text{AA}')(b\sin \quad)}{\frac{1}{2}(\text{SS}')(b\sin \quad)}$$

$$\frac{2a}{2ae} \quad \frac{1}{e}$$

(60) Answer : (c) $\frac{8}{3}$

Major axis is along X–axis

$$\frac{a}{e} \quad ae \quad 4$$

$$a\left(2 \quad \frac{1}{2}\right) \quad 4 \quad a \quad \frac{8}{3}$$

(61) Answer : (d) None of these

Given that $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1$ as $r > 1$

$1 - r < 1$ and $1 + r > 0$

Let $1 - r = -a^2$, $1 + r = b^2$ then we get

$$\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \qquad \frac{x^2}{a^2} \quad \frac{y^2}{b^2} \qquad 1$$

Which is not possible for any values of x and y.

(62) Answer : (d) $be\sqrt{a^2 \quad m^2}$

Since (m, n) lies on an ellipse

$$\frac{m^2}{a^2} \quad \frac{n^2}{b^2} \quad 1$$

$$n = b\sqrt{1 - \frac{m^2}{a^2}}$$

Area of SPS' = $\frac{1}{2}n$ (SS') = $\frac{1}{2}n(2ae)$

$$bae\sqrt{1 \quad \frac{m^2}{a^2}} \quad be\sqrt{a^2 \quad m^2}$$

(63) Answer : (b) $a - ex_1$

$$\frac{PS}{PM} \quad e$$

$$PS \quad e\left|\frac{a}{e} \quad x_1\right| \quad a \quad ex_1$$

(64) Answer : (d) $\frac{}{6}$

Equation of tangents is $\frac{x}{a}\frac{\sqrt{3}}{2} \quad \frac{y}{6}\cdot\frac{1}{2} \quad 1$ and equation of tangent at the point ($a\cos$, $b\sin$)

is $\frac{x}{a}\cos \quad + \frac{y}{b}\sin \quad = 1$. Both are same

$$\cos \quad = \frac{\sqrt{3}}{2} \ \& \ \sin \quad = \frac{1}{2} \qquad \frac{}{6}$$

(65) Answer : (a)

$PS + PS' = 2a = 2\sqrt{20} \quad 4\sqrt{5}$ (Here major axis of an ellipse is along y–axis)

(66) Answer : (c) Depends on whether $a > b$ or $a < b$

(67) Answer : (c) ellipse

$\frac{x}{3} = \cos t + \sin t$ and $\frac{y}{4} = \cos t - \sin t$

$$\frac{x^2}{9} \quad \frac{y^2}{16} \quad 2$$

(68) Answer : (d) None of these

The two curves do not intersect each other

(69) Answer : (a) $\frac{1}{2}$

$\tan 60^o = \frac{OB}{OS}$

$\sqrt{3} \quad \frac{b}{ae}$ Now $e^2 \quad 1 \quad \frac{b^2}{a^2}$ $4e^2 = 1$

$\frac{b}{a} \quad \sqrt{3}e$ $= 1 - 3e^2$ $e \quad \frac{1}{2}$ ($\because 0 \quad e \quad 1$)

(70) Answer : (c) 2

Let the points of intersection of the line and an ellipse be ($a$cos , $b$sin) and

$$\left(a\cos\left(\frac{\ }{2}\right), b\sin\left(\frac{\ }{2}\right)\right)$$

Since they lie on the given line $lx + my + n = 0$.

lacos + mbsin + n = 0 and

– lasin + mbcos + n = 0 squaring and adding

we get $a^2l^2 + b^2m^2 = 2n^2$

$$\frac{a^2l^2 \quad b^2m^2}{n^2} \quad 2$$

(71) Answer : (b) $2ab$

Let PQRS bear rectangle,

Where P is ($a$cos , $b$sin)

Area of rectangle

= 4 cos · sin

= $2ab$ sin2

= $2ab$ (∵ This is *maximum* when sin2 = 1)

(72) Answer : (c) a point if $k = 0$

$2x^2 + 3y^2 - 8x - 18y + 35 = k$

$2(x^2 - 4x) + 3(y^2 - 6y) + 35 = k$

$2(x - 2)^2 + 3(y - 3)^2 = k$

For $k = 0$, we get $2(x - 2)^2 + 3(y - 3)^2 = 0$

Which represents the point (2, 3)

(73) Answer : (b) 45^o

Let be the eccentric angle of the point of contact then tangent at is $\frac{x\cos}{a} \quad \frac{y\sin}{b} \quad 1$.

Also $\frac{x}{\sqrt{2}} \quad \frac{y}{\sqrt{2}} \quad 1$ is the tangent

$$\frac{\cos}{\left(\frac{1}{\sqrt{2}}\right)} \quad \frac{\sin}{\left(\frac{1}{\sqrt{2}}\right)} \quad 1$$

$\cos \quad \frac{1}{\sqrt{2}}$ & $\sin \quad \frac{1}{\sqrt{2}}$

$= 45^{o}$

(74) Answer : (b) $3x^2 + 4y^2 = 12$

$\frac{x^2}{4} \quad \frac{y^2}{3} \quad 1$

$e \quad \frac{1}{2}$ and $x \quad \frac{a}{e} \quad \frac{a}{\frac{1}{2}} \quad 4 \quad a \quad 2 \qquad a^2 = 4$

Now $b^2 = a^2 (1 - e^2) = 4 \left(1 \quad \frac{1}{4}\right) \quad 3$

Equations of an ellipse is $\frac{x^2}{4} \quad \frac{y^2}{3} \quad 1$

$3x^2 + 4y^2 = 12$

(75) Answer : (a) 4

The given ellipse is $\frac{x^2}{16} \quad \frac{y^2}{9} \quad 1$

Here $a^2 = 16$, $b^2 = 9$

$b^2 = a^2 (1 - e^2) \quad \frac{9}{16} \quad 1 - e^2 \quad e \quad \frac{\sqrt{7}}{4}$

Foci are $(\quad \sqrt{7}, 0)$

Radius of the circle = Distance between $(\quad \sqrt{7}, 0)$

and (0, 3) $\sqrt{(\quad \sqrt{7}, 0)^2 \quad (0 \quad 3)^2} \quad \sqrt{7 \quad 9} \quad 4$

(76) Answer : (a) $y = 4(x - 1)$

If $y = mx + C$ is tangent to $y = x^2$ then $x^2 - mx - C = 0$ has equal roots

$m^2 + 4C = 0 \quad C \quad \frac{m^2}{4} \qquad = B^2 - 4AC$

$y = mx - \frac{m^2}{4}$ is tangent to $y = x^2$ $\quad\quad 0 = m^2 = 4C$

This is also tangent to $y = -(x-2)^2$

$$mx - \frac{m^2}{4} = -x^2 + 4x - 4$$

$x^2 + (m-4)x + \left(4 \quad \frac{m^2}{4}\right) \quad 0$ has equal roots

$(m-4)^2 - 4(1)\left(4 \quad \frac{m^2}{4}\right) \quad 0$ $\quad\quad m = 0 : 4$

$m^2 - 8m + 16 - 16 + m^2 = 0$ $\quad\quad y = 0$ or $y = 4x - 4$ are the tangents.

$2m^2 - 8m = 0 \quad 2m\,(m-4) = 0$

(77) Answer : (b) 1

For hyperbola $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$, $b^2 = a^2\,(e_1{}^2 - 1)$

$$e_1{}^2 \quad 1 \quad \frac{b^2}{a^2} \quad \frac{a^2 \quad b^2}{a^2}$$

For conjugate hyperbola $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad\quad 1$

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

$$e_2{}^2 = 1 + \frac{a^2}{b^2} = \frac{a^2 + b^2}{b^2}$$

$$\frac{1}{e_1{}^2} \quad \frac{1}{e_2{}^2} \quad \frac{a^2 \quad b^2}{a^2 \quad b^2} \quad 1$$

(78) Answer : (a) $x^2\text{cosec}^2 \quad - y^2\sec^2 \quad = 1$

The length of transverse axis is = 2sin $\quad = 2a$

$a = \sin$

Also for ellipse $3x^2 + 4y^2 = 12$ or $\frac{x^2}{4} \quad \frac{y^2}{3} \quad 1$

$a^2 = 4$ and $b^2 = 3$

$$e \quad \sqrt{1 \quad \frac{b^2}{a^2}} \quad \sqrt{1 \quad \frac{3}{4}} \quad \frac{1}{2}$$

Focus of ellipse $\left(2 \quad \frac{1}{2}, 0\right) \quad (1, 0)$

As hyperbola is confocal with ellipse, focus of hyperbola = (1, 0)

$ae = 1 \quad \sin \quad e = 1 \quad e = \text{cosec}$

$$b^2 = a^2(e^2 - 1)$$
$$= \sin^2 \quad (\text{cosec}^2 \quad - 1)$$
$$= \cos^2$$

Equation hyperbola is $\frac{x^2}{\sin^2} \quad \frac{y^2}{\cos^2} \quad 1$

$x^2\text{cosec}^2 \quad - y^2\sec^2 \quad = 1$

(79) Answer : (d) hyperbola

$y = \alpha x + \beta$ touches $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$

if $\quad ^2 = a^2 \quad ^2 - b^2$

locus of (,) is $y^2 = a^2x^2 - b^2$

$a^2x^2 - y^2 = b^2$

$$\frac{x^2}{\left(\frac{b^2}{a^2}\right)} \quad \frac{y^2}{b^2} \quad 1, \text{ which is a hyperbola}$$

(80) Answer : (a) $\frac{1 \quad e}{1 \quad e}$

The equation of the chord joining ($a$sec , $b$tan) and ($a$sec , $b$tan) is

$$\frac{x}{a}\cos\left(\frac{}{2}\right) \quad \frac{y}{b}\sin\left(\frac{}{2}\right) \quad \cos\left(\frac{}{2}\right)$$

This passes through (ae, 0)

$$e\cos\left(\frac{\quad}{2}\right) \quad \cos\left(\frac{\quad}{2}\right)$$

$$e \quad \frac{\cos\left(\frac{\quad}{2}\right)}{\cos\left(\frac{\quad}{2}\right)}$$

$$\frac{1 \quad e}{1 \quad e} \quad \frac{\cos\left(\frac{\quad}{2}\right) \quad \cos\left(\frac{\quad}{2}\right)}{\cos\left(\frac{\quad}{2}\right) \quad \cos\left(\frac{\quad}{2}\right)}$$

$$\tan\frac{\quad}{2} \tan\frac{\quad}{2}$$

$$\tan\frac{\quad}{2} \tan\frac{\quad}{2} \quad \frac{1 \quad e}{1 \quad e}$$

(81) Answer : (d) $e \quad \frac{2}{\sqrt{3}}$

Let the hyperbola be $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ and any double ordinate A, B be ($a$sec , $b$tan) and ($a$sec , –$b$tan) respe and O is centre (0, 0).

OAB being equilateral

$$\tan 30^{o} \quad \frac{b\tan}{a\sec} \quad \frac{b}{a}\sqrt{3} \quad \text{cosec}$$

$$3\frac{b^2}{a^2} \quad \text{cosec}^2$$

$3(e^2 - 1) = \text{cosec}^2 \quad 1$

$3(e^2 - 1) \quad 1 \qquad (\because \text{cosec}^2 \quad 1)$

$$e^2 \quad \frac{4}{3} \qquad e \quad \frac{2}{\sqrt{3}}$$

(82) Answer : (a) $\sqrt{\frac{17}{20}}$

$y = mx + 6$ touches the hyperbola

$\frac{x^2}{100} \quad \frac{y^2}{49} \quad 1$ only if $6 \quad \sqrt{100\,m^2 \quad 49}$

$m^2 \quad \frac{36 \quad 49}{100}$ $\qquad \left(\because y \quad mx \quad \sqrt{a^2m^2 \quad b^2}\right)$

$m \quad \sqrt{\frac{85}{100}} \quad \sqrt{\frac{17}{20}}$

(83) Answer : (d) (6, 3), (–2, 3)

$9(x^2 - 4x + 4) - 16(y^2 - 6y + 9) = 252 + 36 - 144$

$9(x - 2)^2 - 16(y - 3)^2 = 144$

$\frac{(x \quad 2)^2}{16} \quad \frac{(y \quad 3)^2}{9} \quad 1$ OR $\frac{X^2}{A^2} \quad \frac{Y^2}{B^2} \quad 1$

$x = 2 = \quad 4$ & $y - 3 = 0$

$x = 6, -2$ and $y = 3$

Vertices are (6, 3), (–2, 3)

(84) Answer : (c) Abscissa of foci

$e^2 \quad 1 \quad \frac{b^2}{a^2} \quad 1 \quad \frac{\sin^2}{\cos^2} \quad \sec^2$

$a^2e^2 = \cos^2 \quad \sec^2 \quad = 1$

Foci (ae, 0) = (1, 0) which is independent of

(85) Answer : (c) $30x \quad 24y \quad \sqrt{161} \quad 0$

Let m be the slope of the tangent to $4x^2 - 9x^2 = 1$

Then m = (slope of the line $5x - 4y + 7 = 0$) = $\frac{5}{4}$

We have $\frac{x^2}{\frac{1}{4}} \quad \frac{y^2}{\frac{1}{9}} \quad 1$ OR $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$

The equations of the tangents are $a^2 \quad \frac{1}{4}$ & $b^2 \quad \frac{1}{9}$

OR $y \quad \frac{5x}{4} \quad \sqrt{\left(\frac{5}{8}\right)^2 \quad \frac{1}{9}}$

$$\frac{5x}{4} \quad \frac{\sqrt{225 \quad 64}}{8(3)}$$

$$y \quad \frac{5x}{4} \quad \frac{\sqrt{161}}{24} \qquad\qquad 24y \quad 30x \quad \sqrt{161}$$

$$30x \quad 24y \quad \sqrt{161} \quad 0$$

(86) Answer : (d) a hyperbola

Let equation of the lines be $y = m_1(x - a)$ and $y = m_2(x - \text{a})$ $\quad m_1m_2 = \text{P}$

$y^2 = m_1m_2\,(x^2 - a^2) = \text{P}(x^2 - a^2)$

Hence, locus of points of intersection is $y^2 = \text{P}(x^2 - a^2)$

or $\text{P}x^2 - y^2 = \text{P}a^2$ which is hyperbola

(87) Answer : (a) $y \quad x \quad \sqrt{a^2 \quad b^2}$

Tangent to $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ is $y \quad m_1x \quad \sqrt{a^2m_1{}^2 \quad b^2}$ ... ... (1)

The other hyperbola $\frac{x^2}{(\quad b)^2} \quad \frac{y^2}{(\quad a^2)} \quad 1$, then any tangent to it is

$y \quad m_2x \quad \sqrt{(\quad b^2)m_2{}^2 \quad (\quad a^2)}$ ... ... (2)

If (1) and (2) are same, then $m_1 = m_2$ and $a^2m_1{}^2 - b^2 = -b^2m_2{}^2 + a^2$

$a^2m_1{}^2 + b^2m_1{}^2 = a^2 + b^2$

$m_1{}^2 = 1$

$m_1 = \quad 1$

(88) Answer : (a) $\left(4, \quad \sqrt{6}\right)$

Equation of tangent to hyperbola $x^2 - 2y^2 = 4$ at any point (x_1, y_1) is $xx_1 - 2yy_1 = 4$

Comparing with $2x \quad \sqrt{6}y \quad 2$ or $4x \quad 2\sqrt{6}y \quad 4$

$x_1 = 4$ and $\quad 2y_1 \quad 2\sqrt{6}$

$\left(4, \quad \sqrt{6}\right)$ is the required point of contact

(89) Answer : (a) $y \quad 3\sqrt{\frac{2}{7}}x \quad \frac{15}{\sqrt{7}}$

$$\frac{x^2}{16} \quad \frac{y^2}{9} \quad 1$$

Equation of tangent to hyperbola having slope *m* is

$$y \quad mx \quad \sqrt{16m^2 \quad 9}$$

It touches the circle Distance of this line from centre of the circle is radius of the circle

$$\frac{\sqrt{16m^2 \quad 9}}{\sqrt{m^2 \quad 1}} \quad 3$$

$$7m^2 = 18$$

$$m \quad 3\sqrt{\frac{2}{7}}$$

Equation of tangents is $y \quad 3\sqrt{\frac{2}{7}}x \quad \frac{15}{\sqrt{7}}$

(90) Answer : (a) (6, –3)

P is nearest to given line if tangent at P is parallel to given line. Now slope of tangent at $P(x_1, y_1)$ is $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} \quad \frac{18x_1}{24y_1} \quad \frac{3x_1}{4y_1}$ which must be equal to $\frac{3}{2}$

$x_1 = -2y_1$ ... ... (1)

Also (x_1, y_1) lies on the curve

$$\frac{x_1^2}{24} \quad \frac{y_1^2}{18} \quad 1 \quad \text{... ... ... (2)}$$

Solving (1) and (2), we get two points (6, –3) and (–6, 3) of which (6, –3) is nearest

(91) Answer : (c) $x \quad \sqrt{3}y \quad 3 \quad 0$

Let at point (x_1, y_1) of parabola $y^2 = 4x$ equation of tangent is

$yy_1 = 2(x + x_1) = 2x - yy_1 + 2x_1 = 0$ ... ... (1)

As it is tangent to the circle $(x - 3)^2 + y^2 = 9$

length of from (3, 0) to equation (1) is 3

$$\left|\frac{6 \quad 2x_1}{\sqrt{4 \quad y_1^2}}\right| \quad 3$$

$36 + 24x_1 + 4x_1^2 = 36 + 9y_1^2$

$9y_1^2 = 4x_1^2 + 24x_1$

Also $y_1^2 = 4x_1$

$9y_1^2 = 36x_1$

$4x_1^2 + 24x_1 = 36x_1$

$4x_1^2 - 12x_1 = 0$

$4x_1 (x_1 - 3) = 0$

$x_1 = 0 \; ; \; 3$

$x_1 = 0 \quad y_1 = 0$ and

$x_1 = 3 \quad y_1 = \quad 2\sqrt{3}$

Equation is $2x - 2\sqrt{3}y + 6 = 0$

(92) Answer : (a) $\dfrac{2b}{\sqrt{a^2 \quad 4b^2}}$

Since both the circles have same radius, tangent pass through the mid point of the centres of the circles, which is $\left(\dfrac{a}{2}, 0\right)$.

Hence $m \quad \dfrac{2b}{\sqrt{a^2 \quad 4b^2}}$

(93) Answer : (b) $9x^2 - 8y^2 - 18x + 9 = 0$

Let a pair of tangents be drawn from point (x_1, y_1) to hyperbola $x^2 - y^2 = 9$

Then chord of contact will be $xx_1 - yy_1 = 9$ ... ... (1)

But given chord of contact is $x = 9$ ... ... (2)

As equations (1) and (2) represent same line, these equations should be identical and hence

$\dfrac{x_1}{1} \quad \dfrac{y_1}{0} \quad \dfrac{9}{9} \qquad x_1 = 1, y_1 = 0$

Equation of pair of tangents drawn from (1, 0) to $x^2 - y^2 = 9$ is

$(x^2 - y^2 - 9)(1^2 - 0^2 - 9) = (1x - 0y - 9)^2$ (using $SS_1 = T^2$)

$(x^2 - y^2 - 9)(-8) = (x - 9)^2$

$-8x^2 + 8y^2 + 72 = x^2 - 18x + 81$

$9x^2 - 8y^2 - 18x + 9 = 0$

(94) Answer : (a) $\dfrac{9}{2}$

Hyperbola $9x^2 - 16y^2 - 18x - 32y - 151 = 0$ can be written as

$9(x^2 - 2x) - 16(y^2 + 2y) = 151$

$9(x - 1)^2 - 16(y + 1)^2 = 151 + 9 - 16 = 144$

$$\frac{(x \quad 1)^2}{16} \quad \frac{(y \quad 1)^2}{9} \quad 1 \text{ OR } \frac{X^2}{16} \quad \frac{Y^2}{9} \quad 1$$

Here $a^2 = 16$, $b^2 = 9$ (where $X = x - 1$ & $Y = y + 1$)

Latus rectum $\quad 2\frac{b^2}{a} \quad \frac{2(9)}{4} \quad \frac{9}{2}$

(95) Answer : (a) $xy \quad \frac{105}{64}$

The family of parabolas is $y \quad \frac{a^3x^2}{3} \quad \frac{a^2x}{2} \quad 2a$

$$\frac{y}{\frac{a^3}{3}} = x^2 + \frac{a^2}{2}\frac{3}{a^3}x - \frac{2a}{\frac{a^3}{3}}$$

$$\frac{3y}{a^3} \quad \frac{6a}{a^3} \quad x^2 \quad 2\left(\frac{3}{4a}\right)x \quad \frac{9}{16a^2} \quad \frac{9}{16a^2}$$

$$\frac{3y}{a^3} \quad \frac{6}{a^2} \quad \frac{9}{16a^2} \quad \left(x \quad \frac{3}{4a}\right)^2$$

$$\left(x \quad \frac{3}{4a}\right)^2 \quad \frac{3y}{a^3} \quad \frac{105}{16a^2} \quad \frac{3}{a^3}\left(y \quad \frac{35}{16}a\right)$$

If (,) be the vertex then $\alpha = \frac{-3}{4a}$ & $\beta = \frac{-35}{16}a$

$$\frac{105}{64}$$

Locus of (,) is $xy \quad \frac{105}{64}$

(96) Answer : (a) $\frac{\quad}{4}$

The angle between the lines represented by

$\sqrt{3}x^2 \quad 4xy \quad \sqrt{3}y^2 \quad 0$ is given by

$$\tan^{\ 1}\frac{2\sqrt{h^2 \quad ab}}{|a \quad b|} \quad \tan^{\ 1}\frac{2\sqrt{2^2 \quad 3}}{|\sqrt{3} \quad \sqrt{3}|} \quad \tan^{\ 1}\frac{1}{\sqrt{3}} \quad \overline{6}$$

Hence, shaded area

$$\frac{\overline{6}}{2} \quad (2^2 \quad 1^2) \quad \overline{4}$$

(97) Answer : (c) $x \quad y \quad 2\sqrt{2}$

Let the equation of the tangent be $\frac{x}{a} \quad \frac{y}{a} \quad 1$

i.e. $x + y = a$ ... ... (1)

Length of perpendicular from the centre (–2, 2)

on equation (1) of radius $\sqrt{4 \quad 4 \quad 4} \quad 2$

i.e. $\frac{| \quad 2 \quad 2 \quad a|}{\sqrt{1 \quad 1}} \quad 2 \qquad a \quad 2\sqrt{2}$

Hence, the equation of the tangent is $x \quad y \quad 2\sqrt{2}$

(98) Answer : (b) $x^2 + y^2 - 3x + 1 = 0$

The circle through points of intersection of the two circles $x^2 + y^2 - 6 = 0$ and $x^2 + y^2 - 6x + 8 = 0$ is

$(x^2 + y^2 + 6) + \lambda\ (x^2 + y^2 - 6x + 8) = 0$

As it passes through (1, 1)

$= 1$

Equation of required circles is $2x^2 + 2y^2 - 6x + 2 = 0$

$x^2 + y^2 - 3x + 1 = 0$

(99) Answer : (a) no value of a

The equation of PQ is $54x + (c - d)y + a + 1 = 0$ ... ... (1)

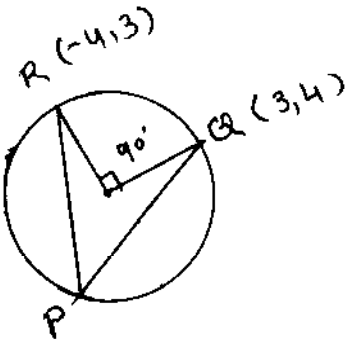
Also equation of PQ is $5x + by - a = 0$ ... ... (2)

$$\frac{5a}{5} \quad \frac{c \quad d}{b} \quad \frac{a \quad 1}{a}$$

$a \quad \frac{a \quad 1}{a}$ $\qquad$ $a^2 + a + 1 = 0$

no value of a $\qquad$ ($\because$ D < 0)

(100) Answer : (c) $\frac{\quad}{4}$



Angle subtended by QR at centre 0 is 90^o

$\left[\because m_1 m_2 \quad \frac{4}{3} \quad \frac{(\ \ 3)}{4} \qquad 1\right]$

Hence, angle at circum ference at P(any where) will be half of $\frac{\quad}{2}$

i.e. $\quad$ QPR = $\frac{\quad}{4}$

(101) Answer : (b) A hyperbola

Let $xy = C^2$ be the rectangular hyperbola, and let P(x_1, y_1) be the point on it.

Let Q(h, k) be the midpoint of PN

Then the coordinates of Q are $\left(x_1, \frac{y_1}{2}\right)$

$x_1 = h$ and $\frac{y_1}{2} \quad k \qquad y_1 = 2k$

But (x_1, y_1) lies on $xy = C^2$

$h(2k) = C^2$

$hk \quad \frac{C^2}{2}$

Hence, the locus of $h(k, k)$ is $xy \quad \frac{C^2}{2}$, which is a rectangular hyperbola

(102) Answer : (d) $k \quad (0, 2)$

We have $\left|\sqrt{x^2 + (y+1)^2} - \sqrt{x^2 + (y-1)^2}\right| = k$

Which is equivalent to $|S_1P - S_2P|$ = constant

Where $S_1 = (0, 1)$, $S_2 = (0, -1)$ and P $\quad (x, y)$

The above equation represent a hyperbola, then we have $k = 2a$

[Where $2a$ is the transverse axis and e is the eccentricity] and $2ae = S_1S_2 = 2$

Dividing, we have $e \quad \frac{2}{k}$

Since, $e > 1$ for a hyperbola, $k < 2$

Also k must be a positive quantity.

So, we have $k \quad (0, 2)$

(103) Answer : (b) sec

Any tangent to hyperbola forms triangle with asymptotes which has constant are ab.

Given $ab = a^2\tan$

$$\frac{b}{a} \quad \tan \qquad \left(\frac{b^2}{a^2} \quad e^2 \quad 1\right)$$

$e^2 - 1 = \tan^2$

$e^2 = 1 + \tan^2 \quad = \sec^2$

$e = \sec$

(104) Answer : (a) ab

P($a\sec$, $b\tan$)

Tangent at P is $\frac{x}{a} \sec \quad - \frac{y}{b}\tan \quad = 1$ ... ... (1)

Asymptotes are $y \quad \frac{b}{a}x$ ... ... (2)

and $y \quad \frac{b}{a}x$ ... ... (3)

Solving (1) and (2) we have

Q $\left(\frac{a}{\sec \quad \tan}, \frac{b}{\sec}\right.$

Solving (1) and (3)

R $\left(\frac{a}{\sec\alpha + \tan\alpha}, \frac{-b}{\sec\alpha + t}\right.$

Then area of OQR

$\frac{1}{2}$ mode of $\begin{vmatrix} 0 \\ \frac{a}{\sec \quad \tan} \\ \frac{a}{\sec \quad \tan} \end{vmatrix}$

$\frac{1}{2}|ab \quad ab| \quad \frac{1}{2} \cdot 2ab$

(105) Answer : (d) $\frac{x}{x_1 \quad x_2}$

The mid point of chord is

The equation of the chord

$x\left(\frac{y_1 \quad y_2}{2}\right) \quad y\left(\frac{x_1}{}\right.$

$2\left(\frac{x_1 \quad x_2}{2}\right)\left(\frac{y_1}{2}\right.$

$x(y_1 + y_2) + y(x_1 +$

$\frac{x}{x_1 \quad x_2} \quad \frac{y}{y_1 \quad y_2}$

(106) Answer : (a) $\frac{2}{3}$

Given hyperbola is $x^2 - 2$

PQ · PR $\quad \frac{|a\sec}{}$

tan

an α

	0	1
—	$\frac{b}{\sec \quad \tan}$	1
—	$\frac{b}{\sec \quad \tan}$	1

ab

$\frac{y}{y_1 \quad y_2} \quad 1$

$\left(\frac{x_1 \quad x_2}{2}, \frac{y_1 \quad y_2}{2}\right)$

in terms of its mid point (T = S')

$\frac{x_2}{2} \quad C^2$

$y_2 \quad C^2$

$x_2) = (x_1 + x_2)(y_1 + y_2)$

1

$y^2 = 2$ or $\frac{x^2}{2} \quad \frac{y^2}{1} \quad 1$

$\frac{\sqrt{2}b\tan \quad |}{\sqrt{3}} \frac{|a\sec \quad \sqrt{2}b\tan \quad |}{\sqrt{3}}$

$$\frac{a^2 \sec^2 \qquad 2b^2 \tan^2}{3}$$

$$\frac{2(\sec^2 \qquad \tan^2 \;)}{3} \qquad \{a \quad \sqrt{2}, b \quad 1\} \qquad \frac{2}{3}$$

<u>Answer Key</u>

(1) (c)	(2) (c)	(3) (d)	(4) (a)	(5) (c)	(6) (a)	(7) (d)	(8) (d)	(9) (a)
(10) (c)	(11) (b)	(12) (d)	(13) (b)	(14) (b)	(15) (a)	(16) (d)	(17) (b)	(18) (a)
(19) (d)	(20) (b)	(21) (a)	(22) (a)	(23) (d)	(24) (a)	(25) (c)	(26) (d)	(27) (b)
(28) (c)	(29) (a)	(30) (d)	(31) (c)	(32) (a)	(33) (a)	(34) (b)	(35) (c)	(36) (b)
(37) (d)	(38) (c)	(39) (a)	(40) (a)	(41) (d)	(42) (d)	(43) (c)	(44) (d)	(45) (d)
(46) (b)	(47) (d)	(48) (a)	(49) (b)	(50) (c)	(51) (a)	(52) (a)	(53) (a)	(54) (d)
(55) (c)	(56) (b)	(57) (a)	(58) (c)	(59) (d)	(60) (c)	(61) (d)	(62) (d)	(63) (b)
(64) (d)	(65) (a)	(66) (c)	(67) (c)	(68) (d)	(69) (a)	(70) (c)	(71) (b)	(72) (c)
(73) (b)	(74) (b)	(75) (a)	(76) (a)	(77) (b)	(78) (a)	(79) (d)	(80) (b)	(81) (d)
(82) (a)	(83) (d)	(84) (c)	(85) (c)	(86) (d)	(87) (a)	(88) (a)	(89) (a)	(90) (a)
(91) (c)	(92) (a)	(93) (b)	(94) (a)	(95) (a)	(96) (a)	(97) (c)	(98) (b)	(99) (a)
(100) (c)	(101) (b)	(102) (d)	(103) (b)	(104) (a)	(105) (d)	(106) (a)		

Unit - 12

Three Dimensional Geometry

Important Point

- Distance formula in R^3: If $\bar{a} = (x_1, y_1, z_1)$ $\bar{b} = (x_2, y_2, z_3)$

$\overrightarrow{AB} = \bar{b} - \bar{a} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$AB = |\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

- Division of line segment :

Suppose position vector of A & B be $\bar{a}$ & $\bar{b}$, respectively if $P(\bar{r})$ divides $\overline{AB}$ from A in λ ratio. where $(P \neq A, P \neq B)$

Co-ordinate of P is $\bar{r} = \dfrac{\lambda\bar{a} + \bar{b}}{\lambda + 1}$, $\lambda \neq 0, -1$

- Co-ordinates of mid point of $\overline{AB} = \dfrac{\bar{a} + \bar{b}}{2}$

- In ΔABC; If $A(\bar{a}), B(\bar{b}), C(\bar{c})$ then posintion vector of centroid is

$\bar{g} = \dfrac{\bar{a} + \bar{b} + \bar{c}}{3}$,

- Co-ordinates of Incentre : In ΔABC, if co-ordinate of position vector A, B & C are $\bar{a}$, $\bar{b}$ & $\bar{c}$ and BC = a, CA = b, AB = c

  Then position vector of incentre is $\dfrac{a\bar{a}+b\bar{b}+c\bar{c}}{a+b+c}$

- For equilateral triangle centroid and Incentre are equal.
- Direction co-sine & direction angle:

  If vector $\bar{r}=(a, b, c)\in R^3$ makes angle α, β, γ with unit vectors i, j & k then α, β, γ are called direction angles and $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called direction co-sine of $\bar{r}$.

  $$l=\cos\alpha=\frac{a}{\sqrt{a^2+b^2+c^2}},\quad m=\cos\beta=\frac{b}{\sqrt{a^2+b^2+c^2}},\quad n=\cos\gamma=\frac{c}{\sqrt{a^2+b^2+c^2}}$$

- If l, m and n are direction co-sine of $\bar{r}=(a, b, c)$, then $l^2+m^2+n^2=1$

  $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

- If unit vector in the direction of $\bar{r}=(a, b, c)$:

  $$\hat{r}=\left(\frac{a}{|\bar{r}|}, \frac{b}{|\bar{r}|}, \frac{c}{|\bar{r}|}\right)=(l, m, n)$$

- Direction ratio : if $\bar{x}\neq 0$ & $m\neq 0$ for $m\bar{x}$, mx_1, mx_2, mx_3 is called direction ratio.
- Vector equation of line:

  If direction of line is $\bar{l}$ passes through

  A($\bar{a}$) then equation of line is : $\bar{r}=\bar{a}+k\bar{l}$, $k\in R$

- Parametric equation of line:

  $x=x_1+kl_2$, $y=y_1+kl_2$, $z=z_1+kl_3$, $k\in R$ are the parametric equations of line

  passing through $\bar{a}=(x_1, y_1, z_1)$ & with direction $\bar{l}=(l_1, l_2, l_3)$

- Cartesian equation of line $\bar{r}=(x, y, z)$, $\bar{a}=(x_1, y_1, z_1)$ & $\bar{l}=(l_1, l_2, l_3)$

$\overline{x} = (x, y, z)$, $\overline{a} = (x_1, y_1, z_1)$ & direction $\overline{l} = (l_1, l_2, l_3)$

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{l_2} = \frac{z - z_1}{l_3}$$

- Equation of line passing through $A(\overline{a})$ and $B(\overline{b})$:

  $\overline{a} = (x_1, y_1, z_1)$ $\overline{b} = (x_2, y_2, z_2)$ & $\overline{r} = (x, y, z)$

  Vector equation of line $\overline{r} = \overline{a} + k(\overline{b} - \overline{a})$ $k \in R$

  Cartesion equation of line $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}$

- Paramaetric equation of line:

  $x = x_1 + k(x_2 - x_1)$, $y = y_1 + k(y_2 - y_1)$, $z = z_1 + k(z_2 - z_1)$, $k \in R$

  If $l_1 = 0$ & $l_2 \neq 0$, $l_3 \neq 0$ then $x = x_1$, $\dfrac{y - y_1}{l_2} = \dfrac{z - z_1}{l_3}$ OR $\dfrac{x - x_1}{0} = \dfrac{y - y_1}{l_1} = \dfrac{z - z_1}{l_3}$

- Angle between two lines in space R^3:

  $\overline{r} = \overline{a} + k\overline{l}$, $\overline{r} = \overline{b} + k\overline{m}$ $k \in R$

  If two lines are parallel & direction of lines $\overline{l}$ & $\overline{m}$ is same of opposite.

  $\overline{l}$ *and* $\overline{m} = \theta$ OR $\overline{l} = k\overline{m}$ $k \in R - \{0\}$

  If two lines are perpendicular then $\overline{l} \cdot \overline{m} = 0$

  If angle between two lines is θ then $\cos\theta = \dfrac{|\overline{l} \cdot \overline{m}|}{|\overline{l}|\ |\overline{m}|}$ $0 < \theta < \dfrac{\pi}{2}$

- To obtain angle between two lines it is not necessary that two lines are intersecting (in R^3 only):

  In R^3 condtion for two lines $\overline{r} = \overline{a} + k\overline{l}$, $\overline{r} = \overline{b} + k\overline{m}$, $k \in R$ to intersect is

  $(\overline{a} - \overline{b}) \cdot (\overline{l} \times \overline{m}) = 0$ where $\overline{l} \neq \overline{0}$, $\overline{m} \neq \overline{0}$

  In R^3, condition for two lines $\overline{r} = \overline{a} + k\overline{l}$ & $\overline{r} = \overline{b} + k\overline{m}$, $k \in R$ to interset in cartesion form

  $\overline{a} = (x_1\ y_1\ z_1)$, $\overline{b} = (x_2, y_2, z_2)$, $\overline{l} = (l_1, l_2, l_3)$

$\bar{m} = (m_1, m_2, m_3)$ is $\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 0$

- Condition that

  lines $\bar{r} = \bar{a} + k\bar{l}$, $\bar{r} = \bar{b} + k\bar{m}$, $k \in R$ $\bar{l} \neq \bar{0}$, $\bar{m} \neq 0$ are co-planer is

  $(\bar{a} - \bar{b}) \cdot (\bar{l} \times \bar{m}) = 0$

- Non-coplaner lines :
  If for any two lines l & m there does not exist plane containing them then they are non-coplanar.
- Condition for two lines to be co-planer or non-coplaner

  $\bar{r} = \bar{a} + k\bar{l}$ & $\bar{r} = \bar{b} + k\bar{m}$, $k \in R$

  $\bar{a} = (x_1, y_1, z_1)$, $\bar{b} = (x_2, y_2, z_2)$, $\bar{l}\,(l_1, l_2, l_3)$, $\bar{m} = (m_1, m_2, m_3)$

  (1) For Co-planer line : $(\bar{a} - \bar{b}) \cdot (\bar{l} \times \bar{m}) = 0$ vector form

  Cartesian form $\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 0$

  (2) For non-co-planerline : $(\bar{a} - \bar{b}).(\bar{\ell} \times \bar{m}) \neq 0$

  Cartesian form $\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{vmatrix} \neq 0$

- Perpendicular distance of a line from point :

  Perpendicular distance of $\bar{r} = \bar{a} + k\bar{l}, k \in R$ from point $P(\bar{p})$ is

  (1) $PM = \dfrac{|\overrightarrow{AP} \times \bar{l}|}{|\bar{l}|} = \dfrac{|(\bar{P} - \bar{a}) \times \bar{l}|}{|\bar{l}|}$

  (2) Cartesian Form $\bar{a} = (x_2, y_2, z_2)$ $P(x_1, y_1, z_1)$, $\bar{l} = (l_1, l_2, l_3)$

  $PM = \left\| \begin{matrix} \bar{l} & \bar{j} & \bar{k} \\ x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & l_2 & l_3 \end{matrix} \right\|$

- Perpendicular distance between parallel lines:

  $\bar{r} = \bar{a} + k\bar{l}$, $\bar{r} = \bar{b} + k\bar{l}$, $k \in R$, is $= \dfrac{|(\bar{b} - \bar{a}) \times \bar{l}|}{|\bar{l}|}$

- Distance between two skew lines

$\bar{r} = \bar{a} + k\bar{l}$ & $\bar{r} = \bar{b} + k\bar{m}$, $k \in R$, then $p = \dfrac{|(\bar{b} - \bar{a}) \cdot (\bar{l} \times \bar{m})|}{|\bar{l} \times \bar{m}|}$

In R^3 relation between two lines $L : \bar{r} = \bar{a} + k\bar{l}$, $k \in R$, $M : \bar{r} = \bar{b} + k\bar{m}$, $k \in R$

using $\bar{l} \times \bar{m}$. we will get relation.

$\bar{\ell} \times \bar{m}$

- $\bar{\ell} \times \bar{m} = \bar{0}$ — lines are parallel OR Co- Inside Lines
  - $(\bar{b} - \bar{a}) \times \bar{\ell} \neq \bar{0}$ — Parallel Lines
  - $(\bar{b} - \bar{a}) \times \bar{\ell} = \bar{0}$ — Co- Inside Lines
- $\bar{\ell} \times \bar{m} \neq \bar{0}$ — Skew OR Intersecting Lines
  - $(\bar{b} - \bar{a}).(\bar{\ell} = \bar{m}) \neq 0$ — Skew Lines
  - $(\bar{b} - \bar{a}).(\bar{\ell} \times \bar{m}) \neq 0$ — Intersecting Lines

Plane :

- Vector equation of plane :

If plane passes through $A(\bar{a})$, $B(\bar{b})$, $C(\bar{c})$ then vector equation is

$\bar{r} = \bar{a} + m(\bar{b} - \bar{a}) + n(\bar{c} - \bar{a})$, $m, n \in R$

- Parametric Form $\bar{r} = \bar{a}l + m\bar{b} + n\bar{c}$ where $l + m + n = 1$
- Cartesian parametric form

$\bar{r} = (x, y, z)$, $\bar{a} = (x_1, y_1, z_1)$, $\bar{b} = (x_2, y_2, z_2)$, $\bar{c}(x_3, y_3, z_3)$

$x = lx_1 + mx_2 + nx_3$ where $l + m + n = 1$, $l, m, n \in R$

$y = ly_1 + my_2 + ny_3$

$z = lz_1 + mz_2 + nz_3$

- Cartesian equation : $(\bar{r} - \bar{a}) \cdot \left[(\bar{b} - \bar{a}) \times (\bar{c} - \bar{a})\right] = 0$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

- If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$, $D(x_4, y_4, z_4)$ are co-planer then

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} = 0$$

- Equation of plane with intercepts a, b, c with X, Y and Z axis repectively is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \ (a, b, c \neq 0)$$

- Equation of plane passing through $A(\bar{a})$ with normal $\bar{n}$ is $\bar{r} . \bar{n} = \bar{a} . \bar{n}$

cartesian form $\bar{r} = (x, y, z)$, $\bar{n} = (a, b, c)$ $\therefore$ $ax + by + cz = d$ $(d = \bar{a} . \bar{n})$

- If angle between two planes is θ

then $\cos\theta = \dfrac{|\bar{n}_1 . \bar{n}_2|}{|\bar{n}_1| \, |\bar{n}_2|}$ $0 \leq \theta < \dfrac{\pi}{2}$

- If planes are perpendicular then $\bar{n}_1 . \bar{n}_2 = 0$
- The equation of plane passing through two parallel lines :

$\bar{r} = \bar{a} + k\bar{l}$, $k \in R$ & $\bar{r} = \bar{b} + k\bar{m}$, $k \in R$

The equation of plane is $(\bar{r} - \bar{a}) . \left[(\bar{b} - \bar{a}) \times \bar{l}\right] = 0$

Cartesian form

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & l_2 & l_3 \end{vmatrix} = 0 \ \left[\bar{a} = (x_1, y_1, z_1), \bar{b} = (x_2, y_2, z_2), \bar{l} = (l_1, l_2, l_3)\right]$$

- The equation of plane passing through two intersecting lines

$\bar{r} = \bar{a} + k\bar{l}$ and $\bar{r} = \bar{b} + k\bar{m}$, $(\bar{r} - \bar{a}) . (\bar{l} \times \bar{m}) = 0$

Cartesian form $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 0$

where $\bar{a} = (x_1, x_2, x_3)$, $\bar{l} = (l_1, l_2, l_3)$ & $\bar{m} = (m_1, m_2, m_3)$

- Perpendicular distance from point $P(\overline{p})$ to plane $\overline{r} \cdot \overline{n} = d$ is $\dfrac{|\overline{p} \cdot \overline{n} - d|}{|\overline{n}|}$

- $$= \frac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{a^2 + b^2 + c^2}}$$ (Cartesian form)

- Perpendicular distance between two planes

  $\overline{r} \cdot \overline{n} = d_1$ and $\overline{r} \cdot \overline{n} = d_2$ is $\dfrac{|d_1 - d_2|}{|\overline{n}|}$

- Angle between line $\overline{r} = \overline{a} + k\overline{l}$, $k \in R$, plane $\overline{r} \cdot \overline{n} = d$

  $$\alpha = \sin^{-1}\frac{|\overline{l} \cdot \overline{n}|}{|\overline{l}|\ |\overline{n}|} \qquad 0 < \alpha < \frac{\pi}{2}$$

- For two plane $\pi_1 : \overline{r} \cdot \overline{n}_1 = d_1$ and $\pi_2 : \overline{r} \cdot \overline{n}_2 = d_1$

  intersection is line then equation of line is $\overline{r} = \overline{a} + k\overline{n}$, $k \in R$, $\overline{n} = \overline{n}_1 + \overline{n}_2$

- For two plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ equation of plane passing through the intersection of two planes

  $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0 \quad , \lambda \neq 0, -1$

Question Bank

1. The point on x-axis equidistance from A(2, -5, 7) and B(1, 3, 6) is ....

   (a) (-16, 0, 0) (b) (16, 0, 0) (c) (6, 0, 0) (d) none of these

2. The equation of the locus of point which are equdistance from (4, 5, 2) and (1, 6, 3) is ....

   (a) $6x - 2y - 2z + 1 = 0$ (b) $6x + 2y - 2z + 1 = 0$

   (c) $6x + 2y + 2z + 1 = 0$ (d) $6x - 2y - 2z - 1 = 0$

3. If the position vector of A, B, C in R^3 are (-1, 2, 0), (1, 2, 3) and (4, 2, 1) then type of ΔABC is ....

   (a) Right angled (b) Isosceles right angled

   (c) Euilateral (d) Isosceles

4. If the vertices of quadrilatral are (1, 1, 1), (-2, 4, 1), (-1, 5, 5), (2, 2, 5) then it is.....

   (a) rectangle (b) square (c) parallelogram (d) rhombus

5. A(1, 1, 2), B(2, 3, 5), C(1, 3, 4) and D(0, 1, 1) forms ..... and its area is ....

   (a) Square, $2\sqrt{3}$ (b) Parallelogram, $2\sqrt{3}$

   (c) Rectangle, $2\sqrt{3}$ (d) Parallelogram, $\sqrt{3}$

6. For A(7, -3, 1) and B(4, 9, 8), the point that divides $\overline{AB}$ from B in the ratio 2:5 is....

   (a) $\left(\frac{34}{7}, \frac{39}{7}, \frac{42}{7}\right)$ (b) $\left(\frac{34}{7}, \frac{39}{7}, \frac{-42}{7}\right)$

   (c) $\left(\frac{-34}{7}, \frac{39}{7}, \frac{-42}{7}\right)$ (d) $\left(\frac{-34}{7}, \frac{-39}{7}, \frac{-42}{7}\right)$

7. For A(1, 5, 6), B(3, 1, 2) and C(4, -1, 0), B divides $\overline{AC}$ from A in ...... ratio

   (A) -2 : 3 (b) 2 : 3 (c) 2 : 1 (d) -2 : 1

8. A(0, -1, 4), B(1, 2, 3), C(5, 4, -1), then the foot of perpendicular from A on $\overline{BC}$ is.......

   (a) (-3, 3, 1) (b) (3, -3, 1) (c) (3, 3, 1) (d) (3, 3, -1)

9. If A(a, 1, 3), B(-1, b, 2), C(1, 0, c) are the vertices of ΔABC whose centroid is (2, 3, 5), then values of a, b, c are respectively ....

   (a) 10, 8, 6 (b) 6, 10, 8

   (c) 8, 6, 10 (d) 6, 8, 10

10. If A(6, 4, 6), B(12, 4, 0), C(4, 2, -1) are the vertices of triangle, then it's incentre is....

(a) $\left(\frac{22}{3}, \frac{10}{3}, \frac{4}{3}\right)$ (b) $\left(\frac{-22}{3}, \frac{10}{3}, \frac{4}{3}\right)$

(c) $\left(\frac{22}{3}, \frac{-10}{3}, \frac{4}{3}\right)$ (d) $\left(\frac{22}{3}, \frac{10}{3}, \frac{-4}{3}\right)$

11. If the mid points of sides of ΔABC are P(9, 2, 5), Q(-7, 6, 1), R(8, -9, 3) then the centroid of ΔABC is ....

(A) $\left(\frac{10}{3}, \frac{-1}{3}, \frac{2}{3}\right)$ (b) $\left(\frac{-10}{3}, \frac{-1}{3}, \frac{-2}{3}\right)$

(c) $\left(-1, -1, \frac{2}{3}\right)$ (d) None of these

12. For ΔABC, A(-1, -2, -3), B(1, 2, 3), C(1, 2, 1) the length of median through A is .... and centroid is ......

(a) $3\sqrt{3}, \left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$ (b) $3\sqrt{5}, \left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$

(c) $\sqrt{5}, \left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$ (d) $3, \left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$

13. The co-ordinates of the points of trisection of $\overline{AB}$ is ..... where A(-5, 7, 2), B(1, 3, 7)

(a) $\left(-1, 4, \frac{16}{3}\right)\left(-3, \frac{11}{2}, \frac{11}{3}\right)$ (b) $\left(1, 4, \frac{16}{3}\right)\left(-3, \frac{11}{2}, \frac{11}{3}\right)$

(c) $\left(-1, 4, \frac{16}{3}\right)\left(-3, \frac{-11}{2}, \frac{-11}{3}\right)$ (d) None of these

14. If $m\angle B = \frac{\pi}{2}$ in ΔABC and P, Q are points of trisection of hypotenuse $\overline{AC}$, then $BP^2 + BQ^2 =$ ....

(a) $\frac{5}{9}\ AC^2$ (b) $\frac{5}{9}AC$ (c) $\frac{25}{81}AC^2$ (D) $\frac{25}{81}AC$

15. If G (0) is centroid of ΔABC, then $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} =$ ....

(a) $\bar{0}$ (b) 0 (c) $\bar{x} + \bar{y} + \bar{z}$ (d) $\frac{\bar{x} + \bar{y} + \bar{z}}{3}$

16. If A - P - B and $\frac{AP}{PB} = \frac{m}{n}$, then for every point 'O' in space ......

(a) (m - n) $\overrightarrow{OP}$ (b) (m + n) $\overrightarrow{OP}$ (c) m $\overrightarrow{OP}$ (d) n $\overrightarrow{OP}$

17. In ΔABC, if mid points of $\overline{AB}$ and $\overline{AC}$ are D and E respectively, then $\overrightarrow{BE} + \overrightarrow{DC} =$ ....

(a) $\frac{3}{2}\overrightarrow{BE}$ (b) $\frac{2}{3}\overrightarrow{BE}$ (c) $\frac{3}{2}BC$ (D) $\frac{2}{3}$ BC

18. In parallelogram ABCD, $AB^2 + BC^2 + CD^2 + DA^2 = k(AC^2 + BD^2)$, then k = ...

(a) 4 (b) 16 (d) 2 (d) 1

19. If sides of regular hexagon ABCDEF, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are $\bar{a}$ and $\bar{b}$ respectively, then $\overrightarrow{AF} =$ ....

(a) $\bar{b} - \bar{a}$ (b) $\bar{a} - \bar{b}$ (c) $\bar{a} + \bar{b}$ (d) $\bar{a}$

20. For regular hexagon ABCDEF, $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} =$ ....

(a) $\bar{0}$ (b) $3\overrightarrow{AD}$ (c) $2\overrightarrow{AD}$ (d) $4\overrightarrow{AD}$

21. For regular hexagon ABCDEF, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{AF} + \overrightarrow{EF} + \overrightarrow{ED} =$ ....

(a) $3\overrightarrow{AD}$ (b) $\overrightarrow{AD}$ (c) $\bar{0}$ (d) $2\overrightarrow{AD}$

22. If the centroid of ΔABC and ΔPQR is G and G’ respectively then $\overrightarrow{AP} + \overrightarrow{BQ} + \overrightarrow{CR} =$ ....

(a) $\overrightarrow{GG'}$ (b) $3\overrightarrow{GG'}$ (c) $2\overrightarrow{GG'}$ (d) $4\overrightarrow{GG'}$

23. If three vertices of rhombus are (6, 0, 1) (8, -3, 7) (2, -5, 10), then forth vertices = ...

(a) (0, -2, -4) (b) (0, -2, 4) (c) (0, 2, 4) (d) (0, 2, -4)

24. If vector $\bar{r}$ forms an angle α, β, γ with x, y, z-axis then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$ ....

(a) 1 (b) 2 (c) -1 (d) -2

25. If α, β, γ are direction co-sines of $\bar{x}$, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma =$ ....

(a) 1 (b) 2 (c) -1 (d) -2

26. If vector $\bar{r}$ form angles $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ with x and z axis respectively, then angle with y-axis is........

(a) $\frac{\pi}{4}, \frac{3\pi}{4}$ (b) $\frac{\pi}{4}, -\frac{\pi}{4}$ (c) $\frac{3\pi}{4}, -\frac{\pi}{4}$ (d) $\frac{\pi}{3}, -\frac{3\pi}{4}$

27. If $\frac{\pi}{4}$ is an angle with positive direction of x-axis in R^3 the no. of such vectors are...

(a) 1 (b) 2 (c) 3 (d) infinite

28. If any vector forms angles $\frac{\pi}{4}, \frac{\pi}{3}$ and $\frac{\pi}{6}$ with axis, then such vector with measure 4 unit is.........

(a) $\left(2, 2\sqrt{3}, 2\sqrt{2}\right)$ (b) $\left(-2, -2\sqrt{3}, 2\sqrt{2}\right)$

(c) $\left(2, 2\sqrt{3}, -2\sqrt{2}\right)$ (d) $\left(-2, -2\sqrt{3}, -2\sqrt{2}\right)$

29. If vector $\bar{x}$ forms an equal angle α with three axis and

$|\bar{x}| = 9$, then $\alpha = ...$ where $0 < \alpha < \frac{\pi}{2}$

(a) $\cos^{-1}\frac{1}{\sqrt{2}}$ (b) $\cos^{-1}\frac{1}{9}$ (c) $\cos^{-1}\frac{1}{\sqrt{3}}$ (d) $\cos^{-1}\frac{1}{3}$

30. For $\bar{x} = (a, 3, -2)$, $\bar{y} = (a, -a, 2)$, if $\bar{x} \perp \bar{y}$, then $a = ...$

(a) 4, 1 (b) 4, -1 (c) -4, -1 (d) -4, 1

31. If angle between two vectors $i + \sqrt{3}j$ and $\sqrt{3}\, i + a\, i$ is $\frac{\pi}{3}$, then a = ...

(a) 0 (b) 3 (c) -3 (d) none of these

32. The unit vector which is perpendicular to (2, -4, 3) and (5, 0, 1), is ....

(a) $\left(\frac{4}{\sqrt{585}}, \frac{13}{\sqrt{585}}, \frac{20}{\sqrt{585}}\right)$ (b) $\left(\frac{-4}{\sqrt{585}}, \frac{13}{\sqrt{585}}, \frac{-20}{\sqrt{585}}\right)$

(c) $\left(\frac{-4}{\sqrt{585}}, \frac{-13}{\sqrt{585}}, \frac{20}{\sqrt{585}}\right)$ (d) $\left(\frac{-4}{\sqrt{585}}, \frac{13}{\sqrt{585}}, \frac{20}{\sqrt{585}}\right)$

33. Vector which is in XY - plane and perpendicular to 4i - 3j + 2k, is ....

(a) $\left(\frac{3}{5}, \frac{4}{5}, 0\right)$ (b) $\left(-\frac{3}{5}, -\frac{4}{5}, 0\right)$

(c) $\pm\frac{1}{5}(3, 4, 0)$ (d) $\pm\frac{1}{5}(-3, -4, 0)$

34. If angle between two unit vectors $\bar{a}$ & $\bar{b}$ is

α, then $|\bar{a} - \bar{b}\cos\alpha| = ...$ $0 < \alpha < \frac{\pi}{2}$

(a) $\sin\alpha$ (b) $\sin\frac{\alpha}{2}$

(c) $\sin 2\alpha$ (d) $\sin^2\frac{\alpha}{2}$

35. If angle between two untis vectors $\bar{a}$ and $\bar{b}$ is θ, then $\cos\frac{\theta}{2} =$ .... $0 < \theta < \pi$

(a) $|\bar{a}+\bar{b}|$ (b) $\frac{1}{2}|\bar{a}+\bar{b}|$ (c) $\frac{1}{2}|\bar{a}+\bar{b}|^2$ (d) $|\bar{a}+\bar{b}|^2$

36. If angle between two units vectors $\bar{a}$ and $\bar{b}$ is θ, then $\sin\frac{\theta}{2} =$ ....

(a) $|\bar{a}+\bar{b}|$ (b) $\frac{1}{2}|\bar{a}-\bar{b}|$ (c) $|\bar{a}-\bar{b}|$ (d) $\frac{1}{2}|\bar{a}+\bar{b}|$

37. $\bar{x} = (2, -6, 3)$, $\bar{y} = (1, 2, -2)$ and $\bar{x}\,\hat{}\,\bar{y} = \theta$, then $\sin\theta =$ ....

(a) $\frac{21}{\sqrt{185}}$ (b) $-\frac{\sqrt{185}}{21}$ (c) $-\frac{21}{\sqrt{185}}$ (d) $\frac{\sqrt{185}}{21}$

38. If angle between $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{6}$ and $|\bar{a}| = 4$, $|\bar{b}| = 2$, then $|\bar{a}\times\bar{b}| =$ ....

(a) 4 (b) 16 (c) 8 (d) 2

39. If angle between $\bar{a}$ and $\bar{b}$ is θ, then $\frac{|\bar{a}\times\bar{b}|}{\bar{a}\,.\,\bar{b}} =$ ....

(a) - cot θ (b) - tanθ (c) tan θ (d) cot θ

40. For vectors $\bar{a}, \bar{b}, \bar{c}$ if each vector is perpendicular to the sum of remaining two vectors and $|\bar{a}| = 3$, $|\bar{b}| = 4$, $|\bar{c}| = 5$, then $|\bar{a}+\bar{b}+\bar{c}| =$ ....

(a) $2\sqrt{2}$ (b) $3\sqrt{2}$ (c) $4\sqrt{2}$ (d) $5\sqrt{2}$

41 For vector a, b, c if each vector forms an angle $\frac{\pi}{3}$ with reamaning two vectors and $|\bar{a}| = 1$, $|\bar{b}| = 2$, $|\bar{c}| = 3$, then $|\bar{a}+\bar{b}+\bar{c}| =$ ....

(a) $\sqrt{17}$ (b) 0 (c) 5 (d) $\sqrt{5}$

42. For unit vectors $\bar{a}, \bar{b}, \bar{c}$, if $|\bar{a}+\bar{b}+\bar{c}| = 1$ and $\bar{a}$ is perpendiculr to $\bar{b}$ also $\bar{c}$ form and angle α, β, with $\bar{a}$ and $\bar{b}$ respectively then cos α + cos β = ...

(a) −1 (b) 1 (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

43. If $(\bar{a}+\bar{b})\,.\,(\bar{a}-\bar{b}) = 63$ and $|\bar{a}| = 8|\bar{b}|$, then $|\bar{a}| =$ ....

(a) 8 (b) 64 (c) 16 (d) 4

44. The angle between two unit vectors $\bar{a}$ and $\bar{b}$ is θ, $|\bar{a}+\bar{b}| < 1$ if .....

(a) $\theta = \frac{\pi}{2}$ (b) $\theta > \frac{\pi}{3}$ (c) $\frac{2\pi}{3} < \theta < \pi$ (d) $\theta = \frac{\pi}{6}$

45. If angle between two unit vectors $\bar{a}$ and $\bar{b}$ is θ, $0 < \theta < \frac{\pi}{2}$ if $|\bar{a} - \bar{b}| < 1$ and θ is in...... interval

(a) $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ (b) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (c) $\left(0, \frac{\pi}{3}\right)$ (d) $\left(0, \frac{\pi}{6}\right)$

46. For vector $\bar{a}$ and $\bar{b}$, $|\bar{a} + \bar{b}| < |\bar{a} - \bar{b}|$, then the angle between $\bar{a}$ and $\bar{b}$ is ......

(a) obtuse (b) Acute (c) Right (d) supplimentary

47. If unit vector $\bar{a}$ and $\bar{b}$ form an angle of $\frac{\pi}{6}$ and $\frac{2\pi}{3}$ with positive direction of x-axis respectively, then $|\bar{a} + \bar{b}| =$ ....

(a) $\sqrt{\frac{2}{3}}$ (b) 2 (c) $\sqrt{2}$ (d) $\sqrt{3}$

48. The unit vector which is perpendicular to the vector (2, 4, -3) and which is in YZ plane is ...

(a) $\pm\left(0, \frac{5}{3}, 4\right)$ (b) $\pm\frac{1}{5}(0, 3, 4)$

(c) $\frac{1}{5}(0, 3, 4)$ (d) $\frac{1}{5}(0, -3, -4)$

49. Equation of line passes through (-3, 4, 7) with direction (5, 2, 8) is ......

(a) $\frac{x-3}{5} = \frac{y-4}{2} = \frac{z-7}{8}$ (b) $\frac{x+3}{5} = \frac{y-4}{2} = \frac{z-7}{8}$

(c) $x - 3 = y - 4 = z - 7$ (d) $x + 3 = y - 4 = z - 7$

50. Equation of line passes through A(-2, 4, 7) and direction (5, -9, 12) is ....

(a) $x = -2 + k5$, $y = 4 - 9k$, $z = 7 - 12k$, $k \in R$

(b) $x = -2k + k5$, $y = 4 - 9k$, $z = 7 + 12k$, $k \in R$

(c) $x = 2 + 5k$, $y = 4 - 9k$, $z = 7 + 12k$, $k \in R$

(d) None of these

51. Equation of line passing through (0, 0, 0) and parallel to Y-axis is.....

(a) $\frac{x}{0} = \frac{y}{1} = \frac{z}{1}$ (b) $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$

(c) $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ (d) $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$

52. Direction cosine of line $\frac{4-x}{7}=\frac{y+9}{5}=\frac{32+8}{2}$ is ....

(a) $-\frac{21}{\sqrt{670}}, \frac{15}{\sqrt{670}}, \frac{2}{\sqrt{670}}$ (b) $\frac{21}{\sqrt{670}}, \frac{15}{\sqrt{670}}, \frac{2}{\sqrt{670}}$

(c) $\frac{21}{\sqrt{670}}, \frac{-15}{\sqrt{670}}, \frac{2}{\sqrt{670}}$ (d) $\frac{-21}{\sqrt{670}}, \frac{-15}{\sqrt{670}}, \frac{-2}{\sqrt{670}}$

53. Direction cosine of line $2x = 3y + 5$, $z = 7 - \frac{y}{5}$ is.....

(a) $\frac{10}{\sqrt{235}}, \frac{15}{\sqrt{235}}, \frac{3}{\sqrt{235}}$ (b) $\frac{-10}{\sqrt{235}}, \frac{-15}{\sqrt{235}}, \frac{-3}{\sqrt{235}}$

(c) $\frac{10}{\sqrt{235}}, \frac{-15}{\sqrt{235}}, \frac{3}{\sqrt{235}}$ (d) None of these

54. Which of the following point is on the line passes through A(1, 2, 0) and B(3, 1, 1)?

(a) (7, -1, 3) (b) (-7, 1, 3) (c) (7, -1, -3) (d) (7, 1, 3)

55. If $l + m + n = 0$, $l^2 - m^2 + n^2 = 0$ and if the direction cosine of two lines are the solution of the given equation, then angle between two line is...........

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

56. Angle between two diagonal of the cube is.........

(a) $\cos^{-1}\frac{1}{\sqrt{3}}$ (b) $\cos^{-1}\frac{1}{3}$ (c) $\cos^{-1}\frac{1}{9}$ (d) $\cos^{-1}\frac{\sqrt{3}}{2}$

57. If any line form an angle $\alpha, \beta, \gamma, \delta$ with the diagonal of cube then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ = ...

(a) $\frac{8}{3}$ (b) $-\frac{8}{3}$ (c) $\frac{4}{3}$ (d) $-\frac{4}{3}$

58. If any line form an angle $\alpha, \beta, \gamma, \delta$ with the diagonal of cube then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$ = ...

(a) $\frac{8}{3}$ (b) $-\frac{8}{3}$ (c) $\frac{4}{3}$ (d) $-\frac{4}{3}$

59. If any line form an angle $\alpha, \beta, \gamma, \delta$ with the diagonal of cube then $\cos2\alpha + \cos2\beta + \cos2\gamma + \cos2\delta$ = ...

(a) $-\frac{4}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $-\frac{8}{3}$

60. If α, β and γ are the direction cosine of the line then $\cos2\alpha + \cos2\beta + \cos2\gamma$ = ...

(a) 1 (b) -1 (c) $\frac{4}{3}$ (d) - $\frac{4}{3}$

61. Line $\frac{x+1}{2}=\frac{y-2}{2}=\frac{z+3}{-1}$ and $\frac{x-1}{3}=\frac{z-2}{1}$, y = -1 angle between two line......

(a) $\cos^{-1}\frac{\sqrt{10}}{90}$ (B) $\cos^{-1}\frac{5}{\sqrt{90}}$ (c) $\cos^{-1}\frac{1}{6}$ (d) $\cos^{-1}\frac{\sqrt{10}}{9}$

62. Line $\frac{\sqrt{2}x-3\sqrt{2}}{1}=\frac{2\sqrt{2}-\sqrt{2}y}{2}$, $z+1=0$ direction cosine ......

(a) $\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0$ (b) $-\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0$ (c) $\frac{1}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, 0$

63. $\frac{2-3x}{6}=\frac{y+1}{2}=\frac{1-z}{-2}$ direction cosine ....

(a) $-2, 2, 2$ (b) $-1,1,1$ (c) $-3,2,2$ (d) $6, 2, -2$

64. Direction cosine of line x = 3 - 2y z = 2y -1 is ....

(a) $\left(\frac{-2}{3},\frac{1}{3},\frac{2}{3}\right)$ (b) $\left(\frac{-2}{3},\frac{-1}{3},\frac{2}{3}\right)$

(c) $\left(\frac{2}{3},\frac{1}{3},\frac{2}{3}\right)$ (d) None of thene

65. The direction of line passes through given point is $2x$ - 3y = 7, z = 3, point (2, -1, 3)

(a) (3, -2, 0) (b) (3, 2, 0) (c) (-3, 2, 0) (d) (-3, -2, 0)

66. Line x = 2y + 1, 2y = 1 - z and $2x$ + y + z = 0, z + 2 = 0 angle between two line....

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

67. Line $\frac{x-1}{c}=\frac{y+3}{-1}=\frac{z-3}{2}$ and $\frac{x-3}{6}=\frac{y-1}{3}=\frac{4-z}{6}$ if direction are same then c = ...

(a) -2 (b) 2 (c) $\frac{1}{3}$ (d) - $\frac{1}{3}$

68. If the direction coside between two lines are (3, 4, -6). Then angle between two line is....

(a) $\cos^{-1}\frac{20}{5246}$ (b) $\cos^{-1}\frac{\sqrt{29}}{\sqrt{5246}}$ (c) $\cos^{-1}\frac{29}{\sqrt{5246}}$ (c) $\cos^{-1}\frac{\sqrt{29}}{5246}$

69. Equation of line passes through (0, 0) and forming an equal angle with the axis is ....

(a) $x = y = z$ (b) $x + y + z = 3$ (c) $x + y + z = 1$ (d) $x = y, z = 3$

70. $\frac{x-5}{7}=\frac{y-5}{k}=\frac{z-2}{5}$ and $\frac{x}{3}=\frac{y-21}{8}=\frac{3z-4}{5}$ are perpendicular then k = ...

(a) $\frac{11}{3}$ (b) - $\frac{11}{3}$ (c) $\frac{3}{11}$ (d) - $\frac{3}{11}$

71. Line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ and $\frac{x-l}{\alpha}=\frac{y-m}{\beta}=\frac{z-n}{\gamma}$ is ....

(a) intersecting (b) parallel

(c) non coplanner (d) perpendicular

72. Lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ $\therefore$ $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting in

(a) (1, 1, 1) (b) (-1, 1, -1) (c) (-1, -1, 1) (d) (-1, -1, -1)

73. The lines $x-3=\frac{y+2}{-1}=z-I$ and $\frac{x}{2}=\frac{z+3}{3}, y+I=0$ intersects at ......

(a) (2, I, 0) (b) (-2, 1, 0) (c) (-2, -1, 0) (d) (2, -1, 0)

74. $x = y = z$ $\therefore$ $x - 1 = y - 2 = z - 3$ then the perpendicular distance between the line = ...

(a) 2 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{2}{\sqrt{3}}$

75. The points which are at 5 unit dist. from (2, -1, 3) to

$\bar{r}=(-2, 2, 3)+k(-1, -2, 1), k \in R$ is ________

(a) (6, -4, 3), (-2, -2, 3) (b) (6, -4, 0) (-2, 2, 3)

(c) (6, -4, 3), (-2, 2, 3) (d) None of these

76. Line $\bar{r}=(1,2,1)+k(-1, -2, 1), k \in R$ the point which is at $\sqrt{6}$ dist. away from (2, 4, 0) is ....

(a) (1, 2, 1) (3, 6, -1) (b) (1, 2, 1) (3,-6,-1)

(c) (-1,-2,1), (3, 6, -1) (d) None of these

77. Perpendicular distance from point (1, 3, 4) to line $\frac{x-5}{2} = \frac{y-6}{-1} = \frac{z+7}{3}$ is ....

(a) $\frac{\sqrt{1398}}{7}$ (b) $\frac{\sqrt{1398}}{14}$ (c) $\sqrt{\frac{1398}{7}}$ (d) $\frac{1398}{7}$

78. Foot of perpendicular and perpendicular distance from P(2, -1, 5) and

line $\frac{x-1}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$ is .....

(a) (-1, -2, 3), $\sqrt{14}$ (b) (1, 2, 3), 14

(c) (-1, -2, -3), $\sqrt{14}$ (d) (1, 2, 3), $\sqrt{14}$

79. Foot of perpendicular and perpendicular distance from P(1, 0, 3) and line $\bar{r} =$ (4, 7, 1) +
k(1, 2, -2), k ∈ R is ....

(a) $\sqrt{13}, \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ (b) $13, \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

(c) $\sqrt{13}, \left(\frac{-5}{3}, \frac{-7}{3}, \frac{-17}{3}\right)$ (d) $13 \left(\frac{-5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

80. The Equation of line passing through (1, 2, 1) and $\frac{2x-1}{3} = \frac{1-y}{3} = \frac{3z-2}{5}$ is ....

(a) $\frac{2x-2}{3} = \frac{2-y}{3} = \frac{3z-3}{5}$ (b) $\frac{2x+2}{3} = \frac{2+y}{3} = \frac{3z+3}{5}$

(c) $\frac{2x-1}{-3} = \frac{1-y}{-3} = \frac{3z-2}{-1}$ (d) None of these

81. $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-5}{0} = \frac{y-3}{2} = \frac{z-2}{3}$. Then the equation of line passing through (3, -1, 11) and perpendicular to given line is....

(a) $\frac{x-3}{1} = \frac{y+1}{-6} = \frac{z+11}{4}$ (b) $\frac{x-3}{1} = \frac{y+1}{-6} = \frac{z-11}{4}$

(c) $\frac{x+3}{1} = \frac{y+1}{-6} = \frac{z+11}{4}$ (d) $\frac{x-3}{-1} = \frac{y+1}{6} = \frac{z+1}{4}$

82. P (1, 6, 3) to $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ on then image of p is ____________

(a) (-1, 0, -7) (b) (-1, 0, 7) (c) (1, 0, 7) (d) (1, 0, -7)

83. The equation of line passes through (1, 2, 3) and $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-1}{3}=\frac{y}{2}=\frac{z}{6}$ and perpendicular to given two line is...........

(a) $\frac{x-1}{14}=\frac{y-2}{-9}=\frac{z-3}{-4}$ (b) $\frac{x+1}{14}=\frac{y+2}{-9}=\frac{z+3}{4}$

(c) $\frac{x-1}{14}=\frac{y-2}{9}=\frac{z-3}{-4}$ (d) $\frac{x+1}{-14}=\frac{y+2}{9}=\frac{z+3}{4}$

84. The direction cosine of $x = a$y + b, z = cy + d ....

(a) $\pm\frac{a}{\sqrt{a^2+c^2+1}}, \pm\frac{1}{\sqrt{a^2+c^2+1}}, \pm\frac{c}{\sqrt{a^2+c^2+1}}$

(b) $\frac{a}{\sqrt{a^2+c^2+1}}, \frac{1}{\sqrt{a^2+c^2+1}}, \frac{c}{\sqrt{a^2+c^2+1}}$

(c) $\frac{-a}{\sqrt{a^2+c^2+1}}, \frac{-1}{\sqrt{a^2+c^2+1}}, \frac{-c}{\sqrt{a^2+c^2+1}}$ (d) None of these

85. If the lines

$l : x = a$y + b z = cy + b &

m : $x = a$’y + b z = c’y + d’

are perpendicular to each otehr then aa’ + cc’ + 3 = ...

(a) 2 (b) -2 (c) 0 (d) 1

86. Lines $\bar{r}=(1, 3, 5)+k(-1, 2, 3), k\in R$ and

$\bar{r}=(1, 3, 1)+k(1, -2, 3), k\in R$ are.........

(a) coincident (b) parallel

(c) skew (d) pependicular

87. Lines $\bar{r}=(2, 1, 3)+k(1, -1, 1)$ and $\bar{r}=(3, 0, 4)+k(-1, 1, -1)$ are .... (k ∈ R).

(a) coincident (b) skew (c) Intersecting (d) Parallel

88. Lines $\bar{r} = (1, 2, 6) + k(1, 3, 5)$ and $\bar{r} = (-1, 3, 5) + k(2, 1, 1)$, $k \in R$ are.....

(a) parallel (b) Intersecting (c) coincident (d) skew

89. Lines $\{(k + 3), -k-1, k + 1) / k \in R\}$, $\{(2k, 0, 3k-3) / k \in R\}$ are.......

(a) parallel (b) Intersecting (c) coincident (d) skew

90. $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$ lines are ....

(a) parallel (b) coincident (c) Intersecting (d) skew

91. The shortest distance between two lines $\frac{x-1}{1} = \frac{y+1}{3} = z$ and $\frac{x-1}{3} = \frac{y-2}{1}, z = 2$ is .....

(a) $\frac{7}{14}$ (b) $\frac{\sqrt{7}}{74}$ (c) $\frac{7}{\sqrt{74}}$ (d) $\sqrt{\frac{7}{74}}$

92. shortest distance between two lines
$x = 1 + t$, $y = 1 + 6t$, $z = 2t$, $t \in R$ and
$x = 1 + 2k$, $y = 5 + 15k$, $z = -2 + 6k$, $k \in R$ is..........

(a) 4 (b) 6 (c) 2 (d) 1

93. shortest distance between two lines

$\bar{r} = (4, -1, 0) + k(1, 2, -3)$, $k \in R$ and

$\bar{r} = (1, -1, 2) + k(2, 4, -5)$, $k \in R$ is ....

(a) $\frac{6}{\sqrt{5}}$ (b) $\frac{6}{5}$ (c) $\frac{\sqrt{6}}{5}$ (d) $\sqrt{\frac{6}{5}}$

94. Line L : $\bar{r} = (8, -9, 10) + k(3, -16, 7), k \in R$ and

M : $\bar{r} = (15, 29, 5) + k(3, 8, -5)$, $k \in R$. If $P \in L, Q \in M$, where $\overline{PQ}$ is shortest distance between L and M then PQ = ...

(a) $\sqrt{14}$ (b) 14 (c) $\frac{1}{14}$ (d) $\frac{1}{\sqrt{14}}$

95. For Lines : L : $\frac{x-23}{-6} = \frac{y-19}{-4} = \frac{z-25}{3}$ and M : $\frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2}$ and $P \in L$, $Q \in M$, $\overleftrightarrow{PQ} \perp L$ and $\overleftrightarrow{PQ} \perp M$, then PQ = ...

(a) $\sqrt{26}$ (b) $\frac{1}{26}$ (c) $\frac{1}{\sqrt{26}}$ (d) 26

96. If lengths of eages of cube are a, b, c, then the shortest distance between diagonal $\overleftrightarrow{OO'}$ and eage which is non-coplaner $\overleftrightarrow{OO'}$ to $\overleftrightarrow{AB'}$ is ......

(a) $\frac{ac}{\sqrt{a^2+c^2}}$ (b) $\frac{ab}{\sqrt{a^2+b^2}}$

(c) $\frac{bc}{\sqrt{b^2+c^2}}$ (d) $\frac{abc}{\sqrt{a^2+b^2+c^2}}$

97. If length each sides of cube is one unit, then shortest distance between diagonal $\overleftrightarrow{OO'}$ and
one eage $\overleftrightarrow{AB'}$ which is non-coplaner to $\overleftrightarrow{OO'}$ is ....

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 2

98. The equaton of plane passing through A(1, 2, 3), B(2, 1, 0), C(3, 3, -1) is ......

(a) $7x + 2y - 3z = 12$ (b) $7x - 2y + 3z = 12$

(c) $x + y + z = 12$ (d) $7x - 2y - 3z = 12$

99. If intercepts on axis are 3, -4, 7, then ..... point is on the plane.

(a) (2, -3, 1) (b) (1, 1, -2) (c) (1, -1, -3) (d) None of these

100. If $4x - 81y + 9z = 1$ is equation plane, then sum of its intercepts is....

(a) $\frac{1017}{2916}$ (b) $\frac{1017}{2916}$ (c) $\frac{101}{2916}$ (d) $\frac{-1017}{2916}$

101. The equation of plane which is passing through (2, 1, 3) and having equal X and Y-intercept and Z-intercept 14 is.........

(a) $11x - 11y + 3z = 42$ (b) $11x + 11y + 3z = 42$

(c) $11x + 11y - 3z = 42$ (d) $11x + 11y + 3z + 42 = 0$

102. The angle between $2x - y + 3 = 2$ and $x + y + 2z = 3$ is .....

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $-\frac{\pi}{3}$ (d) $\frac{4\pi}{3}$

103. The angle between line $\bar{r} = (-1, 1, 2) + k(3, 2, 4), k \in R$ and plane $2x + y - 3z + 4 = 0$ is...

(a) $\cos^{-1}\left(\frac{4}{\sqrt{406}}\right)$ (b) $\sin^{-1}\left(\frac{4}{\sqrt{406}}\right)$ (c) $\sin^{-1}\frac{1}{9}$ (d) $\cos^{-1}\frac{1}{\sqrt{19}}$

104 The angle between line $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and plane $2x-2y+z=1$ is ....

(a) $\sin^{-1}\frac{1}{\sqrt{19}}$ (b) $\cos^{-1}\left(\frac{1}{9}\right)$ (c) $\sin^{-1}\left(\frac{1}{9}\right)$ (d) $\cos^{-1}\frac{1}{\sqrt{19}}$

105. The foot of the perpendicular and perpendicular distance from point (1, 2, 3) to plane x - 2y + 2z = 5 is .... and .... respectively

(a) $\left(\frac{11}{9}, \frac{14}{9}, \frac{31}{9}\right), \frac{3}{2}$ (b) $\left(\frac{-11}{9}, \frac{-14}{9}, \frac{-31}{9}\right), \frac{2}{3}$

(c) $\left(\frac{11}{9}, \frac{14}{9}, \frac{31}{9}\right), \frac{2}{3}$ (d) $\left(\frac{11}{9}, \frac{14}{9}, \frac{-31}{9}\right), \frac{2}{3}$

106. The equation of the line of the intersection of the planes x + 2y - 3z = 6 and 2x - y + z = 7 is.....

(a) $\frac{x-4}{1}=\frac{y-1}{7}=\frac{z}{5}$ (b) $\frac{x+4}{1}=\frac{y-1}{7}=\frac{z}{5}$

(c) $\frac{x+1}{1}=\frac{y+1}{7}=\frac{z}{5}$ (d) $\frac{x-1}{-1}=\frac{y-1}{-7}=\frac{z}{5}$

107. The Image of point (1, 3, 4) with respect to the plane 2x - y + z +3 = 0 is....

(a) (3, 5, 2) (b) (-3, -5, 2) (c) (-3, -5, -2) (d) (-3, 5, 2)

108. The perpendicular distance and foot of perpendicular from A(2, -1, 1) to the plane 2x - 3y + 4z = 44 is ....

(a) $\sqrt{29}, \ (4, -4, -6)$ (b) $\sqrt{29}, \ (4, -4, 6)$

(c) $\sqrt{29}, \ (4, 4, 6)$ (d) $\sqrt{29}, \ (-4, -4, 6)$

109. If plane 2x -2 y + z = -3 express in form of $x \cos\alpha + y\cos\beta + z\cos\gamma = p$, then prpedicular distance from origin to the plane is ...... foot of perpendicular is .... and direction cosine is.........

(a) $1, \left(-\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right), -\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$ (b) $2, \left(-\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right), -\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$

(c) $1, \left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right), \frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ (d) None of these

110. For points A(1, 2, 3), B(5, 4, 1), the equation of plane which is perpendicular bisector of $\overline{AB}$ is..................

(a) x + 2y - 7 + z =0 (b) 2x + y - z = 7

(c) x + 2y + z + 7 = 0 (d) 2x - 2y - z = 7

111. The equation of plane which is perpendicular to the planes $3x + y + z = 0$ and $x + 2y + 3z = 5$ and passing through (1, 3, 5) is ......

(a) $x + 2y + z = 0$ (b) $x - 2y - z = 0$

(c) $x - 2y + z = 0$ (d) $x + 2y - z = 0$

112. If two planes $\bar{r}.(2, -b, 1) = 4$ and $\bar{r}.(4, -1, -c) = 6$ are parallel then b, c = ...

(a) $-\frac{1}{2}, -2$ (b) $\frac{1}{2}, 2$ (c) $-\frac{1}{2}, 2$ (d) $\frac{1}{2}, -2$

113. If perpendicular distance between two planes $3x - 2y + z = 1$ and $6x - 4y + 2z = k$ is $\frac{3}{2\sqrt{14}}$ then $k =$ ....

(a) 5, -1 (b) -5, 1 (c) -5, -1 (d) 5, -1

114. If lines $\frac{x-1}{2} = \frac{y-3}{4} = z$ and $\frac{x-4}{3} = \frac{1-y}{2} = \frac{z-1}{1}$ are co-planer, then the equation of plane containing these two lines is ....

(a) $6x + y + 16z = 9$ (b) $6x + y - 16z = 9$

(c) $6x - y - 16z = 9$ (d) $6x - y + 16z = 9$

115. The equation of plane passing through the lines

$\frac{x}{2} = \frac{y-1}{1} = \frac{z+2}{2}$ and $\frac{2x+3}{4} = \frac{3-y}{-1} = \frac{z}{2}$ is ....

(a) $4x + 11y + 14z = 36$ (b) $4x + 14y - 11z = 36$

(c) $4x - 14y - 11z = 36$(d) $4x - 14y + 11z = 36$

116. The equation of plane passing through point (1, -1, 2) and

$\bar{r} = (1, 1, 1) + k(2, 1, 2), k \in R$ is................

(a) $5x - 2y - 4z + 1 = 0$ (b) $5x + 2y + 4z + 1 = 0$

(c) $5x - 2y + 4z + 1 = 0$ (d) $5x - 2y + 4z = 1$

117.The equtiion of plane passing through the lines

$L: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $M: \frac{x-1}{2} = \frac{y}{3} = \frac{z-5}{4}$ is.........

(a) $7x + 2y + 2z - 3 = 0$ (b) $7x - 2y + 2z - 3 = 0$

(c) $7x - 2y - 2z + 3 = 0$ (d) $7x + 2y - 2z + 3 = 0$

118. The equation of plane passing through the lines

$L: \frac{x+3}{2} = \frac{y+3}{3} = \frac{z-7}{-3}$ and $M: \frac{x+1}{4} = \frac{y+1}{5} = \frac{z+1}{-1}$ is.........

(a) 6x + 5y - z = 0 (b) 6x - 5y - z = 0

(c) 6x - 5y + z = 0 (d) 6x + 5y + z = 0

119. The equation of plane passing through the point A(1, 2, 3), B(3, -1, 2) also perpendicular to $x + 3y + 2z = 7$ is.......

(a) $3x + 5y - 9z + 14 = 0$ (b) $3x - 5y - 9z + 14 = 0$

(c) $3x - 5y + 9z + 14 = 0$ (d) $3x + 5y + 9z + 14 = 0$

120. The equation of planes which are parallel to plane $x + 2y + 2z = 1$ and at 2 unit distant from it are ....

(a) $x + 2y + 2z = 7$

(b) $x + 2y + 2z = 5$

(c) $x + 2y + 2z = 7$ and $x + 2y + 2z = -5$

(d) $x + 2y + 2z = -7$ and $x + 2y + 2z = 5$

121. The equation of plane passing through $(1, 6, -4)$ and containing

$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{-1}$ is ....

(a) $25x + 14y + 8z = 77$ (b) $25x + 14y - 8y = 77$

(c) $25x - 14y - 8z = 77$ (d) $25x + 14y + 8y = -77$

122. Equation of plane parallel to $2x + 4y + 8y = 17$ containing and line

$\frac{x-3}{2} = y = \frac{z-8}{-1}$ is ....

(a) $x - 2y - 48 = 35$ (b) $x - 2x - 4z = 35$

(c) $x + 2y + 4z = 35$ (d) $x + 2y - 4z = 35$

123. The equation of plane passing through the intersection of planes $x + y + z + 1 = 0$ and $x - 3y + z + 3 = 0$ and parallel to $2x = y = 2z$ is ....

(a) $x - y + z + 2 = 0$ (b) $x - y - z - 2 = 0$

(c) $x + y - 3 + 2 = 0$ (d) $x + y + z + 2 = 0$

124. The equation of plane passing thorugh the intersection of the planes $x - y + z = 1$ and $x + y - z = 1$ and perpendicular to $x - 2y + z = 2$ is ....

(a) $x + 3y + z = 3$ (b) $3x + y - z = 3$

(c) $x - 3y - z = 3$ (d) $x - 3y + z = 3$

125. If y intercept of plane $(x - y + z - 1) + \lambda (x + y - z - 1) = 0$ is 3 unit then λ = ...

(a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

126. If the equation of plane is at 3p distance from origin which intersect the axis at A, B, C then the centroid of ΔABC from an equation......

(a) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=-\frac{1}{p^2}$ (b) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

(c) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}+\frac{1}{p^2}=1$ (d) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=0$

127. The equation of the plane which intersect the axis at A, B, C and the centroid of ΔABC is (2, 1, 3) is ....

(a) $3x + 6y + 2z + 18 = 0$ (b) $3x + 6y + 2z = 18$

(c) $3x + 6y + z = 0$ (d) $x + y + z = 18$

128. The equation of the plane which intersects the axis at A, B, C and the centrioid of ΔABC is (α, β, γ) is ....

(a) $x + y + z = 3\alpha\beta\gamma$ (b) $x + y + z = 3$

(c) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$ (d) $x + y + z = \alpha\beta\gamma$

129. The locus of point of the plane passing thorugh (α, β, γ) and intersect the axis in A, B, C and the plane which is parallel to such plane is..............

(a) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$ (b) $\frac{\alpha}{x}+\frac{\beta}{y}+\frac{\gamma}{z}=1$

(c) $x + y + z = 1$ (d) $x + y + z = \alpha\beta\gamma$

130. If perpendicular distance from (0, 0, 0) to the variable plane is p and variable palne intersects the axis in A, B, C, the centroid of ΔABC is on

$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=$ ....

(a) $\frac{a}{p^2}$ (b) $\frac{p^2}{9}$ (c) $\frac{p}{9}$ (d) $\frac{9}{p}$

131. The distance of a variable plane from origin to plane is p and the Variable plane intersects the axis in A, B, C, then the point of intersection of given plane and the plane parallel to the co-ordinate plane is on $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=$ ....

(a) p^2 (b) $\frac{1}{p^2}$ (c) p (d) $\frac{1}{p}$

132. Line of intersection of the planes $2x + y + 2z = 1$, $x + 2y - 2z = 1$ and $6x + 2y + 3z = 1$, $6x + 2y - 3z = 1$ is ..... and point of intersection is ....

(a) intersecting (1, 1, 1)

(b) Perpendicular (-1, 1, 1)

(c) non-coplaner lines, does not exist

(d) Parallel, does not exist

133. Perpendicular distance between line $\bar{r} = (2, -2, 3) + k(1, -1, 4)$, $k \in R$ and $x + 5y + z = 5$ is ....

(a) $\frac{10}{3}$ (b) $\frac{10}{3\sqrt{3}}$ (c) $\frac{10}{\sqrt{3}}$ (d) 10

ANSWER

1.	(B)	36.	(D)	71.	(A)	106.	(A)
2.	(A)	37.	(D)	72.	(D)	107.	(D)
3.	(B)	38.	(A)	73.	(D)	108.	(B)
4.	(B)	39.	(C)	74.	(B)	109.	(A)
5.	(B)	40.	(D)	75.	(C)	110.	(B)
6.	(A)	41.	(C)	76.	(A)	111.	(C)
7.	(C)	42.	(A)	77.	(C)	112.	(B)
8.	(C)	43.	(A)	78.	(B)	113.	(A)
9.	(D)	44.	(C)	79.	(A)	114.	(B)
10.	(A)	45.	(A)	80.	(A)	115.	(D)
11.	(D)	46.	(A)	81.	(B)	116.	(A)
12.	(B)	47.	(C)	82.	(C)	117.	(C)
13.	(D)	48.	(B)	83.	(A)	118.	(B)
14.	(A)	49.	(B)	84.	(A)	119.	(A)
15.	(A)	50.	(B)	85.	(A)	120.	(C)
16.	(B)	51.	(B)	86.	(B)	121.	(A)
17.	(A)	52.	(A)	87.	(A)	122.	(C)
18.	(C)	53.	(D)	88.	(D)	123.	(A)
19.	(A)	54.	(A)	89.	(B)	124.	(B)
20.	(D)	55.	(B)	90.	(D)	125.	(B)
21.	(D)	56.	(B)	91.	(C)	126.	(B)
22.	(B)	57.	(C)	92.	(C)	127.	(B)
23.	(B)	58.	(A)	93.	(A)	128.	(C)
24.	(B)	59.	(A)	94.	(B)	129.	(B)
25.	(C)	60.	(B)	95.	(D)	130.	(A)
26.	(A)	61.	(B)	96.	(B)	131.	(B)
27.	(D)	62.	(A)	97.	(B)	132.	(C)
28.	(A)	63.	(B)	98.	(B)	133.	(B)
29.	(A)	64.	(D)	99.	(D)		
30.	(B)	65.	(B)	100.	(A)		
31.	(A)	66.	(D)	101.	(B)		
32.	(D)	67.	(B)	102.	(B)		
33.	(C)	68.	(C)	103.	(B)		
34.	(A)	69.	(A)	104.	(C)		
35.	(B)	70.	(B)	105.	(C)		

Hint

1. Point on X-axis which is equidistant from A(2, -5, 7) and B(1, 3, 6) is P(x, 0, 0)

(b) $AP^2 = PB^2$

$\therefore (x-2)^2 + 25 + 49 = (x-1)^2 + 9 + 36$

$-2x = -32 \qquad x = 16 \qquad (16, 0, 0)$

2. $A(4, 5, 21), B(1, 6, 3), P(x, y, 3), AP^2 = BP^2$

(a) $(x-4)^2 + (y-5)^2 + (z-2)^2 = (x-1)^2 + (y-6)^2 + (z-3)^2$

$\therefore 6x - 2y - 2z + 1 = 0$

3. $A(-1, 2, 0), B(1, 2, 3), (4, 2, 1)$

(b) $\Delta ABC \qquad AB = \sqrt{13}$, $BC = \sqrt{13}$, $CA = \sqrt{26}$

$AB = BC$ and $AB^2 + BC^2 = CA^2$

$\therefore$ Isosceles right angled

4. $A(1, 1, 1)\ B(-2, 4, 1), (-1, 5, 5), D(2, 2, 5)$

(b) $AB = \sqrt{18} = 3\sqrt{2}, BC = \sqrt{18} = 3\sqrt{2}, CD = \sqrt{18} = 3\sqrt{2}$

$AD = \sqrt{18} = 3\sqrt{2}$

and $AC = \sqrt{36} = 6, BD = \sqrt{36} = 6$

$AB = BC = CD = AD$ and $AB^2 + BC^2 = AC^2$

and $BC^2 + CD^2 = BD^2$

$\therefore$ vertices of square.

5. $A(1, 1, 2), B(2, 3, 5), C(1, 3, 4), D(0, 1, 1)$

(b) $\overrightarrow{AB} = (1, 2, 3), \left|\overrightarrow{AB}\right| = \sqrt{1+4+9} = \sqrt{14}$

$\overrightarrow{BC} = (-1, 0, 1)\ \left|\overrightarrow{BC}\right| = \sqrt{1+0+1} = \sqrt{2}$

$\overrightarrow{CD} = (-1, -2, 3),\ \left|\overrightarrow{CD}\right| = \sqrt{1+4+9} = \sqrt{14}$

$\overrightarrow{AD} = (-1, 0, -1),\ \left|\overrightarrow{AD}\right| = \sqrt{2}$

$\overrightarrow{AC} = (0, 2, 2),\ \left|\overrightarrow{AC}\right| = \sqrt{4+4} = \sqrt{8}$

$\overrightarrow{BD} = (-2, -2, -4),\ \left|\overrightarrow{BD}\right| = \sqrt{4+4+16} = \sqrt{24}$

$AB = CD$ and $BC = AD$

$AB^2 + BC^2 \neq AC^2$ It form parallelogram

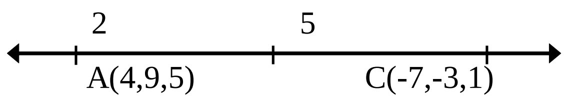
Area of $\square^m ABCD = \left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$

$= \left|(-2, -2, 2)\right|$

$= \sqrt{4+4+4} = 2\sqrt{3}$ unit.

6. $A(7, -3, 1),\ B(4, 9, 8)$

(a) co-ordinates of point diving $\overline{AB}$



$= \left(\frac{2(7)+5(4)}{2+5}, \frac{2(-3)+5(9)}{2+5}, \frac{2(1)+5(8)}{2+5}\right)$

$= \left(\frac{34}{7}, \frac{39}{7}, \frac{42}{7}\right)$

7. $A(1, 5, 6), B(3, 1, 2), C(4, -1, 0)$, B divides $\overline{AC}$ in ratio is $\lambda : 1$ then

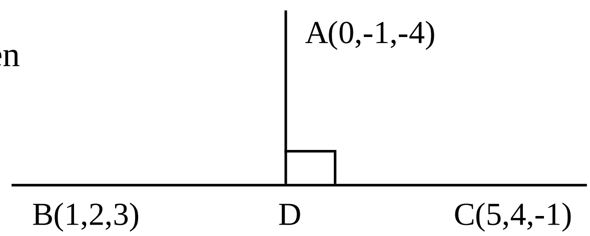
(c) $3 = \frac{4\lambda+1}{\lambda+1}, \quad 1 = \frac{-\lambda+5}{\lambda+1}, \quad 2 = \frac{0+6}{\lambda+1}$

$\lambda = 2 \quad \lambda = 2 \quad \lambda = 2$

8. $A(0, -1, -4), B(1, 2, 3), C(5, 4, -1)$

(c) D divides, $\overline{BC}$ from B in ratio $\lambda : 1$ then

$D = \left(\frac{5\lambda+1}{\lambda+1}, \frac{4\lambda+2}{\lambda+1}, \frac{-\lambda+3}{\lambda+1}\right)$



$\overrightarrow{BC} = (4, 2, -4)$

$\overrightarrow{AD} = \left(\frac{5\lambda + 1}{\lambda + 1}, \frac{5\lambda + 3}{\lambda + 1}, \frac{3\lambda + 7}{\lambda + 1}\right)$

$\overrightarrow{BC} \perp \overrightarrow{AD}$

$\therefore \overrightarrow{BC} \cdot \overrightarrow{AD} = 0$

$4\left(\frac{5\lambda + 1}{\lambda + 1}\right) + 2\left(\frac{5\lambda + 3}{\lambda + 1}\right) + (-4)\left(\frac{3\lambda + 7}{\lambda + 1}\right) = 0$

$18\lambda = 18$

$\lambda = 1$

Foot of perpendicular D(3, 3, 1)

9. $(2, 3, 5) = \left(\frac{a - 1 + 1}{3}, \frac{1 + 3b0}{3}, \frac{2 + 3 + C}{3}\right)$

(d) $\frac{a}{3} = 2$ $\frac{b + 1}{3} = 3$ $\frac{C + 5}{3} = 5$

$a = 6$ $b = 8$ $C = 10$

10. $A(6, 4, 6)$, $B(12, 4, 0)$ $C(4, 2, -2)$

(a) $a = BC = \sqrt{64 + 4 + 4} = \sqrt{72}$

$b = AC = \sqrt{4 + 4 + 64} = \sqrt{72}$

$c = AB = \sqrt{36 + 0 + 36} = \sqrt{72}$

$a = b = c = \sqrt{72}$ ΔABC, is equilateral triangle and in incentre and centroid are equal

$\therefore$ centriod $= \left(\frac{6 + 12 + 4}{3}, \frac{4 + 4 + 2}{3}, \frac{6 + 0 - 2}{3}\right) = \left(\frac{22}{3}, \frac{10}{3}, \frac{4}{3}\right)$

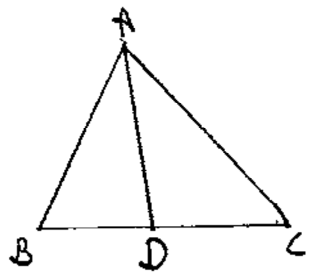
11. The centroid of triangle and centroid of triangle form by mid point of given.Triangle are equal

(d) $\therefore$ Centroid of ΔABC = centroid of ΔPQR

$= \left(\frac{9 + (-7) + 8}{3}, \frac{2 + 6 + (-9)}{3}, \frac{5 + 1 + 3}{3}\right)$

$\left(\frac{10}{3}, \frac{-1}{3}, 3\right)$

12. A $(-1, -2, -3)$, B $(1, 2, 3)$, C $(1, 2, 1)$



(b) Centroid of ΔABC G$\left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$

Mid point of $\overline{BC}$ is D $(1, 2, 2)$

Length of median = $\overline{AD}$

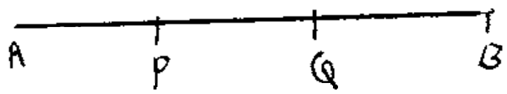
$\therefore$ AD = $\sqrt{4+16+25} = \sqrt{45}$

$= 3\sqrt{5}$ unit.

13. A $(-5, 7, 2)$, B $(1, 3, 7)$ is P and Q are points of trisection then

(d) Q divides $\overline{AB}$ from A side in ratio 2:1.

$$Q = \left(\frac{2(1)+1(-5)}{2+1}, \frac{2(3)+1(7)}{2+1}, \frac{2(7)+1(2)}{2+1}\right)$$



$\left(-1, \frac{13}{3}, \frac{16}{3}\right)$

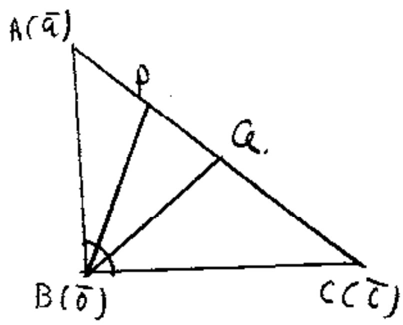
P is mid point of $\overline{AC}$

Co-ordi of P $\left(\frac{-1-5}{2}, \frac{7+\frac{13}{3}}{2}, \frac{\frac{16}{3}+2}{2}\right)$

$\left(-3, \frac{17}{3}, \frac{11}{3}\right)$

14. Suppose the position vector of A$(\bar{a})$, B$(\bar{o})$, C$(\bar{c})$, in ΔABC

(a) P and Q divide $\overline{AC}$ from A in ratio 1:2 and 2:1



$\therefore P\left(\frac{2\bar{a}+\bar{c}}{3}\right), Q\left(\frac{\bar{a}+2\bar{c}}{3}\right)$

But $\bar{a}\cdot\bar{c} = \overrightarrow{AB}\cdot\overrightarrow{BC} = 0$

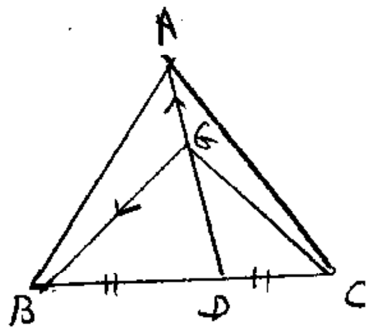
$$BP^2 + BQ^2 = \frac{1}{9}\left|2\bar{a}+\bar{c}\right|^2 + \frac{1}{9}\left|\bar{a}+2\bar{c}\right|^2$$

$= \frac{1}{9}\left[5\,|\,\bar{a}\,|^2 + 5\,|\,\bar{c}\,|^2\right]$

$= \frac{5}{9}\left[AB^2 + BC^2\right]$

$= \frac{5}{9}\,AC^2 \qquad AB^2 + BC^2 = AC^2 \qquad m\angle B = \frac{\pi}{2}$

15. In ΔABC the position vector of A, B, C is $\bar{X}, \bar{Y}, \bar{Z},$ respectively and G is centroid with position vector ΔABC

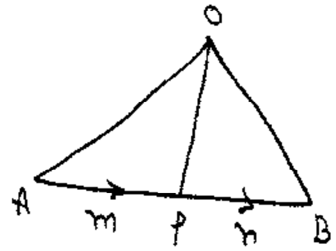


(a) $\left(\bar{O}\right) = \left(\frac{\bar{X} + \bar{Y} + \bar{Z}}{3}\right)$

$\therefore\ \bar{X} + \bar{Y} + \bar{Z} = \bar{O}$

$\therefore\ \overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \bar{O}$

16. Here the direction of $\overrightarrow{AP}$ and $\overrightarrow{PB}$ are same and $\frac{AP}{PB} = \frac{m}{n}$



(b) $\therefore\ n\,\overrightarrow{AP} = m\,\overrightarrow{PB}$

$n\left(\overrightarrow{OP} - \overrightarrow{OA}\right) = m\left(\overrightarrow{OB} - \overrightarrow{OP}\right)$

$\therefore\ (m + n)\,\overrightarrow{OP} = n\,\overrightarrow{OA} + m\,\overrightarrow{OB}$

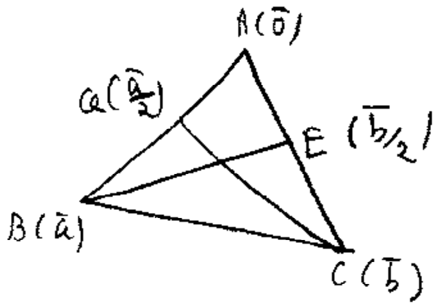
17. In ΔABC the position vector are $A\left(\bar{O}\right)\ B\left(\bar{a}\right)\ C\left(\bar{b}\right)$

(a) The mid point of $\overline{AB}$ and $\overline{AC}$ are D and E respectively $D\left(\frac{\bar{a}}{2}\right),\ E\left(\frac{\bar{b}}{2}\right)$

$\overrightarrow{BE} + \overrightarrow{DC} = \left(\frac{\bar{b}}{2} - \bar{a}\right) + \left(\bar{b} - \frac{\bar{a}}{2}\right)$

$= \frac{1}{2}\left(3\bar{b} - 3\bar{a}\right) = \frac{3}{2}\left(\bar{b} - \bar{a}\right)$

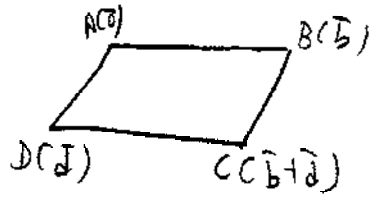
$= \frac{3}{2}\,\overrightarrow{BC}$



18. In parallelogram, $A\left(\bar{O}\right)\ B\left(\bar{a}\right),\ d\left(\bar{d}\right)$ then $C\left(\bar{b} + \bar{d}\right)$.

(c) $AB^2 + BC^2 + CD^2 + DA^2$

$= \left| \bar{b} \right|^2 + \left| \bar{d} \right|^2 + \left| -\bar{b} \right|^2 + \left| -\bar{d} \right|^2$



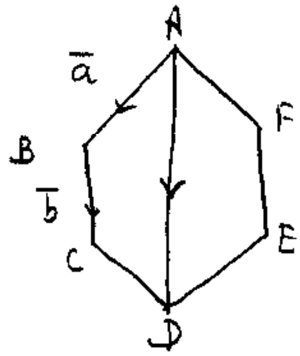
$= 2\left(\left| \bar{b} \right|^2 + \left| \bar{d} \right|^2 \right)$

$AC^2 + BD^2 = \left| \bar{b} + \bar{d} \right|^2 + \left| \bar{d} - \bar{b} \right|^2 = 2\left(\left| \bar{b} \right|^2 + \left| \bar{d} \right|^2 \right)$

$AB^2 + BC^2 + CD^2 + DA^2 = K\left(AC^2 + BD^2\right)$

$\therefore K = 2$

19. Here $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}$



(a) $\bar{a} + \bar{b} + \overrightarrow{AF} = 2\overrightarrow{BC}$

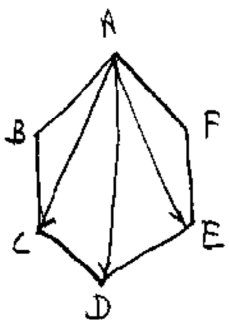
$\therefore \bar{a} + \bar{b} + \overrightarrow{AF} = 2\bar{b}$

$\therefore \overrightarrow{AF} = \bar{b} - \bar{a} \quad \left(\because \overrightarrow{AD} = 2\overrightarrow{BC}\right)$

20. $\overrightarrow{AB} = \overrightarrow{ED},\ \overrightarrow{AF} = \overrightarrow{CD}$

(b) $\overrightarrow{AE} + \overrightarrow{ED} = \overrightarrow{AD}$

and $\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}$



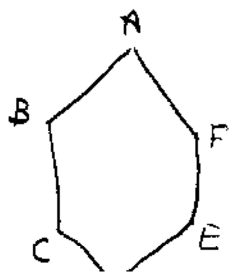
Here $AB^2 + AC^2 + AD^2 + \overrightarrow{AE} + \overrightarrow{AF}$

$= \overrightarrow{ED} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{CD}$

$= \left(\overrightarrow{AE} + \overrightarrow{ED}\right) + \left(\overrightarrow{AC} + \overrightarrow{CD}\right) + \overrightarrow{AD}$

$= \overrightarrow{AD} + \overrightarrow{AD} + \overrightarrow{AD} = 3\overrightarrow{AD}$

21. In regular hexagon, ABCDEF



(d) $\overrightarrow{AB} = \overrightarrow{ED},\ \overrightarrow{BC} = \overrightarrow{FE}$

$\overrightarrow{CD} = \overrightarrow{AF}$

$\therefore \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}$

$\therefore \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{AF} + \overrightarrow{FE} + \overrightarrow{ED}$

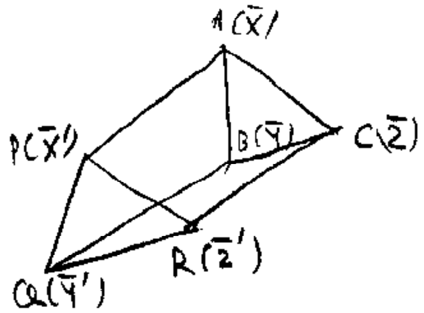
$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{CD} + \overrightarrow{BC} + \overrightarrow{AB}$

$= 2\left(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}\right)$

$= 2\,\overrightarrow{AD}$

22. In ΔABC and ΔPQR If the centroid are G and G' respectively

(b) if position-vector of A, B, C are $\overline{X}, \overline{Y}, \overline{Z}$ respectively and position-vector of P, Q, R are $\overline{x}', \overline{y}', \overline{z}'$ respectively then position vectors of G and G' are $\frac{\overline{X}+\overline{Y}+\overline{Z}}{3}$ and $\frac{\overline{X}'+\overline{Y}'+\overline{Z}'}{3}$



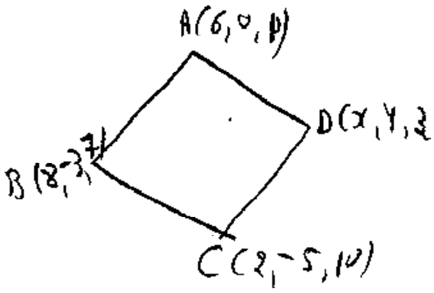
$\overline{AP} + \overline{BQ} + \overline{CR} = \left(\overline{X}' - \overline{X}\right) + \left(\overline{Y}' - \overline{Y}\right) + \left(\overline{Z}' - \overline{Z}\right)$

$= 3\left[\frac{\left(\overline{X}' + \overline{Y}' + \overline{Z}'\right)}{3} - \frac{\overline{X}+\overline{Y}+\overline{Z}}{3}\right]$

$= 3\,\overrightarrow{G}G'$

23. A $(6, 0, 1)$, B $(8, -3, 7)$, C $(2, -5, 10)$

(b) $\overrightarrow{AB} = (2, -3, 6)$ $\left|\overrightarrow{AB}\right| = \sqrt{4+9+36} = 7$



$\overrightarrow{BC} = (-6, -2, 3)$ $\left|\overrightarrow{BC}\right| = 7$

$AB = BC$

A, B, C are three vertices, $D\,(x, y, z)$ is forth vertices.

□^m ABCD $\therefore$ $\overrightarrow{AD} = \overrightarrow{BC}$

$(x-6, y-0, z-1) = (-6, -2, 3)$

$x = 0, y = -2, z = 4$

$(0, -2, 4)$

24. (b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

25. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(c) $\frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

26. $\cos\alpha = \cos\frac{\pi}{3} = \frac{1}{2}, \cos\gamma = \cos\frac{2\pi}{3} = \frac{-1}{2}$

(a) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\frac{1}{4} + \cos^2\beta + \frac{1}{4} = 1 \ \cos^2\beta = \frac{1}{2}$

$\cos\beta = \pm\frac{1}{\sqrt{2}}$

$\beta = \frac{\pi}{4}$ & $\beta = \frac{3\pi}{4}$

27. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad \cos\alpha = \frac{1}{2}$

(d) $\frac{1}{4} + \cos^2\beta + \cos^2\gamma = 1, \ \cos^2\beta + \cos^2\gamma = \frac{3}{4}$

There are many such values exist satisfing above $\cos\beta$, and $\cos\gamma$

$\therefore$ Infinite vectors.

28. Direction cosine $\cos\frac{\pi}{3}, \cos\frac{\pi}{6}, \cos\frac{\pi}{4}$

(a) $\therefore \frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$

$\frac{\overline{X}}{(\overline{X})} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right)$ yLk $(\overline{X}) = 4$

$\overline{X} = (2, 2\sqrt{3}, 2\sqrt{2})$

29. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \qquad \alpha = \beta, \quad \alpha = \gamma$

(c) $\therefore 3\cos^2\alpha = 1 \qquad \cos\alpha = \pm\frac{1}{\sqrt{3}}$

$\alpha = \cos^{-1}\frac{1}{\sqrt{3}} \qquad \alpha = \pi - \cos^{-1}\frac{1}{\sqrt{3}}$

$0 < \alpha < \frac{\pi}{2}$

$\therefore\ \alpha = \cos^{-1} \frac{1}{\sqrt{3}}$

30. $\overline{X} = (a, 3, -2),\ \overline{Y} = (a, -a, 2)\ \ \overline{X} \perp \overline{Y} \Leftrightarrow \overline{X} \cdot \overline{Y} = 0$

(b) $(a, 3, -2) \cdot (a, -a, 2) = 0$

$a^2 - 3a - 4 = 0,\ (a - 1)(a + 11) = 0$

$a = 4,$ or $a = -1$

31. $\overline{X} = i + \sqrt{3}, = (1, \sqrt{3}),\ \overline{Y} = \sqrt{3}\ i + a\ J = (\sqrt{3}, a)$

(a) $\overline{X}\overline{Y} = \frac{\pi}{3}, \cos(\overline{X}\,\overline{Y}) = \cos\frac{\pi}{3} = \frac{1}{2}$

$\frac{\overline{X} \cdot \overline{Y}}{|\overline{X}||\overline{Y}|} = \frac{1}{2}$

$\therefore \frac{\sqrt{3} + a\sqrt{3}}{\sqrt{1+3}\sqrt{3+a^2}} = \frac{1}{2},$

$\therefore \sqrt{3}\,(a+1) = \sqrt{3 + a^2}$

$3a^2 + 6a + 3 = 3 + a^2$

$\therefore 2a\,(a + 3) = 0,$

$\therefore a = 0, a = -3$

For, $a = 0$, $\frac{\sqrt{3} + \sqrt{3}\,(0)}{2\sqrt{3+0}} = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$

For $a = -3$, $\frac{\sqrt{3} + \sqrt{3}\,(-3)}{2\sqrt{3+9}} = -\frac{-2\sqrt{3}}{2\sqrt{12}} = \frac{1}{2} \neq \frac{1}{2}$

$a = 0$ is possible, $a = -3$ is not possible.

32. $\overline{X} = (2, -4, 3),\ \overline{Y} = (5, 0, 1)$

(d) $\overline{X} \times \overline{Y} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 2 & -4 & 3 \\ 5 & 0 & 1 \end{vmatrix} = i\,(-4-0) - J\,(2-15) + K\,(0+20)$

$= (-4, 13, 20)$

$$\left|\overline{X} \times \overline{Y}\right| = \sqrt{16 + 169 + 400} = \sqrt{585}$$

vector perpendicular to both vector is $= \pm \dfrac{\overline{X} \times \overline{Y}}{\left|\overline{X} \times \overline{Y}\right|}$

$$= \pm \left(\frac{-4}{\sqrt{585}}, \frac{13}{\sqrt{585}}, \frac{20}{\sqrt{585}}\right)$$

33. Suppose unit vector in XY - plane is (a, b, o) which is perpendiculat to $(4, -3, 2)$

(c) $(a, b, 0)(4, -3, 2) = 0$

$4a - 3b = 0 \qquad a = \dfrac{3b}{4} \qquad ...(1)$

(a, b, o) is unit vector

$a^2 + b^2 = 1 \qquad ....(2)$

$\dfrac{9b^2}{16} + b^2 = 1, \quad 25b^2 = 16, \qquad b = \pm \dfrac{4}{5}$

$\therefore a = \pm \dfrac{3}{5}$

$\therefore \pm \dfrac{1}{5}(3, 4, 0)$

34. $\overline{a}, \overline{b},$ are unit vectors. $|\overline{a}| = |\overline{b}| = 1 \quad \overline{a}\hat{\,}\overline{b} = \alpha$

(a) $\cos\alpha = \overline{a} \cdot \overline{b}$

$\left|\overline{a} - \overline{b}\cos\alpha\right|^2 = |a|^2 - 2\overline{a}.\overline{b}\cos\alpha + |\overline{b}|^2 \cos^2\alpha$

$= 1 - 2\overline{a} \cdot \overline{b}\cos\alpha + \cos^2\alpha$

$= 1 - 2.\cos\alpha\cos\alpha + \cos^2\alpha$

$= 1 - \cos^2\alpha$

$= \sin^2\alpha$

$\therefore |\overline{a} - \overline{b}\cos\alpha| = \sin\alpha \qquad 0 < \alpha < \dfrac{\pi}{2}$

35. $|\overline{a}| = |\overline{b}| = 1, \cos\theta = \overline{a}.\overline{b}$

(b) $\left|\bar{a}+\bar{b}\right|^2 = |\bar{a}|^2 + 2\,\bar{a}.\bar{b} + |\bar{b}|^2$

$= 1 + 2\cos\theta + 1$

$= 2\left(2\cos^2\frac{\theta}{2}\right) \qquad 0 < \theta < \pi$

$\cos\frac{\theta}{2} = \frac{1}{2}|\bar{a}+\bar{b}| \qquad 0 < \frac{\theta}{2} < \frac{\pi}{2}$

36. $\left|\bar{a}\right| = \left|\bar{b}\right| = 1 \quad \cos\theta = \bar{a}\,.\,\bar{b}$

(d) $\left|\bar{a}-\bar{b}\right|^2 = \left|\bar{a}\right|^2 - 2\bar{a}.\,\bar{b} + |\bar{b}|^2$

$= 1 - 2\cos\theta + 1$

$= 2\,(1-\cos\theta)$

$= 2.2\,\sin^2\frac{\theta}{2} \qquad 0 < \theta < \pi$

$\sin\frac{\theta}{2} = \frac{1}{2}\left|\bar{a}-\bar{b}\right| \qquad 0 < \frac{\theta}{2} < \frac{\pi}{2}$

37. $\bar{X} = (2, -6, 3),\ \bar{Y} = (1, 2, -2)$

(d) $\bar{X}\times\bar{Y} = \begin{vmatrix} i & j & k \\ 2 & -6 & 3 \\ 1 & 2 & -2 \end{vmatrix} = 6i + 7J + 10k$

$\left|\bar{X}\times\bar{Y}\right| = \sqrt{36+49+100} = \sqrt{185}$

$\left|\bar{X}\right| = \sqrt{4+36+9} = 7,\ \left|\bar{Y}\right| = \sqrt{1+4+4} = 3$

$\sin\theta = \frac{|\bar{X}\times\bar{Y}|}{|\bar{X}||\bar{Y}|} = \frac{\sqrt{185}}{21}$

38. Angle between $\bar{a}$ & $\bar{b}$ is $\frac{\pi}{6}$ and $|\bar{a}| = 4,\ |\bar{b}| = 2$

(a) $\therefore\ |\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$

$|\bar{a}\times\bar{b}| = (4)(2)\frac{1}{2} \qquad \sin\frac{\pi}{6} = \frac{1}{2}$

$|\bar{a}\times\bar{b}| = 4$

39. $\dfrac{|\bar{a} \times \bar{b}|}{\bar{a} \cdot \bar{b}} = \dfrac{|\bar{a}||\bar{b}| \sin\theta}{|\bar{a}||\bar{b}| \cos\theta} = \tan\theta$

(c)

40. $|\bar{a}| = 3,\ |\bar{b}| = 4, |\bar{c}| = 5$

(d) $\bar{a} \cdot (\bar{b} + \bar{c}) = 0,\ \bar{b} \cdot (\bar{c} + \bar{a}) = 0,\ \bar{c} \cdot (\bar{a} + \bar{b}) = 0$

$2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}) = 0$

$|\bar{a} + \bar{b} + \bar{c}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a})$

$= 9 + 16 + 25 = 50$

$|\bar{a} + \bar{b} + \bar{c}| = 5\sqrt{2}$

41. $|\bar{a}| = 1,\qquad |\bar{b}| = 2,\qquad |\bar{c}| = 3$

(c) $|\bar{a} + \bar{b} + \bar{c}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a})$

$= 1 + 4 + 9 + 2\left(|\bar{a}||\bar{b}| \cos\theta + |\bar{b}| + |\bar{c}| \cos\theta + |\bar{c}||\bar{a}| \cos\theta\right)$

$= 14 + 2\left(1(2) + 2(3) + 3(1)\right) \cos\frac{\pi}{3}\Big)$

$= 14 + 2(11)\frac{1}{2}$

$= 25 \qquad\qquad |\bar{a} + \bar{b} + \bar{c}| = 5$

42. $|\bar{a}| = |\bar{b}| = |\bar{c}| = 1$ and $|\bar{a} + \bar{b} + \bar{c}|^2 = 1$

(a) $(\bar{a} + \bar{b} + \bar{c})(\bar{a} + \bar{b} + \bar{c}) = 1$

$|\bar{a} + \bar{b} + \bar{c}|^2 = 1 + 1 + 1 + 2\left(|\bar{b}||\bar{c}| \cos\beta + |\bar{c}||\bar{a}| \cos\alpha\right)$

$1 = 3 + 2(\cos\beta + \cos\alpha)$

$2(\cos\alpha + \cos\beta) = -2$

$\cos.\alpha + \cos\beta = -1$

43. $(\bar{a} + \bar{b})(\bar{a} - \bar{b}) = 63$

(a) $|\bar{a}|^2 - \bar{a}\bar{b} + \bar{b}\bar{a} - |\bar{b}|^2 = 63$

$|\bar{a}|^2 - |\bar{b}|^2 = 63$

$|\bar{a}| = 8|\bar{b}|$

$\therefore |\bar{a}|^2 - \frac{1}{64}|\bar{a}|^2 = 63$

$\therefore |\bar{a}|^2 \left(\frac{63}{64}\right) = 63$ $\quad |\bar{a}|^2 = 64$

$\therefore |\bar{a}| = 8$

44. $|\bar{a} + \bar{b}| < 1 \quad |\bar{a} + \bar{b}|^2\ 2$

(c) $|\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a}\,.\,\bar{b} < 1$

$1 + 1 + 2(1)(1)\cos\theta < 1$

$2\cos\theta < -1$

$\cos\theta < \frac{-1}{2} \quad -1 < \cos\theta < \frac{-1}{2}$

$\pi > \theta > \frac{2\pi}{3} \quad \frac{2\pi}{3} < \theta < \pi$

In 2nd quadrant cos is decresing funtion.

45. $\left|\bar{a} - \bar{b}\right| < 1$

(c) $\therefore \left|\bar{a} - \bar{b}\right|^2 < 1$

$|\bar{a}|^2 - 2\bar{a}.\bar{b} + |\bar{b}|^2 < 1$

$1 + 1 - 2\cos\theta < 1$

$\frac{1}{2} < \cos\theta < 1$

$\therefore \frac{\pi}{3} < \theta < 0$

46. $|\bar{a} + \bar{b}| < |\bar{a} - \bar{b}|$

(a) $|\bar{a} + \bar{b}|^2 < |\bar{a} - \bar{b}|^2$

$|\bar{a}|^2 + 2\bar{a}.\bar{b} + |\bar{b}|^2 < |\bar{a}|^2 - 2\bar{a}\bar{b} + |\bar{b}|^2$

$4\bar{a}\bar{b} < 0, \bar{a}\bar{b} < 0$

$|\bar{a}|\,|\bar{b}|\cos\theta < 0$ and $|\bar{a}|\,|\bar{b}| > 0$

$\cos\theta < 0$ {¤

$\therefore$ Angle between $\bar{a}$ & $\bar{b}$ is obtuse.

47. $\bar{a}$ & $\bar{b}$ from an angle $\frac{\pi}{6}$ & $\frac{2\pi}{3}$ with positive direction of X-axis.

(c) $\hat{\bar{a}\,\bar{b}} = \frac{2\pi}{3} - \frac{\pi}{6} = \frac{\pi}{2}$

$\bar{a}\,.\,\bar{b} = 0$

$|\bar{a} + \bar{b}|^2 = |\bar{a}|^2 + 2\bar{a}\,.\,\bar{b} + |\bar{b}|^2$

$= 1 + 0 + 1$

$= 2$

$|\bar{a} + \bar{b}| = \sqrt{2}$

48. Take and unit vector in YZ plane say $(0, a, b)$ which is perpendicular to $(2, 4, -3)$

(b) $\therefore\ (0, a, b)\cdot(2, 4 - 3) = 0$

$\therefore\ 4a - 3b = 0 \qquad \therefore\ a = \frac{3b}{4}$

But, $\sqrt{a^2 + b^2} = 1 \qquad \therefore\ a^2 + b^2 = 1$ and $a = \frac{3b}{4}$

$\therefore\ \frac{9b^2}{16} + b^2 = 1$

$\therefore\ b^2 = \frac{16}{25} \qquad \therefore b = \pm\frac{4}{5}$ and $a = \pm\frac{3}{5}$

Required unit vector $= \pm\frac{1}{5}(0, 3, 4)$

49. $\bar{a} = (-3, 4, 7),\ \bar{\ell} = (5, 2, 8)$

(b) $\bar{r} = \bar{a} + k\,\bar{\ell}, \quad k \in R$

$(x, y, z) = (-3, 4, 7) + k\,(5, 2, 8)$

$\frac{x+3}{5} = \frac{y-4}{2} = \frac{z-7}{8}$

50. $\bar{a} = (-2, 4, 7)\ \ \bar{\ell} = (5, -9, 12)$

(b) $x = x_1 + k\ell_1$ $y = y_1 + k\ell_2$ $z = z_1 + k\ell_3$

$x = -2 + 5k,$ $y = 4 - 9k,$ $z = 7 + 12k,$ $k \in R$

51. Line is parallel to Y-axis

(b) $\therefore$ If direction of line is in the Y-axis, $\bar{\ell} = (0, 1, 0)$

point $\bar{a} = (0, 0, 0)$

$$\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$$

52. $$\frac{x-4}{-7} = \frac{y-(-9)}{5} = \frac{z-\left(\frac{-8}{3}\right)}{\frac{2}{3}}$$

(a) $\bar{\ell} = \left(-7, 5, \frac{2}{3}\right)$, $|\bar{\ell}| = \sqrt{49 + 25 + \frac{4}{9}} = \sqrt{\frac{670}{9}}$

Direction cosine $\dfrac{\frac{-7}{\sqrt{670}}}{3}$, $\dfrac{\frac{5}{\sqrt{670}}}{3}$, $\dfrac{\frac{\frac{2}{3}}{\sqrt{670}}}{3}$

$\therefore \dfrac{-21}{\sqrt{670}}, \quad \dfrac{15}{\sqrt{670}}, \quad \dfrac{2}{\sqrt{670}}$

53. $\dfrac{2x-5}{3} = y \qquad y = 35 - 5z$

(d) $$\therefore \frac{x-\frac{5}{2}}{\frac{3}{2}} = y = \frac{z-7}{-\frac{1}{5}} \quad \bar{\ell} = \left(\frac{3}{2}, 1, \frac{-1}{5}\right)$$

$$|\bar{\ell}| = \sqrt{\frac{9}{4} + 1 + \frac{1}{25}} = \frac{\sqrt{329}}{10}$$

Direction cosine $\dfrac{15}{\sqrt{329}}, \quad \dfrac{10}{\sqrt{329}} \quad \dfrac{-2}{\sqrt{329}}$

54. $\bar{a} = (1, 2, 0), \quad \bar{b} = (3, 1, 1), \quad \bar{b} - \bar{a} = (2, -1, 1)$

(a) $\bar{r} = (1, 2, 0) + k(2, -1, 1)$

$\bar{r} = (1 + 2k,\ 2 - k,\ k)$ .....(1)

putting in eq (1) (7, -1, 3) for

$(7, -1,\ 3) = (1 + 2k,\ 2 - k,\ k)$

$k = 3,\ \ k = 3,\ \ k = 3$

point $(7, -1, 3)$ in a the line

55. $\ell + m + n = 0$ .....(1) $\ell^2 - m^2 + n^2 = 0$ ....(2)

(b) $m = -(\ell + m)$ put in (2)

$\therefore\ \ell^2 - (\ell + n)^2 + n^2 = 0$

$\ell^2 - \ell^2 - n^2 - 2\ell n + n^2 = 0,\ \ell n = 0$, $\ell = 0$, or $n = 0$

$\ell = 0$ Then from eq^n (1) $m = -n$

$\therefore$ direction cosine (0, -n, n)

$n = 0$ Then from (1) $m = -\ell$

$\therefore$ direction cosine $(\ell, -\ell, 0)$

$$\cos\theta = \frac{(0, -n, n).(\ell_1 - \ell_1\ 0)}{\sqrt{n^2 + n^2}\sqrt{\ell^2 + \ell^2}} = \frac{n\ell}{2n\ell} = \frac{1}{2}$$

$$\cos\theta = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$

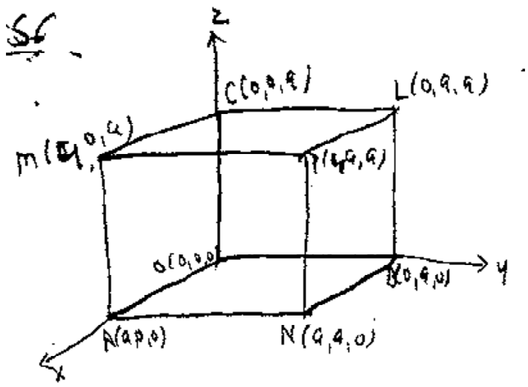
56. If O is one vectices of cube and $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are direction with X, Y, Z, axis, $OA = OB = OC = a$

(b) diagonal $\overline{AL}$, $\overline{BM}$, $\overline{CN}$ & $\overline{OP}$

$\overrightarrow{AL} = (-a, a, a)$, $\overrightarrow{BM} = (a, -a, a)$

Angle between $\overrightarrow{AL}$ and $\overrightarrow{BM}$ is Q.

$$\cos\theta = \frac{\left|\overrightarrow{AL}\ .\ \overrightarrow{BM}\right|}{\left|\overrightarrow{AL}\right|\left|\overrightarrow{BM}\right|} = \frac{\left|-a^2 - a^2 + a^2\right|}{\sqrt{3a^2}\ \sqrt{3a^2}}$$

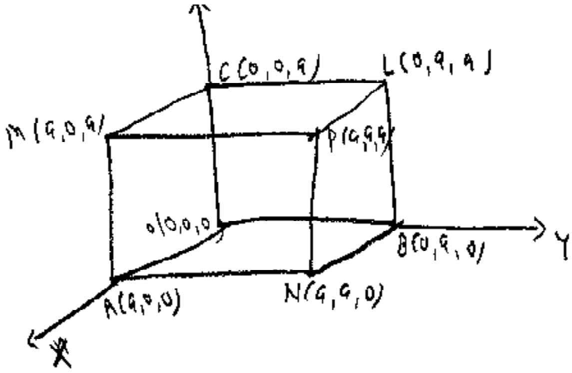


$$\cos\theta = \frac{1}{3} \qquad \theta = \cos^{-1}\frac{1}{3}$$

$$\cos^{-1}\frac{1}{3}$$

57. For cube,

(c) OA = OB = OC = a (side)



$\overline{AL}$, $\overline{BM}$, $\overline{CN}$, & $\overline{OP}$ Four diagonal

$\overrightarrow{OP} = (a, a, a)$

$\overrightarrow{AL} = (-a, a, a)$

$\overrightarrow{BM} = (a_1 - a, a)$

$\overrightarrow{CN} = (a, a, -a)$

Here ℓ, m and n are direction co-sine of Line, Diagonal $\overrightarrow{OP}$, $\overrightarrow{AL}$, $\overrightarrow{BM}$, $\overrightarrow{CN}$ form an angle α, β, γ & δ with line then.

$$\cos\alpha = \frac{\overrightarrow{OP}.\ \ell}{|\overrightarrow{OP}|\,|\ \ell\ |} = \frac{(a, a, a)\cdot(\ell, m, n)}{\sqrt{3a^2}\ \sqrt{\ell^2 + m^2 + n^2}} = \frac{a\ (\ell + m + n)}{\sqrt{3}a\ \sqrt{\ell^2 + m^2 + n^2}}$$

$$\cos\alpha = \frac{\ell + m + n}{\sqrt{3}\ \sqrt{\ell^2 + m^2 + n^2}} = \frac{\ell + m + n}{\sqrt{3}} \quad (\because \ell^2 + m^2 + n^2 = 1)$$

$$\cos\beta = \frac{-\ell + m + n}{\sqrt{3}},\ \cos\gamma = \frac{\ell - m + n}{\sqrt{3}}\ \cos\delta = \frac{\ell + m - n}{\sqrt{3}}$$

$$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$$

$$= \frac{4}{3}\left(\ell^2 + m^2 + n^2\right) = \frac{4}{3}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$$

58. We know $\cos^2\alpha + \cos^2\beta + \cos\delta + \cos^2\gamma = \frac{4}{3}$

(a) $\therefore \sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = 4 - \frac{4}{3}$

$$= \frac{8}{3}$$

59. We know $\cos^2\alpha\ \cos^2\beta + \cos^2\gamma + \cos^2 d = \frac{4}{3}$

(a) $\therefore \cos 2\alpha + \cos 2\beta + \cos 2\gamma + \cos 2\gamma = 2\cos^2\alpha - 1 + 2\cos^2\beta - 1$

$$+ 2\cos^2\gamma - 1 + 2\cos^2\delta - 1$$

$$= 2\left(\frac{4}{3}\right) - 4$$

$$= -\frac{4}{3}$$

60. Here α, β, γ are direction cosines of line.

(b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$$

$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$$

61. $\bar{\ell} = (2, 2, -1), \ \bar{m} = (3, 0, 0)$

(b) $\cos\alpha = \dfrac{|\ \bar{\ell}\,.\,\bar{m}\ |}{|\bar{\ell}|.\,|\bar{m}|}$ $\cos\alpha = \dfrac{5}{3\sqrt{10}}$

$$\alpha = \cos^{-1}\frac{5}{\sqrt{90}}$$

62. $\dfrac{x-3}{\frac{1}{\sqrt{2}}} = \dfrac{y-2}{-\sqrt{2}} = \dfrac{z+1}{0}$ $\bar{\ell} = \left(\dfrac{1}{\sqrt{2}}, -\sqrt{2}, 0\right)$

(a) $|\ \bar{\ell}\ | = \sqrt{\dfrac{1}{2} + 2 + 0} = \sqrt{\dfrac{5}{2}}$

unit vector in the direction of $\bar{\ell} = \dfrac{\bar{\ell}}{|\ \bar{\ell}\ |}$

$$= \left(\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}, 0\right)$$

direction cosine of line $\dfrac{1}{\sqrt{5}}, \dfrac{-2}{\sqrt{5}}, 0$

63. $\dfrac{x-\frac{2}{3}}{-2} = \dfrac{y-(-1)}{2} = \dfrac{z-1}{2}$

(b) direction of line $\bar{\ell} = (-2, 2, 2)$

$\therefore (-1, 1, 1)$

$\therefore -1 : 1 : 1$

64. $\dfrac{x-3}{-2} = \dfrac{y}{1} = \dfrac{z+1}{2}$, $\bar{\ell} = (-2, 1, 2)$

(d) unit vector direction $= \dfrac{\bar{\ell}}{|\bar{\ell}|}$ $|\bar{\ell}| = \sqrt{4+1+4} = 3$

$= \left(\dfrac{-2}{3}, \dfrac{1}{3}, \dfrac{2}{3}\right)$

direction cosine $\dfrac{-2}{3} : \dfrac{1}{3} : \dfrac{-2}{3}$

65. Line $\dfrac{x-\frac{7}{2}}{\frac{1}{2}} = \dfrac{y}{\frac{1}{3}} = \dfrac{z-3}{0}$ $\bar{x} = \left(\dfrac{1}{z}, \dfrac{1}{3}, 0\right)$

(b) $\therefore$ Direction of Line $(3, 2, 0)$

66. $\dfrac{x-1}{2} = \dfrac{y}{1} = \dfrac{z-1}{-2}$, $\dfrac{x+1}{\frac{-1}{2}} = \dfrac{y}{1} = \dfrac{z+2}{0}$

(d) $\bar{\ell} = (2, 1, -2)$, $\bar{m} = \left(\dfrac{-1}{2}, 1, 0\right)$

$\bar{\ell} \cdot \bar{m} = -1 + 1 - 0 = 0$

$\therefore$ Lines are perpendicular to each other $\dfrac{\pi}{2}$

67. $\dfrac{x-1}{-c} = \dfrac{y+3}{-1} = \dfrac{z-3}{2}$ $\bar{\ell} = (-c, -1, 2)$

(b) $\dfrac{x-3}{6} = \dfrac{y-1}{3} = \dfrac{z-4}{-6}$ $\bar{m} = (6, 3, -6)$

$\bar{\ell} = k\bar{m}$ $\quad (-C, -1, 2) = k(6, 3, -6)$

$-c = 6k \quad 3k = -1 \quad -6k = 2$

$C = -6\left(\frac{-1}{3}\right) = 2$

$C = 2$

68. $\bar{\ell} = (3, 4, -6)$, $\bar{m} = (9, 2, 1)$, $\bar{\ell}.\bar{m} = 27 + 8 - 6 = 29$

(c) $|\bar{\ell}| = \sqrt{9 + 16 + 36} = \sqrt{61}$, $(\bar{m}) = \sqrt{81 + 4 + 1} = \sqrt{86}$

$\alpha = \cos^{-1}\left|\frac{\bar{\ell}.\bar{m}}{|\bar{\ell}||\bar{m}|}\right| = \cos^{-1}\frac{29}{\sqrt{5246}}$

69. Lines forming an equal angle with the axis

(a) $\alpha = \beta = \gamma$

Direction cosine of line $= (\cos\alpha, \cos\beta, \cos\gamma) = \cos\beta(1, 1, 1)$

equation of line parsing through $\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1}$

$\therefore x = y = z$

70. Line $\frac{x-5}{7} = \frac{y-5}{K} = \frac{z-2}{5}$, $\bar{\ell} = (7, K, 5)$

(b) $\frac{x}{3} = \frac{y-21}{8} = \frac{z-\frac{4}{3}}{\frac{5}{3}}$ $\quad \bar{m} = \left(3, 8, \frac{5}{3}\right)$

line are perpendicular to, $\bar{\ell}.\bar{m} = 0$

$21 + 8K + \frac{25}{3} = 0$

$8K = \frac{-88}{3}$

$K = \frac{-11}{3}$

71. Line's $\bar{r} = (\alpha, \beta, \gamma) + K(\ell, m, n)$, $\bar{a} = (\alpha, \beta, \gamma)$ $\bar{\ell} = (\ell, m, n)$

(a) $\bar{r} = (\ell, m, n) + K(\alpha, \beta, \gamma)$ $\bar{b} = (\ell, m, n)$, $\bar{m} = (\alpha, \beta, \gamma)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ \ell & m & n \\ \alpha & \beta & \gamma \end{vmatrix} \neq 0$$

$(\ell \neq m \neq n, \ \& \ \alpha \neq \beta \neq \gamma \ (\alpha, \beta, \gamma) \neq (\ell, m, n))$

$\overrightarrow{AB} = (\ell - \alpha, m - \beta, n - \gamma)$

$$\overrightarrow{AB} . (\bar{\ell} \times \bar{m}) = \begin{vmatrix} \ell - m & m - \beta & n - \gamma \\ \ell & m & n \\ \alpha & \beta & \gamma \end{vmatrix}$$

$$= \begin{vmatrix} \ell & m & n \\ \ell & m & n \\ \alpha & \beta & \gamma \end{vmatrix} = 0$$ lines are Intercting.

72. Line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$, $\bar{a} = (1, 2, 3)$, $\bar{\ell} = (2, 3, 4)$

(d) $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$, $\bar{b} = (4, 1, 0)$, $\bar{m} = (5, 2, 1)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix} = (-5, 18, -11) \neq \bar{0}$$

$\bar{a} - \bar{b} = (-3, 1, 3)$

$(\bar{a} - \bar{b}) \cdot (\bar{\ell} \times \bar{m}) = 15 + 18 - 33 = 0$

Lines are intersection $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = m$

$(2m+1, 3m+2, 4m+3)$ $\quad m \in R \quad \ldots(1)$

$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = n$ $\quad n \in R$

$(5n+4, 2n+1, n)$ $\quad \ldots(2)$

Two lines are intersecting

$(2m+1, 3m+2, 4m+3) = (5n+4, 2n+1, n)$

$m = -1, \quad n = -1$

$\therefore$ putting eq^n (1) (2) Intersection $(-1, -1, -1)$

73. Line $\frac{x-3}{1} = \frac{y+2}{-1} = \frac{z-1}{1}$, $\bar{a} = (3, -2, 1)$. $\bar{\ell} = (1, -1, 1)$

(d) $\frac{x}{2} = \frac{z+3}{3} = \frac{y-1}{0}$

$\bar{b} = (0, 1, -3)\ \bar{m} = (2, 0, 3)$

$(3+k, -2-k, 1+k) = (2m, -1, -3+3m)$

$3+k = 2m, -2-k = -1 \quad 1+k = -3+3$

$k = -1$

$3-1 = 2m$

$m = 1 \qquad 1+(-1) = -3+3m \quad m = 1$

Line Intersection $(3-1, -2+1, 1-1)\ 3 = 3m$

$m = 1$

$(2, -1, 0)$

74. $x = y = 3$, $\bar{\ell} = (1, 1, 1)$, $\bar{a} = (0, 0, 0)$

(b) $x-1 = y-2 = z-3$, $\bar{m} = (1, 1, 1)\bar{b} = (1, 2, 3)$

$\bar{b} - \bar{a} = (1, 2, 3)$

$$(\bar{b}-\bar{a}) \times \bar{\ell} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = (-1, 2, -1), \quad |\bar{\ell}| = \sqrt{1+1+1} = \sqrt{3}$$

$$\left|(\bar{b}-\bar{a}) \times \bar{\ell}\right| = \sqrt{1+4+1} = \sqrt{6}$$

Perpendicular distance between two lines $= \frac{|(\bar{b}-\bar{a}) \times \bar{\ell}|}{|\bar{\ell}|}$

$$= \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$$

75. $\bar{r} = (-2, 2, 3) + k(4, -3, 0)$, $k \in R$

(c) $\bar{r} = (2, -1, 3)$ put $(2, -1, 3) = (-2 + 4k, 2 - 3k, 3)$

$K = 1$ $K = 1$ $2 = -2 + 4k, \ -1 = 2 - 3k$

$\therefore (2, -1, 3)$ is on the line

$\bar{a} = (-2, 2, 3), \bar{\ell} = (4, -3, 0)$

puting $(2, -1, 3)$ in the equation of line

$$\frac{x-2}{4} = \frac{4+1}{-3} = \frac{z-3}{0} = k$$

$\bar{r} = (2, -1, 3) + k(4, -3, 0)$

$$|K|^2 = \frac{(x-2)^2 + (y+1)^2 + (z-3)^2}{16+9}$$

$|K|^2 = \frac{AP^2}{25}$ where $P(x, y, 3)$ is a point on the line

$|K|^2 = 1$ $K = \pm 1$

$AP = 5$ given.

$K = 1$ put in (1) $\bar{r} = (2, -1, 3) + (4, -3, 0) = (6, -4, 3)$

$K = -1$ put in (1) $\bar{r} = (2, -1, 3) + (-4, 3, 0) = (-2, 2, 3)$

point $(6, -4, 3), (-2, 2, 3)$

76. Line $\bar{r} = (1, 2, 1) + k(-1, -2, 1)$ ....(1) $k \in R$

(a) $\bar{r} = (2, 4, 0)$ put

$(2, 4, 0) = (1 - k, 2 - 2k, 1 + k)$

$k = -1,$ $k = -1,$ $k = -1$ point is on equ.. (1)

$(2, 4, 0)$ eq (1) $\frac{x-2}{-1} = \frac{y-4}{-2} = \frac{z-0}{1} = k$

$$|K|^2 = \frac{(x-2)^2 + (y-4)^2 + (z-0)^2}{1+4+1}, \quad |K|^2 = \frac{AP^2}{6}$$

$AP = \sqrt{6}$

$K = 1$ for $\bar{r} = (2, 4, 0) + (-1, -2, 1)$

$\bar{r} = (1, 2, 1)$

$K = -1$ for $\bar{r} = (2, 4, 0) - 1(-1, -2, 1)$

$= (3, 6, -1)$

point on the line $(1, 2, 1), (3, 6, -1)$

77. $P(1, 3, 4)$ line $\dfrac{x-5}{2} = \dfrac{y+6}{-1} = \dfrac{z+7}{3}$

(c) $\bar{a} = (5, -6, -7), \bar{\ell} = (2, -1, 3)$

$$\overrightarrow{AP} \times \bar{\ell} = \begin{vmatrix} i & j & k \\ -4 & 9 & 11 \\ 2 & -1 & 3 \end{vmatrix} = (38, 34, -14)$$

$|\bar{\ell}| = \sqrt{4+1+4} = \sqrt{14}$

Perpindicular distance from point $= \dfrac{\left|\overrightarrow{AP} \times \ell\right|}{|\bar{\ell}|} = \dfrac{\sqrt{1444+1156+196}}{\sqrt{14}}$

$$= \sqrt{\frac{1398}{7}} \text{ unit}$$

78. $\dfrac{x-11}{10} = \dfrac{y+2}{-4} = \dfrac{z+8}{-11} = K;\ \bar{a} = (11, -2, -8)$

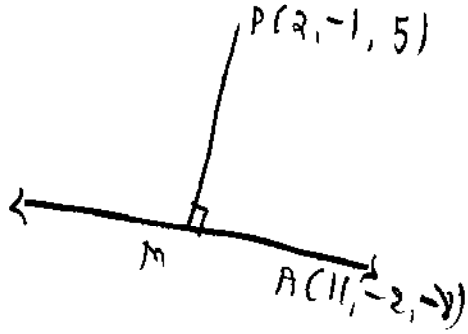
(b) $\bar{\ell} = (10, -4, -11)$

$\bar{r} = (10K+11, -4K-2, -11K-8)$

$P(2, -1, 5)$ foot of perpendicular is on line,

$M = (10K+11, -4K-2, -11K-8)$

$\overrightarrow{PM} = (10K+9, -4K-1, -11K-13)$



$\overrightarrow{PM} \cdot \bar{\ell} = 0$,

$10(10K+9) - 4(-4K-1) - 11(-11K-13) = 0$

$237K = -237$

$\therefore K = -1$

$K = -1$ foot of perpendicular to $M = (1, 2, 3)$

Perpendicular distance $2 = |\overrightarrow{PM}| = \sqrt{1+9+4} = \sqrt{14}$

79. $\bar{r} = (4, 7, 1) + K(1, 2, -2)$ $k \in R$

(a) $\bar{r} = (4 + k, 7 + 2k, 1 - 2k)$

$A(1, 0, 3)$

Position of vector of M = $(4 + k, 7 + 2k, 1 - 2k)$

$\overrightarrow{AM} = (3 + k, 7 + 2k, -2 - 2k)$ .... (1)

$\overrightarrow{AM} \perp \bar{\ell} \qquad \bar{\ell} = (1, 2, -2)$

$\overrightarrow{AM} . \bar{\ell} = 0$

$(3 + k, 7 + 2k, -2 - 2k) . (1, 2, -2) = 0$

$3 + k + 14 + 4k + 4 + 4k = 0$

$9k + 21 = 0$

$k = \frac{-7}{3}$

Perpidicular point

$M = \left(4 - \frac{7}{3}, 7 \frac{-14}{3}, 1 + \frac{14}{3}\right) = \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

$K = \frac{-7}{3}$ puting in eq^n (1) $\overrightarrow{AM} = \left(\frac{2}{3}, \frac{7}{3}, \frac{8}{3}\right)$

perpendicular distance $AM = |\overrightarrow{AM}| = \sqrt{\frac{4}{9} + \frac{49}{9} + \frac{64}{9}} = \sqrt{\frac{117}{9}} = \sqrt{13}$

perpendicular distance $\sqrt{13}$, foot of perpendicular $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

80. $\frac{x - \frac{1}{2}}{\frac{3}{2}} = \frac{y - 1}{-3} = \frac{z - \frac{2}{3}}{\frac{5}{3}}, \qquad \bar{a} = \left(\frac{1}{2}, 1, \frac{2}{3}\right)$

(a) $\bar{\ell} = \left(\frac{3}{2}, -3, \frac{5}{3}\right)$

Line $\bar{\ell}$ is paralled to given line

$\therefore$ direction of $\bar{\ell}$ is similar to $\bar{\ell}$ direction of $\bar{\ell}$

passes through $(1, 2, 3)$ line equation

$\bar{r} = (1, 2, 1) + K \left(\frac{3}{2}, -3, \frac{5}{3}\right)$

$$\frac{2x - 2}{3} = \frac{2 - y}{3} = \frac{3z - 3}{5}$$

$$\frac{2x - 2}{3} = \frac{2 - y}{3} = \frac{3z - 3}{5}$$

81. $\bar{r} = (0, 2, 3) + K(2, 3, 4)$, $\bar{a} = (0, 2, 3)$, $\bar{\ell} = (2, 3, 4)$

(b) $\bar{r} = (5, 3, 2) + K(0, 2, 3)$, $\bar{b} = (5, 3, 2)$ $\bar{m} = (0, 2, 3)$

If direction of $\bar{n}$ is given line then

$$\bar{n} = \bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 0 & 2 & 3 \end{vmatrix} = (1, -6, 4)$$

Line $\bar{c} = (3, -1, 11)$ line passess through point $\bar{c}$

$\bar{r} = \bar{c} + k\bar{n}$ $k \in R$

$$\frac{x - 3}{1} = \frac{y + 1}{-6} = \frac{z - 11}{4}$$

82. Line $(x, y, z) = (0, 1, 2) + k(1, 2, 3)$

(c) foot of perpendicular $M = (k, 2k + 1, 3k + 2)$

$\overrightarrow{PM} = (k - 1, 2k - 5, 3k - 1)$, $\bar{\ell} = (1, 2, 3)$

$\overrightarrow{PM} \perp \bar{\ell}$, $\overrightarrow{PM} \cdot \bar{\ell} = 0$

$(k - 1) + 2(2k - 5) + 3(3k - 1) = 0$

$k - 1 + 4k - 10 + 9k - 3 = 0$

$k = 1$

$\therefore M(1, 3, 5)$ $Q(x_1, y_1, z_1)$, $P(1, 6, 3)$ which, is a image of Q.

$\therefore$ M is a middle point of $\overline{PQ}$

$$1 = \frac{x_1 + 1}{2}, \quad 3 = \frac{y_1 + 6}{2}, \quad 5 = \frac{z_1 + 3}{2}$$

$x_1 = 1$, $y_1 = 0$ $z_1 = 7$

There a fore image = $(1, 0, 7)$

83. $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ $\quad \bar{a}=(0,0,0) \quad \bar{\ell}=(1,2,-1)$

(a) $\frac{x-1}{3}=\frac{y}{2}=\frac{z}{6}$ $\quad \bar{b}=(1,0,0) \quad \bar{m}=(3,2,6)$

$$\bar{n}=\bar{\ell}\times\bar{m}=\begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 3 & 2 & 6 \end{vmatrix}=(14,-9,-4)$$

A line passess through $\bar{c}=(1,2,3)$ and with the direction $\bar{n}=(14,-9,-4)$

$\bar{r}=\bar{c}+k\bar{n}, \quad k\in R$

$$\frac{x-1}{14}=\frac{y-2}{-9}=\frac{z-3}{-4}$$

84. $\frac{x-b}{14}=y=\frac{z-d}{c}$ $\quad \bar{a}=(a,0,d),\ \bar{\ell}=(a,1,c)$

(a) $|\bar{\ell}|=\sqrt{a^2+1+c^2}$

$\therefore \bar{\ell}$ is unit vector in the direction $=\pm\frac{\bar{\ell}}{|\bar{\ell}|}$

$$=\pm\left(\frac{a}{\sqrt{a^2+c^2+1}},\ \frac{1}{\sqrt{a^2+c^2+1}},\ \frac{c}{\sqrt{a^2+c^2+1}}\right)$$

Direction cosine

$$\pm\frac{a}{\sqrt{a^2+c^2+1}},\ \pm\frac{1}{\sqrt{a^2+c^2+1}},\ \pm\frac{c}{\sqrt{a^2+c^2+1}}$$

85. Line L : $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$ $\quad \bar{a}=(b,0,d)$

(a) M : $\frac{x-b}{a'}=\frac{y}{1}=\frac{z'-d'}{c'}$ $\quad \bar{\ell}=(a,1,c)$

$\bar{b}=(b,0,d')$

$\bar{m}=(a',1,c')$

$L\perp M$

$\therefore \bar{\ell}.\bar{m}=0$

$(a, 1, c).\ (a', 1, c^1) = 0$

$aa' + 1 + cc' = 0$

$aa' + cc' + 3 = 2$

86. $\bar{r} = (1, 3, 5) + K(-1, 2, 3),\qquad \bar{a} = (1, 3, 5),\ \bar{\ell} = (-1, 2, 3)$

(b) $\bar{r} = (1, 3, 1) + K(1, -2, -3)\quad \bar{b} = (1, 3, 1),\ \bar{m} = (1, -2, -3)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ -1 & 2 & 3 \\ 1 & -2 & -3 \end{vmatrix} = (0, 0, 0) = \bar{0}$$

$\therefore$ Two lines are parallel or coincide

$\overrightarrow{AB} = \bar{b} - \bar{a} = (0, 0, -4),\quad |\bar{\ell}| = \sqrt{1 + 4 + 9} = \sqrt{14}$

$$\hat{\ell} = \frac{\bar{\ell}}{|\bar{\ell}|} = \left(-\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

$$\overrightarrow{AB} \times \hat{\ell} = \begin{vmatrix} i & j & k \\ 0 & 0 & -4 \\ \frac{-1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{3}{\sqrt{14}} \end{vmatrix} = \left(\frac{8}{\sqrt{14}}, \frac{4}{\sqrt{14}}, 0\right)$$

$$\left|\overrightarrow{AB} \times \hat{\ell}\right| = \frac{\sqrt{64 + 16}}{\sqrt{14}} = \sqrt{\frac{80}{14}} = \frac{4\sqrt{5}}{\sqrt{14}} \neq 0$$

$\therefore$ Perpendicular distance between two given lines is not zero.

$\therefore$ lines are parallel

$\therefore$ Not coincide

87. $\bar{r} = (2, 1, 3) + k(1, -1, 1),\quad \bar{a} = (2, 1, 3),\ \bar{\ell} = (1, -1, 1)$

(a) $\bar{r} = (3, 0, 4) + k(-1, 1 -1),\quad \bar{b} = (3, 0, 4),\ \bar{m} = (-1, 1, -1)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{vmatrix} = (0, 0, 0) = \bar{0}$$

$\therefore$ Two lines are parallel or coincide

$\overrightarrow{AB} = \bar{b} - \bar{a} = (1, -1, 1),\qquad \bar{\ell} = (1, -1, 1)$

$\overrightarrow{AB} \times \overline{\ell} = \overline{0}$, $\quad \left| \overrightarrow{AB} \times \overline{\ell} \right| = 0$

$\therefore$ Distance between two lines in zero

$\therefore$ Lines are coinsident

88. $\overline{r} = (1, 2, 6) + K(1, 3, 5)$, $\quad \overline{a} = (1, 2, 6)$, $\overline{\ell} = (1, 3, 5)$

(d) $\overline{r} = (-1, 3, 5) + K(2, 1, 1)$, $\quad \overline{b} = (-1, 3, 5)$, $\overline{m} = (2, 1, 1)$

$$\overline{\ell} \times \overline{m} = \begin{vmatrix} \overline{I} & \overline{J} & \overline{K} \\ 1 & 3 & 5 \\ 2 & 1 & 1 \end{vmatrix} = (-2, 9, -5) \neq \overline{0}$$

Line are either Intersecting or skew

$\overrightarrow{AB} = \overline{b} - \overline{a} = (-2, 1, -1)$

$\overrightarrow{AB} \,.\, (\overline{\ell} \times \overline{m}) = (-2, 1, -1).\ (-2, 9, -5)$

$= 4 + 9 + 5 = 18 \neq 0$

$\therefore$ Lines are skew.

89. $\overline{r} = (3, -1, 1) + K(1, -1, 1)$, $\quad \overline{a} = (3, -1, 1)$, $\overline{\ell} = (1, -1, 1)$

(b) $\overline{r} = (0, 0, -3) + K(2, 0, 3)$, $\quad \overline{b} = (0, 0, -3)$, $\overline{m} = (2, 0, 3)$

$$\overline{\ell} \times \overline{m} = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 0 & 3 \end{vmatrix} = (-3, -1, 2) \neq \overline{0}$$

Lines are intersecting or skew

$\overrightarrow{AB} = \overline{b} - \overline{a} = (-3, 1, -4)$

$\overrightarrow{AB} \cdot (\overline{\ell} \times \overline{m}) = (-3, 1, -4) \cdot (-3, -1, 2) = 9 - 1 - 8 = 0$

$\therefore$ Lines are intersecting.

90. $\dfrac{x-1}{3} = \dfrac{y+1}{2} = \dfrac{z-1}{5}$, $\quad \overline{a} = (1, -1, 1)$, $\quad \overline{\ell} = (3, 2, 5)$

(d) $\dfrac{x+2}{4} = \dfrac{y-1}{3} = \dfrac{z+1}{2}$ $\quad \overline{b} = (-2, 1, -1)$ $\quad \overline{m} = (4, 3, -2)$

$\overline{a} - \overline{b} = (3, -2, 2)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ 3 & 2 & 5 \\ 4 & 3 & -2 \end{vmatrix} = (-19, 26, 1) \neq \bar{0}$$

Lines are skew or Intersecting.

$\overrightarrow{AB} \cdot (\bar{\ell} \times \bar{m}) = (3, -2, 2) \cdot (-19, 26, 1)$

$= -57 - 52 + 2$

$= 107 \neq 0$

$\therefore$ Lines are skew.

91. $\dfrac{x-1}{1} = \dfrac{y+1}{3} = \dfrac{z}{1}$ $\quad \bar{a} = (1, -1, 0)$ $\bar{\ell} = (1, 3, 1)$

(c) $\dfrac{x-1}{3} = \dfrac{y-2}{1} = \dfrac{z-2}{0}$ $\quad \bar{b} = (1, 2, 2)$ $\quad \bar{m} = (3, 1, 0)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ 1 & 3 & 1 \\ 3 & 1 & 0 \end{vmatrix} = (-1, 3, -8) \neq \bar{0}$$

$\bar{b} - \bar{a} = (0, 3, 2)$

shortest distance $= \dfrac{|(\bar{b} - \bar{a}) \cdot (\bar{\ell} \times \bar{m})|}{|\bar{\ell} \times \bar{m}|}$

$(\bar{b} - \bar{a}) \cdot (\bar{\ell} \times \bar{m}) = (0, 3, 2) \cdot (-1, 3, -8) = 0 + 9 - 16 = -7 \neq 0$

$|\bar{\ell} \times \bar{m}| = \sqrt{1 + 9 + 64} = \sqrt{74}$

shortest distance $= \dfrac{|-7|}{\sqrt{74}} = \dfrac{7}{\sqrt{74}}$ unit.

92. $x - 1 = \dfrac{y-1}{6} = \dfrac{z}{2},$ $\quad \bar{a} = (1, 1, 0),$ $\bar{\ell} = (1, 6, 2)$

(c) $\dfrac{x-1}{2} = \dfrac{y-5}{15} = \dfrac{z+2}{6},$ $\quad \bar{b} = (1, 5, -2),$ $\quad \bar{m} = (2, 15, 6)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ 1 & 6 & 2 \\ 2 & 15 & 6 \end{vmatrix} = (6, -2, 3) \neq \bar{0}$$

$\bar{b} - \bar{a} = (0, 4, -2)$, $(\bar{b} - \bar{a}) \cdot (\bar{\ell} \times \bar{m}) = (0, 4, -2).(6, -2, 3)$

$|\bar{\ell} \times \bar{m}| = \sqrt{36 + 4 + 9} = \sqrt{49} = 7$

shortest distance $= \frac{|-14|}{7} = 2$ unit.

93. $\bar{r} = (4, -1, 0) + K(1, 2, -3)$, $\bar{a} = (4, -1, 0)$ $\bar{\ell} = (1, 2, -3)$

(a) $\bar{r} = (1, -1, 2) + K(2, 4, -5)$, $\bar{b} = (1, -1, 2), \bar{m} = (2, 4, -5)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = (-2, -1, 0) \neq \bar{0}$$

$\bar{b} - \bar{a} = (-3, 0, 2)$

$(\bar{b} - \bar{a}) \cdot (\bar{\ell} \times \bar{m}) = (-3, 0, 2) \cdot (-2, -1, 0) = -6 + 0 - 0$

$= -6$

$\bar{\ell} \times \bar{m} = \sqrt{4 + 1 + 0} = \sqrt{5}$

shortest distance $= \frac{6}{\sqrt{5}}$ unit.

94. Line $L : \bar{r} = (8, -9, 10) + K(3, -16, 7) = (8 + 3K_1, -9 - 16K_1)$ $k \in R$

(b) $P \in L, P = (8 + 3K_1, -9 - 16k_1, 10 + 7K_1)$ ...(1)

line $M : \bar{r} = (15 + 3K_2, 29 + 8K_2, 5 - 5K_2)$ $K_2 \in R$

$Q \in M$, $Q(15 + 3K_2, 29 + 8K_2, 5 - 5K_2)$ ...(2)

$\overrightarrow{PQ} = (7 + 3K_2 - 3K_1, 38 + 8K_2 + 16K_1, -5 - 5K_2 - 7K_1)$

$\bar{\ell} = (3, -16, 7)$ íkÚkk $\bar{m} = (3, 8, -5)$

PQ is shortest distance between L and M

$\overrightarrow{PQ} \perp L$ and $\overrightarrow{PQ} \perp M$

$\overrightarrow{PQ} \cdot \bar{\ell} = 0$

$3(7 + 3K_2 - 3K_1) - 16(38 + 8K_2 + 16K_1) + 7(-5 - 5K_2 + 7K_1) = 0$

$\therefore 77K_2 + 157K_1 + 311 = 0$ ...(3)

similarly $\overrightarrow{PQ} . \bar{m} = 0$

$3(7 + 3K_2 - 3K_1) + 8(38 + 8K_2 + 16K_1) - 5(-5 - 5K_2 - 7K_1)$

$49K_2 + 77K_1 + 1750 = 0$ ...(4)

solving (3) and (4)

put $K_1 = 1$ in eq^n (1) $P(5, 7, 3)$

$K_2 = -2$ in eq^n (2) $Q(9, 13, 15)$

$\overrightarrow{PQ} = (4, 6, 12)$

$|\overrightarrow{PQ}| = \sqrt{16 + 36 + 144} = \sqrt{196} = 14$ unit.

95. Line $L : \dfrac{x-23}{-6} = \dfrac{y-19}{-4} = \dfrac{z-25}{3} = K_1$ $K \in R$

(d) $P \in L : P(-6K_1 + 23, -4K_1 + 19, 3K_1 + 25)$ ...(1)

$M : \dfrac{x-12}{-9} = \dfrac{y-1}{4} = \dfrac{z-5}{2} = K_2, \quad K_2 \in R$

$Q \in M$

$(-9K_2 + 12, 4K_2 + 1, 2K_2 + 5)$ $\overrightarrow{PQ} = (-9k_2 + 6k_1 - 11, 4k_2 + 4k_1 - 18, 2k_2 - 3k_1 - 20)$

$\bar{\ell} = (-6, -4, 3), \quad \bar{m} = (-9, 4, 2)$

$\overrightarrow{PQ} \cdot \bar{\ell} = 0, -6(-9K_2 + 6K_1 - 11) - 4(4K_2 + 4K_1 - 18) + 3(2K_2 - 3K_1 - 20) = 0$

$44K_2 - 61K_1 + 78 = 0$ ...(3)

$\overrightarrow{PQ} . \bar{m} = 0, -9(-9K_2 + 6K_1 - 11) + 4(4K_2 - 4K_1 - 18) + 2(2K_2 - 3K_1 - 20) = 0$

$101K_2 - 44K_1 - 13 = 0$ ...(4)

solving eq^n (3) and (4),

$K_2 = 1, K_1 = 2$

put $K_1 = 2$ eq^n (1) $P(11, 11, 31)$

$K_2 = 1$ eq^n (2) $Q(3, 5, 7)$

$\overrightarrow{PQ} = (-8, -6, -24)$

$\left|\overrightarrow{PQ}\right| = \sqrt{64 + 36 + 576} = \sqrt{676}$

$\left|\overrightarrow{PQ}\right| = 26$ unit.

96. The eq^n of $\overrightarrow{OO'}$ is $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$

(b) $\overrightarrow{OO'}$ is non-complanar edge to $\overrightarrow{OO'}$ then

$$\frac{x-a}{0} = \frac{y}{0} = \frac{z}{c}$$

$\bar{a} = (0, 0, 0)$, $\bar{b} = (a, 0, 0)$

$\bar{\ell} = (a, b, c)$, $\bar{m} = (0, 0, c)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ a & b & c \\ 0 & 0 & c \end{vmatrix} = (bc, -ca, 0)$$

and, $\bar{b} - \bar{a} = (a, 0, 0)$ $|\bar{\ell} \times \bar{m}| = c\sqrt{a^2 + b^2}$

$(\bar{b} - \bar{a}).(\bar{\ell} \times \bar{m}) = abc$

shortest distance between two skew lines $= \dfrac{abc}{c\sqrt{a^2 + b^2}}$

$$= \frac{ab}{\sqrt{a^2 + b^2}}$$

97. $\overrightarrow{OO'}$ $\dfrac{x}{1} = \dfrac{y}{1} = \dfrac{z}{1}$ and $\overline{AB}'$ $\dfrac{x-1}{0} = \dfrac{y}{0} = \dfrac{z}{1}$

(b) $\bar{a} = (0, 0, 0)$, $\bar{b} = (1, 0, 0)$

$\bar{\ell} = (1, 1, 1)$, $\bar{m} = (0, 0, 1)$

$\bar{b} - \bar{a} = (1, 0, 0)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = (1, -1, 0)$$

$(\bar{b} - \bar{a}).(\bar{\ell} \times \bar{m}) = 1$ $|\bar{\ell} \times \bar{m}| = \sqrt{1+1} = \sqrt{2}$

shortest distance between skew lines $= \dfrac{1}{\sqrt{2}}$

98. $\bar{a} = (1, 2, 3)$, $\bar{b} = (2, 1, 0)$, $\bar{c} = (3, 3, -1)$

(b) eq^n of plane $\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & -1 & -3 \\ 2 & 1 & -4 \end{vmatrix} = 0$

$7x - 2y + 3z = 12$, $2(x-1)-5(y-2)+z-3=0$

99. eq^n of plane which intersect the axis is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

(d) $\frac{x}{3} + \frac{y}{-4} + \frac{z}{7} = 1$

$28x - 21y - 12z = 84$ ...(1)

put points in above eq^n (1)

(A) $(2, -3, 1) \Rightarrow 56 + 63 - 12 \neq 84$

(B) $(1, 1, -2) \Rightarrow 28 - 21 + 24 \neq 84$

(C) $(1, -1, -3) \Rightarrow 28 + 21 + 36 = 85 \neq 84$

(D) None of them

100. $4x - 81y + 9z = 1$ which eq^n of plane with interecept on axis

(a) 4x-81y+9z = 1 comparing eq^n

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

X- axis $a = \frac{1}{4}$

Y- axis $b = \frac{-1}{81}$

Z- axis $c = \frac{1}{9}$

$a + b + c = \frac{1}{4} - \frac{1}{81} + \frac{1}{9} = \frac{729 - 36 + 324}{2916}$

$= \frac{1017}{2916}$

101. Suppose X-intercept = Y-Intercept = a

(b) Z-Intercept = 14

$$\frac{x}{a}+\frac{y}{a}+\frac{z}{14}=1 \qquad ...(1)$$

point $(2, 1, 3)$ is on the plane from eq^n (1)

$$\frac{2}{a}+\frac{1}{a}+\frac{3}{14}=1 \qquad \frac{3}{a}=\frac{11}{14}, \qquad a=\frac{42}{11}$$

req. eq^n $\frac{11x}{42}+\frac{11y}{42}+\frac{z}{14}=1$

$\therefore$ $11x+11y+3z=42$

102. For plane : $2x-y+z=2$ $\qquad \bar{n}_1=(2,-1,1) \qquad |\bar{n}_1|=\sqrt{6}$

(b) $\quad x+y+23=3 \qquad \bar{n}_2=(1,1,2) \qquad |n_2|=\sqrt{6}$

$$\cos\alpha=\frac{|\bar{n}_1\,\bar{n}_2|}{|\bar{n}_1||\bar{n}_2|},\ \cos\alpha=\frac{(2-1+2)}{\sqrt{6}\,\sqrt{6}}=\frac{3}{6}=\frac{1}{2}$$

$$\alpha=\frac{\pi}{3}$$

103. Line $\bar{r}=(-1,1,2)+K(3,2,4) \qquad k\in R$

(b) $\quad \bar{a}=(-1,1,2), \qquad \bar{\ell}=(3,2,4)$

For plane $2x+y-3z+4=0 \qquad \bar{n}=(2,1,-3)$

Angle between line and plane is α

$$\sin\alpha=\frac{|\bar{\ell}\,.\,\bar{n}|}{|\bar{\ell}||\bar{n}|} \qquad \bar{\ell}.\bar{n}=(3,2,4)\,.\,(2,1,-3)$$

$$=6+2-12$$

$$=-4$$

$$|\bar{\ell}|=\sqrt{9+4+16}=\sqrt{29}$$

$$|\bar{n}|=\sqrt{4+1+9}=\sqrt{14}$$

$$\sin\alpha=\left|\frac{4}{\sqrt{29}\,\sqrt{14}}\right|=\frac{4}{\sqrt{406}}$$

$$\alpha=\sin^{-1}\left(\frac{4}{\sqrt{406}}\right)$$

104. Line $\frac{x}{2}=\frac{y}{2}=\frac{z}{1} \qquad \bar{\ell}=(2,2,1)$

(c) plane $2x - 2y + z = 1$ $\quad \bar{n} = (2, -2, 1)$

$\bar{\ell}.\bar{n} = (2, 2, 1).(2, -2, 1) = 4 - 4 + 1 = 1$

$|\bar{\ell}| = \sqrt{4+4+1} = 3 \qquad |\bar{n}| = \sqrt{4+4+1} = 3, \quad \text{Sin}\,\alpha = \frac{\bar{\ell}\cdot\bar{n}}{|\bar{\ell}||\bar{n}|}$

$\sin\alpha = \frac{1}{9}, \quad \alpha = \sin^{-1}\frac{1}{9}$

105. A $(1, 2, 3)$ $\quad x - 2y + 2z - 5 = 0,\ \bar{n} = (1, -2, 2), \quad d = 5$

(c) Distance from point to plane $= \frac{|1-4+6-5|}{\sqrt{1+4+4}} = \frac{|-2|}{\sqrt{9}} = \frac{2}{3}$

position vector of foot of perpendicular $\bar{a} + k_1\,\bar{n}$

where $K_1 = \frac{d - \bar{a}\,.\,\bar{n}}{|\bar{n}|^2} \qquad d - \bar{a}.\bar{n} = 5 - (1, 2, 3).(1 - 2, 2)$

$= 5 - (1 - 4 + 6)$

$= 2$

$K_1 = \frac{2}{9} \qquad |\bar{n}|^2 = 9$

position vector $= \bar{a} + k_1\bar{n} = (1, 2, 3) + \frac{2}{9}(1, -2, 2)$

$= \left(\frac{11}{9}, \frac{14}{9}, \frac{31}{9}\right)$

106. plane : $x + 2y - 3z = 6 \qquad \bar{n}_1 = (1, 2, 3), \qquad d_1 = 6$

(a) $2x + y + z = 7 \qquad \bar{n}_2 = (2, -1, 1), d_2 = 7$

$$\bar{n} = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 2 & -1 & 1 \end{vmatrix} = (-1, -7, -5)$$

$\therefore$ Direction of requi line $= (1, 7, 5)$

To obtain (common point) point of intersection of two plane put $z = 0$

$x + 2y = 6,\ 2x - y = 7$

solving this eq^n $x = 4,\ y = 1$

common point $(4, 1, 0)$

eq^n of line $\frac{x-4}{1} = \frac{y-1}{7} = \frac{z}{5}$

107. eq^n of plane $2x - y + z + 3 = 0$, $\bar{n} = (2, -1, 1)$, $d = -3$

(d) point $A(1, 3, 4)$. $\bar{a} = (1, 3, 4)$

Co-ordio of $M = \bar{a} + k_1 \bar{n}$

$$k_1 = \frac{d - \bar{a}\,\bar{n}}{|\bar{n}|^2}$$

$$k_1 = \frac{-3-(2-3+4)}{6} = -1$$

$M = (1, 3, 4) - 1(2, -1, 1) = (-1, 4, 3)$

position vector of B is $= (x, y, z)$ them

$$\frac{x+1}{2} = -1, \quad \frac{y+3}{2} = 4, \quad \frac{z+4}{2} = 3$$

$x = -3 \quad y = 5, z = 2$

point $(1, 3, 4)$ is image is, $(-3, 5, 2)$

108. $\bar{a} = (2, -1, 2)$ plane $2x - 3y + 4z = 44$

(b) $\bar{n} = (2, -3, 4)$, $d = 44$

M is foot of perpendicular from a them

$\bar{m} = \bar{a} + k_1 \bar{n}$ $\qquad K_1 = \frac{d - \bar{a}\,\bar{n}}{|\bar{n}|^2}$

$$K_1 = \frac{44 - (2, -3, 4)(2, -1, 2)}{4 + 9 + 16}$$

$$= \frac{44 - (4+3+8)}{29} = \frac{29}{29} = 1$$

$\bar{m} = \bar{a} + k_1 \bar{n} = (2, -1, 2) + (2, -3, 4) = (4 - 4, 6)$

Direction line $\bar{\ell}$ passing through A is $\overrightarrow{AM}$

$\bar{\ell} = \overrightarrow{AM} = (4, -4, 6) - (2, -1, 2) = (2, -3, 4)$

Lingth of perpendicular $= \sqrt{4 + 9 + 16} = \sqrt{29}$

length $= \sqrt{29}$ foot of perpendicular $= (4, -4, 6)$

109. The eq^n of plane $2x - 2y + z = -3$

(a) $-x\left(\frac{2}{3}\right) + y\left(\frac{2}{3}\right) + z\left(\frac{-1}{3}\right) = 1$

$x\left(-\frac{2}{3}\right) + y\left(\frac{2}{3}\right) + z\left(-\frac{1}{3}\right) = 1$

comparing with eq^n $x\cos\alpha + y\cos\beta + z\cos\gamma = P$

$\cos\alpha = \frac{-2}{3}, \quad \cos\beta = \frac{2}{3}, \quad \cos\gamma = \frac{-1}{3} \qquad P = 1$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{4}{9} + \frac{4}{9} + \frac{1}{9} = 1$

and $P = 1 \geq 0$

$\therefore$ perpendicular distance from origin to plane = 1

position vector of foot of perpendicular $(P\cos\alpha,\ P\cos\beta,\ P\cos\gamma)$

$= \left(\frac{-2}{3}, \frac{2}{3}, \frac{-1}{3}\right)$

Direction cosine : $\cos\alpha, \ \cos\beta, \cos\gamma$

$\therefore \frac{-2}{3}, \frac{2}{3}, \frac{-1}{3}$

110. $A(1, 2, 3)\ B(5, 4, 1)$ mid point of $\overline{AB}$ is M

(b) $M\left(\frac{5+1}{2}, \frac{4+2}{2}, \frac{3+1}{2}\right), \quad M(3, 3, 2)$

plane is passing through

$M(3, 3, 2)$ and perpendicular to $\overline{AB}$

$\overrightarrow{AB} = (4, 2, -2), \ (\bar{r} - \bar{a})\,\bar{n} = 0$

$\bar{n} = \overrightarrow{AB} = (4, 2, -2), \bar{r} = (x, y, z). \bar{a} = (3, 3, 2)$

$(x-3, y-3, z-2).(4, 2, -2) = 0$

$4x - 12 + 2y - 6 - 2z + 4 = 0$

$2x + y - z = 7$

111. $3x + y - z = 0$ $\qquad \bar{n}_1 = (3, 1, -1),$ $\qquad d_1 = 0$

(c) $x + 2y + 3z = 5$ $\qquad \bar{n}_2 = (1, 2, 3),$ $\qquad d_2 = 5$

Required plane is perpendicular to given plane

$$\bar{n} = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ 1 & 2 & 3 \end{vmatrix} = (5, -10, 5)$$

plane passess through $\bar{a} = (1, 3, 5)$

$(x-1, y-3, z-5) . (5, -10, 5) = 0$

$5x - 5 - 10y + 30 + 5z - 25 = 0$

$x - 2y + z = 0$

112. Plane $\bar{r}\ (2, -b, 1) = 4$ $\qquad \bar{n}_1 = (2, -b, 1)$

(b) $\bar{r}\ (4, -1, c) = 6$ $\qquad \bar{n}_2 = (4, -1, c)$

planes are prallel $\bar{n}_1 = k\ \bar{n}_2$

$(2, -b, 1) = k\ (4, -1, c)$

$2 = 4k \qquad -b = -k \qquad 1 = kc$

$k = \frac{1}{2} \qquad b = \frac{1}{2} \qquad 2 = c$

$b = \frac{1}{2}$

113. Plane : $3x - 2y + z = 1$ $\qquad 6x - 4y + 2z - k = 0$

(a) now above plane is parallel to $6x - 4y + 2z - 2 = 0$

$6x - 4y + 23 - k = 0$

perpendicular distance between two plane $= \dfrac{|2 - k|}{\sqrt{36 + 36 + 4}}$

$$\frac{3}{2\sqrt{14}} = \frac{|k-2|}{\sqrt{56}}$$

$K - 2 = 3 \qquad k - 2 = -3$

$K = 5 \qquad k = -1$

114. Line : $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{1}$, $\quad \bar{a} = (1, 3, 0)$, $\bar{\ell} = (2, 4, 1)$

(b) $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{1}$, $\quad \bar{b} = (4, 1, 1)$, $\bar{m} = (3, -2, 1)$

$\bar{a} - \bar{b} = (-3, 2, 1)$ and, $\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ 2 & 4 & 1 \\ 3 & -2 & 1 \end{vmatrix}$

$= (6, 1, -16)$

$(\bar{a} - \bar{b}).(\bar{\ell} \times \bar{m}) = (-3, 2, -1).(6, 1, -16)$

$= -18 + 2 + 16 = 0$

Lines are coplaner eq^n of plane $(\bar{r} - \bar{a}).(\bar{\ell} \times \bar{m}) = 0$

$$\begin{vmatrix} x-1 & y-3 & z-0 \\ 2 & 4 & 1 \\ 3 & -2 & 1 \end{vmatrix} = 0$$

$(x-1)(6) - (y-3)(-1) + z(-16) = 0$

$6x - 6 + y - 16z = 0$

$6x + y - 16z = 9$

115. $\frac{x}{2} = \frac{y-1}{1} = \frac{z+2}{2}$ $\quad \bar{a} = (0, 1, -2)$, $\quad \bar{\ell} = (2, 1, 2)$

(b) $\frac{x + \frac{3}{2}}{2} = \frac{y-3}{1} = \frac{z}{2}$, $\bar{b} = \left(\frac{-3}{2}, 3, 0\right)$, $\quad \bar{m} = (2, 1, 2)$

$\bar{\ell} = \bar{m}$ line are parallel

eq^n of plane : $\begin{vmatrix} x-0 & y-1 & z+2 \\ -\frac{3}{2}-0 & 3-1 & 0+2 \\ 2 & 1 & 2 \end{vmatrix} = 0$

$x(4-2) - (y-1)(-3-4) + (z+2)\left(\frac{-3}{2} - 4\right) = 0$

$2x + 7y - 7 - \frac{11z}{2} - \frac{22}{2} = 0$

$4x + 14y - 14 - 11z - 22 = 0$

$4x + 14y - 11z - 36 = 0$

116. Line $\bar{r} = (1, 1, 1) + k\,(2, 1, 2)$, $\bar{a} = (1, 1, 1)$

(a) $\bar{\ell} = (2, 1, 2)$

$\bar{b} = (1, -1, 2)$

$\overrightarrow{AB} = \bar{b} - \bar{a}$

$= (0, -2, 1)$

Normal of plane $\bar{n} = \overrightarrow{AB} \times \bar{\ell}$

$$= \begin{vmatrix} i & j & k \\ 0 & -2 & 1 \\ 2 & 1 & 2 \end{vmatrix} = (-5, 2, 4)$$

eq^n of plane $\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$(x, y, z).(-5, 2, 4) = (1, 1, 1).(-5, 2, 4)$

$-5x + 2y + 4z = -5 + 2 + 4$

$5x - 2y - 4z + 1 = 0$

117. $L: \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$, $\quad \bar{a} = (1, 2, 3)$, $\quad \bar{\ell} = (2, 3, 4)$

(c) $M: \dfrac{x-1}{2} = \dfrac{y}{3} = \dfrac{z-5}{4}$, $\quad \bar{b} = (1, 0, 5)$, $\quad \bar{m} = (2, 3, 4)$

$\bar{\ell} = \bar{m}$ nkðkLke $\bar{\ell} \times \bar{m} = \bar{0}$

$(1, 2, 3) \in L$, But $\dfrac{1-1}{2}, \dfrac{2}{3}, \dfrac{3-5}{4}$ Not equal

$(1, 2, 3) \notin M$

L and M are parallel line

eq^n of plane $(\bar{r} - \bar{b}).\left[(\bar{b} - \bar{a}) \times \bar{\ell}\right] = 0$

$$\begin{vmatrix} x-1 & y & z-5 \\ 0 & -2 & 2 \\ 2 & 3 & 4 \end{vmatrix} = 0, \ (x-1)(-8-6) - y(-4) + (z-5)(4)$$

$7x - 2y - 2z + 3 = 0$ plane eq^n

118. L : $\frac{x+3}{2} = \frac{y+5}{3} = \frac{z-7}{-3}$

(b) $\bar{a} = (-3, -5, 7)$

$\bar{\ell} = (2, 3, -3)$

$\frac{x+1}{4} = \frac{y+1}{5} = \frac{z+1}{-1}$, $\bar{b} = (-1, -1, -1)$

$\bar{m} = (4, 5, -1)$

$$(\bar{b} - \bar{a}) \cdot (\bar{\ell} \times \bar{m}) = \begin{vmatrix} 2 & 4 & -8 \\ 2 & 3 & -3 \\ 4 & 5 & -1 \end{vmatrix} = 24 - 40 + 16 = 0$$

Lines are co-planer eq^n of plane

$(\bar{r} - \bar{a}) \cdot (\bar{\ell} \times \bar{m}) = 0$

$$\begin{vmatrix} x+3 & y+5 & z-7 \\ 2 & 3 & -3 \\ 4 & 5 & -1 \end{vmatrix} = 0$$

$12x + 36 - 10y - 50 - 2z + 14 = 0$

$6x - 5y - z = 0$

119. $\bar{a} = (1, 2, 3)$, $\bar{b} = (3, -1, 2)$ plane $x + 3y + 2z = 7$

(a) plane in point $\bar{a}$ and $\bar{b}$

$\bar{n} = (1, 3, 2)$

$d = 7$

$\bar{b} - \bar{a} = (2, -3, -1)$

Normal of plane = $\bar{m} = \overrightarrow{AB} \times \bar{n} = \begin{vmatrix} i & j & k \\ 2 & -3 & -1 \\ 1 & 3 & 2 \end{vmatrix}$

$= (-3, -5, 9)$

eq^n of plane : $(x, y, z) \cdot (-3, -5, 9) = (1, 2, 3) \cdot (-3, -5, 9)$

$(x, y\ z) \cdot (-3, -5, 9) = (1, 2, 3) \cdot (-3, -5, 9)$

$-3x - 5y + 9z = -3 - 10 + 27$

$3x + 5y - 9z + 14 = 0$

120. Plane : $\pi_1 : x + 2y + 2z = 1 \qquad ...(1)$

(c) eq^n of parallel plane $\pi_1 \qquad \pi_2 : x + 2y + 2z = k, \quad k \in R - \{-1\}$

$\therefore$ perpendicular dist is 2 unit .

$2 = \dfrac{|1 - k|}{\sqrt{1 + 4 + 4}} \qquad |1 - k| = 6$

$1 - k = -6$ or $1 - k = -6$

$K = -5,$ or $K = 7$

$x + 2y + 2z = 7$ and $x + 2y + 2z = -5$

121. Point $A(1, 6, -4)$ line $\dfrac{x-1}{2} = \dfrac{y-2}{-3} = \dfrac{z-3}{-1}$

(a) $\bar{a} = (1, 2, 3), \ \bar{\ell} = (2, -3, -1)$

$A(\bar{b}), \ \bar{b} = (1, 6, -4)$

normal to plane $\bar{n} = \overrightarrow{AB} \times \bar{\ell}$

$\overrightarrow{AB} = \bar{b} - \bar{a} = (0, 4, -7)$

$$\bar{n} = \begin{vmatrix} i & j & k \\ 0 & 4 & -7 \\ 2 & -3 & -1 \end{vmatrix} = (-25, -14, -8)$$

eq^n of plane $\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$(x, y, z)(-25, -14, -8) = (1, 2, 3)(-25, -14, -8)$

$-25x - 14y - 8z = -25 - 28 - 24$

$25x + 14y + 8z = 77$

122. Plane $2x + 4y + 8z = 17 \qquad \bar{n} = (2, 4, 8) \quad d = 17$

(c) line : $\dfrac{x-3}{2} = y = \dfrac{z-8}{-1} \qquad \bar{a} = (3, 0, 8) \quad \bar{\ell} = (2, 1, -1)$

Direction of line $\bar{\ell} = (2, 1, -1)$

$\bar{\ell}.\bar{n} = (2, 1, -1).(2, 4, 8) = 4 + 4 - 8 = 0$

point on line does not satisfy the eq^n of plane.

$\therefore$ Line is parallel to plane Eq^n of plane paralles to $2x + 4y + 8z = 17$

is $2x + 4y + 8z = k \qquad k \in R - \{-17\}$

point on line p $(3+2t,\ t,\ 8-t)$ satisfies

plane $2x + 4y + 8z = k$

$2(3+2t) + 4t + 8(8-t) = k$

$6 + 4t + 4t + 64 - 8t = k$

$k = 70$

eq^n of plane $2x + 4y + 8z = 70$

$x + 2y + 4z = 35$

123. $\pi_1 : x + y + z + 1 = 0 \qquad \pi_2 : x - 3y + z + 3 = 0$

(c) $\ell(x + y + z + 1) + m(x - 3y + z + 3) = 0 \qquad ...(1)$

$x(\ell + m) + y(\ell - 3m) + z(\ell + m) + \ell + 3m = 0$

$\bar{n} = (\ell + m,\ \ell - 3m,\ \ell + m)$

$\frac{x}{1} = \frac{y}{2} = z$ Direction of Line $\bar{\ell} = (1, 2, 1)$

Line two plane $\bar{\ell}.\bar{n} = 0$ and $\bar{a}.\bar{n} \neq d$

$\ell + m + 2\ell - 6m + \ell + m = 0$

$4\ell - 4m = 0$

$\bar{\ell} = \bar{m}$, so $m = 1$, put eq^n (1)

$x + y + z + 1 + x - 3y + z + 3 = 0$

$x - y + z + 2 = 0$

124. plane $\pi_1 : x + y + z - 1 = 0, \qquad \pi_2 : x + y + z - 1 = 0$

(b) $\ell(x - y + z - 1) + m(x + y - z - 1) = 0 \qquad ...(1)$

$(\ell + m)x + (-\ell + m)y + (\ell - m)z - \ell - m = 0$

plane $x - 2y + z = 2$ parpendicular this plane

$\bar{n}_1 = (\ell + m,\ m - \ell,\ \ell - m),\quad \bar{n}_2 = (1, -2, 1)$

$\bar{n}_1 . \bar{n}_2 = 0$

$(\ell + m) + (-2)(m - \ell) + 1(\ell - m) = 0$

$4\ell - 2m = 0$

$$\frac{\ell}{m} = \frac{1}{2}$$

$\ell = 1$, so $m = 2$ put

eq^n $(x - y + z - 1) + 2(x + y - z - 1) = 0$

$3x + y - z = 3$

125. plane $\pi_1 : x - y + z - 1 = 0, \qquad \pi_2 : x + y - z - 1 = 0$

(b) $(x - y + z - 1) + \lambda(x + y - z - 1) = 0 \qquad ...(1)$

$(1 + \lambda)x + (\lambda - 1)y + (1 - \lambda)z - 1 - \lambda = 0$

$$\frac{x}{1} + \frac{y}{-\left(\frac{1+\lambda}{1-\lambda}\right)} + \frac{z}{\frac{1+\lambda}{1-\lambda}} = 1$$

$$\text{Y - Intercept} = -\frac{1+\lambda}{1-\lambda} = 3$$

$-1 - \lambda = 3 - 3\lambda$

$\lambda = 2$

126. plane Intersect $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, C)$

(b) eq^n plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Parpendicular distance from $(0,0,0)$ is 3P

$$\frac{|-1|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = 3P$$

$$\frac{1}{a^2} - 1\frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2} \qquad ...(1)$$

Centroid of ΔABC, $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ Cheek which of the options satify eq^n

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{P^2}$$

$$\frac{1}{\frac{a^2}{9}} + \frac{1}{\frac{b^2}{9}} + \frac{1}{\frac{c^2}{9}} = 9\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)$$

$= 9\left(\frac{1}{a P_2}\right)$

$= \frac{1}{P_2}$

ΔABC Centoid is on eq^n

$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{P^2}$

127. Hear $A(a, 0, 0)$ $B(0, b, 0)$, $C(0, 0, C)$ $\therefore G = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

(b) G is $(2, 1, 3)$ given

$a = 6,\ b = 3\ c = 9$

plane eq^n, $\frac{x}{6} + \frac{y}{3} + \frac{z}{9} = 1$

$\therefore 3x + 6y + 2z = 18$

128. plane Intersects in $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$ centrioid of

(c) $\Delta ABC = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (\alpha, \beta, \gamma)$

$a = 3\alpha,\ b = 3\beta,\ c = 3\gamma$

$\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$

eq^n plane, $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

129. plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ Intersect axis in $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, C)$ and passing through (α, β, γ) then $\frac{\alpha}{a} + \frac{\beta}{b} + \frac{\gamma}{c} = 1$ hear forn A, B,C, prallle plane are

(b) $x = a,\ y = b\ \ z = c$

point of Intersection is $(x, y, z) = (a, b, c)$

$\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\gamma}{z} = 1$ $\frac{\alpha}{a} + \frac{\beta}{b} + \frac{\gamma}{c} = 1$

130. Point on axis $A(a, 0, 0), B(0, b, 0), C(0, 0, c)$ Centroid of ΔABC

$$(x_1, y_1, z_1) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

(a) A,B,C Satisfied plane

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ distance from $(0, 0, 0)$ to plane is P.

$$P = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\therefore \frac{1}{P^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

$$\frac{1}{P^2} = \frac{1}{9x_1^2} + \frac{1}{9y_1^2} + \frac{1}{9z_1^2}$$

Centroid of ΔABC is on $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{9}{P^2}$

131. eq^n plane is

(b) $x\cos\alpha + y\cos\beta + z\cos\gamma = P$

Which Intersect axis $A\left(\frac{P}{\cos\alpha}, 0, 0\right), B\left(0, \frac{P}{\cos\beta}, 0\right), C\left(0, 0, \frac{P}{\cos\gamma}\right)$

from A, B, C eq^n of parallel plane

$$x = \frac{P}{\cos\alpha} \quad y = \frac{P}{\cos\beta}, \quad z = \frac{P}{\cos\gamma}$$

Intersection of planes

$$(x_1, y_1, z_1) = \left(\frac{P}{\cos\alpha}, \frac{P}{\cos\beta}, \frac{P}{\cos\gamma}\right)$$

$$\cos\alpha = \frac{P}{x_1}, \quad \cos\beta = \frac{P}{y_1} \quad \cos\gamma = \frac{P}{z_1}$$

α, β, γ is diretion cosine

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\frac{P^2}{x_1^2} + \frac{P^2}{y_1^2} + \frac{P^2}{z_1^2} = 1$$

$$\frac{1}{x_1^2} + \frac{1}{y_1^2} + \frac{1}{z_1^2} = \frac{1}{P^2}$$

A, B and C passing - cordinate plane is parllel to plane intersection, point on

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{P^2}$$

132. plane $\pi_1 : 2x + y + 2z = 1$ $\quad \bar{n}_1 = (2, 1, 2),$ $\quad d_1 = 1$

(c) $\pi_2 : x + 2y - 2z = 1$ $\quad \bar{n}_2 = (1, 2, -2),$ $\quad d_2 = 1$

$$\bar{\ell} = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 1 & 2 & -2 \end{vmatrix} = (-6, 6, 3)$$

Take $z = 0$ in π_1 and π_2 eq^n plane

$\therefore 2x + y = 1$ and $x + 2y = 1$

soliving eq^n of plane $x = \frac{1}{3}, y = \frac{1}{3}$, $\bar{a} = \left(\frac{1}{3}, \frac{1}{3}, 0\right)$

is common point $\left(\frac{1}{3}, \frac{1}{3}, 0\right)$

eq^n of common line $\bar{r} = \bar{a} + k\bar{\ell}$ $\qquad k \in R$

$$\bar{r} = \left(\frac{1}{3}, \frac{1}{3}, 0\right) + K\,(-6, 6, 3) \qquad ...(1)$$

$\pi_3 : 6x + 2y + 3z = 1$ $\qquad \bar{n}_3 = (6, 2, 3)$

$\pi_4 : 6x + 2y - 3z = 1$ $\qquad \bar{n}_4 = (6, 2, -3)$

$$\bar{m} = \bar{n}_3 \times \bar{n}_4 = \begin{vmatrix} i & j & k \\ 6 & 2 & 3 \\ 6 & 2 & -3 \end{vmatrix} = (-12, 36, 0)$$

for plane π_3 & π_4 take $x = 0$

$\therefore 2y + 3z = 1, \quad 2y - 3z = 1$

solving eq^n, $y = \frac{1}{2}$, $z = 0$

point of Intersection $\bar{b} = \left(0, \frac{1}{2}, 0\right)$

which on Both plane

$\therefore$ eq^n of common line π_3 and π_4

$\bar{r} = \bar{b} + k\,\bar{m}$

$\bar{r} = \left(0, \frac{1}{2}, 0\right) + k\,(-12, 36, 0)$ ...(2)

from eq^n (1) and (2)

$\bar{\ell} = (-6, 6, 3)$

$\bar{m} = (-12, 36, 0)$

$$\bar{\ell} \times \bar{m} = \begin{vmatrix} i & j & k \\ -6 & 6 & 3 \\ -12 & 36 & 0 \end{vmatrix} = (-108, -36, -144) \neq \bar{0}$$

$\therefore$ lines are not parallel

$\bar{\ell} \times \bar{m} \neq \bar{0}$ line are non coplaner

$$(\bar{a} - \bar{b}).(\bar{\ell} \times \bar{m}) = \left(\frac{1}{3}, \frac{-1}{6}, 0\right).(-108, -36, -144)$$

$= -36 + 6 - 0 = -30 \neq 0$ $\therefore$ line are non coplaner (skew line)

133. line $\bar{r} = (2, -2, 3) + K\,(1, -1, 4)$ $k \in R$

(b) plane $\bar{r}.(1, 5, 1) = 5$

$\bar{r} = (1, -1, 4)$, $\bar{n} = (1, 5, 1)$

$\bar{\ell}\,.\,\bar{n} = (1, -1, 4).(1, 5, 1)$

$= 1 - 5 + 4$

$= 0$

$\therefore$ line is parallel to plane

parpendicular, distance from $(2, -2, 3)$ to plane $x + 5y + z - 5 = 0$

$$P = \frac{|2 + 5(-2) + 3 - 5|}{\sqrt{1 + 25 + 1}} = \frac{10}{\sqrt{27}}$$

$= \frac{10}{3\sqrt{3}}$ unit

ANSWERS

1.	(B)	39.	(C)	77.	(C)	115.	(B)
2.	(A)	40.	(D)	78.	(B)	116.	(A)
3.	(B)	41.	(C)	79.	(A)	117.	(C)
4.	(B)	42.	(A)	80.	(A)	118.	(B)
5.	(B)	43.	(A)	81.	(B)	119.	(A)
6.	(A)	44.	(C)	82.	(C)	120.	(C)
7.	(C)	45.	(C)	83.	(A)	121.	(A)
8.	(C)	46.	(A)	84.	(A)	122.	(C)
9.	(D)	47.	(C)	85.	(A)	123.	(A)
10.	(A)	48.	(B)	86.	(B)	124.	(B)
11.	(D)	49.	(B)	87.	(A)	125.	(B)
12.	(B)	50.	(B)	88.	(D)	126.	(B)
13.	(D)	51.	(B)	89.	(B)	127.	(B)
14.	(A)	52.	(A)	90.	(D)	128.	(C)
15.	(A)	53.	(D)	91.	(C)	129.	(B)
16.	(B)	54.	(A)	92.	(C)	130.	(A)
17.	(A)	55.	(B)	93.	(A)	131.	(B)
18.	(C)	56.	(B)	94.	(B)	132.	(C)
19.	(A)	57.	(C)	95.	(D)	133.	(B)
20.	(B)	58.	(A)	96.	(B)		
21.	(D)	59.	(A)	97.	(B)		
22.	(B)	60.	(B)	98.	(B)		
23.	(B)	61.	(B)	99.	(D)		
24.	(B)	62.	(A)	100.	(A)		
25.	(C)	63.	(B)	101.	(B)		
26.	(A)	64.	(D)	102.	(B)		
27.	(D)	65.	(B)	103.	(B)		
28.	(A)	66.	(D)	104.	(C)		
29.	(C)	67.	(B)	105.	(C)		
30.	(B)	68.	(C)	106.	(A)		
31.	(A)	69.	(A)	107.	(D)		
32.	(D)	70.	(B)	108.	(B)		
33.	(C)	71.	(A)	109.	(A)		
34.	(A)	72.	(D)	110.	(B)		
35.	(B)	73.	(D)	111.	(C)		
36.	(D)	74.	(B)	112.	(B)		
37.	(D)	75.	(C)	113.	(A)		
38.	(A)	76.	(A)	114.	(B)		

JEE UNIT – 13 SOME IMPORTANT POINT

VECTOR : The quantity has magnitude and direction is called a vector. e.g. velocity, acceleration, force are denoted by small sign like $(-)$ above the letter.

NOTE:-

$R^2 = \{(x, y) \mid x \in R, y \in R\}$

$R^3 = \{(x, y, z) \mid x \in R,\ y \in R,\ z \in R\}$.

R^2 and R^3 as vector space denoted by $\bar{x}, \bar{y}, \bar{z}$.

EQUALITY OF VECTORS:-

If $\bar{x} = (x_1,\ y_1,\ z_1)$ and $\bar{y} = (x_2,\ y_2,\ z_2)$.

If $\bar{x} = \bar{y} \Leftrightarrow (x_1,\ y_1,\ z_1) = (x_2,\ y_2,\ z_2)$

$\Leftrightarrow x_1 = x_2,\ y_1 = y_2,\ z_1 = z_2$

ADDITION OF VECTORS:-

If $\bar{x} = (x_1,\ y_1,\ z_1)$ and $\bar{y} = (x_2,\ y_2,\ z_2)$.

$\bar{x} + \bar{y} = (x_1,\ y_1,\ z_1) + (x_2,\ y_2,\ z_2) = (x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2)$

MAGNITUDE OF A VECTOR :- if $\bar{x} = (x_1,\ x_2, x_3,)$ Then magnitude of $\bar{x} = \sqrt{x_1^2 + x_2^2 + x_3^2}$. It is denoted by $|\bar{x}|$.

so $|\bar{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2}$.

NOTE:- The vector whose magnitude 1(one) is called a unit vector.

DIRECTION OF VECTORS:- Let $\bar{x}$ and $\bar{y}$ be non zero vectors of R^2 or R^3 and $k \in R$.

(1) if $\bar{x} = k\bar{y}, k > 0$, then $\bar{x}$ and $\bar{y}$ having same direction

(2) if $\bar{x} = k\bar{y}, k < 0$, then $\bar{x}$ and $\bar{y}$ having opposite direction

(3) for any non zero scalar $k \in R$ and vectors $\bar{x}$ and $\bar{y}$

if $\bar{x} \neq k\bar{y}$, then $\bar{x}$ and $\bar{y}$ having different direction.

COLLINEAR VECTORS:-

if non zero vectors $\bar{x}$ and $\bar{y}$ are same or opposite direction,they are called collinear vector.

NOTE:- if $\bar{x} = k\bar{y}$ if and only if $\bar{x}$ and $\bar{y}$ are collinear vector.

THEOREM : 1 : non zero vectors $\bar{x}$ and $\bar{y}$ are equal if and only if $|\bar{x}| = |\bar{y}|$ and $\bar{x}$ and $\bar{y}$ having same direction.

THEOREM : 2 : if $\bar{x} \neq \bar{0}$ then there is a unique unit vector in the direction of $\bar{x}$.

NOTE:-

* if $\bar{x}$ is a any non zero vector, then $\frac{1}{|\bar{x}|} \cdot \bar{x}$ is a unit vector in the direction of $\bar{x}$ and it is denoted by $\hat{x}$.

* if $\bar{y} = \frac{k\,\bar{x}}{|\bar{x}|}, k > 0$ has same direction to the direction $\bar{x}$ and has magnitude k.

* if $\bar{y} = \frac{k\bar{x}}{|\bar{x}|}, k < 0$ has opposite direction to the direction of $\bar{x}$ and has magnitude k.

THEOREM : 3

(1) every vector of R^2 can be uniquely expressed as a linear combination of $\hat{\imath}$ and $\hat{\jmath}$

(2) every vector of R^3 can be uniquely expressed as a linear combination of $\hat{\imath}$ and $\hat{\jmath}$ and $\hat{k}$.

TRIANGLE LAW OF VECTOR ADDITION :-

Let the position vectors of points $A,\ B,\ C$ be $\bar{a},\ \bar{b}$ and $\bar{c}$ respectively. then $\overrightarrow{AB} + \overrightarrow{BC} = (\bar{b} - \bar{a}) + (\bar{c} - \bar{b}) = \bar{c} - \bar{a} = \overrightarrow{AC}$

PARALLELOGRAM LAW OF VECTOR ADDITION :-

Let the position vectors of points O, $A,\ B,\ C$ be $\bar{o}$ $\bar{a},\ \bar{b}$ and $\bar{c}$ respectively. then $\overrightarrow{OA} = \bar{a}$ and $\overrightarrow{OB} = \bar{b}$ are two distinct vectors.then $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} = \bar{a} + \bar{b}$

INNER PRODUCT OF VECTOR R^2 and R^3:

if $\bar{x} = (x_1, x_2)$ and if $\bar{y} = (y_1, y_2)$ are vector in R^2

then inner product is difined as $x_1y_1 + x_2y_2$. and is dinoted by * $\bar{x} \cdot \bar{y}$ So $\bar{x} \cdot \bar{y} = x_1y_1 + x_2y_2$ and

* if $\bar{x} = (x_1, x_2, x_3)$ and if $\bar{y} = (y_1, y_2, y_3)$ are vector in R^3

then inner product is difined as $\bar{x} \cdot \bar{y} = x_1\, y_1 + x_2y_2 + x_3\, y_3$

OUTER PRODECT OF VECTORS IN R^3 :-

if $\bar{x} = (x_1, x_2, x_3)$ and $\bar{y} = (y_1, y_2, y_3)$ are vector in R^3 then outer prodect of $\bar{x}$ and $\bar{y}$ is dinoted by

$$\bar{x} \times \bar{y} = (x_2y_3 - x_3\, y_2,\ x_3\, y_1 - x_1\, y_3,\ x_1\, y_2 - x_2\, y_1)$$

નોંધ : બહિર્ગુણનને સદિશ ગુણાકારની પ્રક્રિયા અથવા ક્રોસ ગુણાકાર પણ કહે છે.

DIFFERENCE BETWEEN INNER PRODUCT AND OUTER PRODUCT:-

(1) inner product is a scalar quantity,while outer product is a vector quantity.

(2) inner product is diffined in R^2 as well as R^3, while outer product is not difined in R^2.

(3) inner product is commulative,while outer product is note commulative.

BOX PRODUCT AND VECTOR TRIPLE PRODUCT :-

if $\bar{x},\bar{y},\bar{z} \in R^3$ then $\bar{x} \cdot (\bar{y} \times \bar{z})$ is called the box product of $\bar{x}$, $\bar{y}$ and $\bar{z}$. and it is dinoted by $[\bar{x}\ \bar{y}\ \bar{z}]$

NOTE:- $[\bar{x},\bar{y},\bar{z}] = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$

* The product of vector $\bar{x},\bar{y}$ and $\bar{z}$ namely $\bar{x} \times (\bar{y} \times \bar{z})$ is called triple product.

NOTE:-

* $\bar{x} \times (\bar{y} \times \bar{z}) = (\bar{x} \cdot \bar{z})\bar{y} - (\bar{x} \cdot \bar{y})\bar{z}$

* $(\bar{x} \times \bar{y}) \times \bar{z} = (\bar{z} \cdot \bar{x})\bar{y} - (\bar{z} \cdot \bar{y})\bar{x}$

* $\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$ and $\hat{\imath} \cdot \hat{\jmath} = 0, \hat{\jmath} \cdot \hat{k} = 0, \hat{k} \cdot \hat{\imath} = 0$

* $\hat{\imath} \times \hat{\jmath} = \hat{k},\ \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$ and $\hat{\imath} \times \hat{\jmath} = \bar{0}, \hat{\jmath} \times \hat{\jmath} = \bar{0}, \hat{k} \times \hat{k} = \bar{0}$

* $\hat{\jmath} \times \hat{\imath} = -k,\ \hat{k} \times \hat{\jmath} = -\hat{\imath},\ \hat{\imath} \times \hat{k} = -\hat{\jmath}$

LAGRANGE'S IDENTITY : if $x_1,x_2,x_3,\ y_1,y_2,y_3 \in R$ then $(x_1y_1 + x_2y_2 + x_3y_3)^2 + (x_1\,y_2 - x_2\,y_1)^2 + (x_1\,y_3 - x_3\,y_1)^2 + (x_2\,y_3 - x_3\,y_2)^2 = (x_1^2 + x_2^2 + x_3^2)\,(y_1^2 + y_2^2 + y_3^2)$

this identity is known as Lagrange's identity.

* if $\bar{x} = (x_1, x_2, x_3)$ and $\bar{y} = (y_1, y_2, y_3)$ then

$$|\bar{x} \cdot \bar{y}|^2 + |\bar{x} \times \bar{y}|^2 = |\bar{x}|^2 |\bar{y}|^2$$

CAUCHY-SCHWARTZ IDENTITY :

for any two vectors $\bar{x}$ and $\bar{y}$ in R^2 or R^3 so $|\bar{x} \cdot \bar{y}| \leq |\bar{x}||\bar{y}|$

This inequality is known as Cauchy-Schwartz inequality.

TRIANGULAR INEQUALITY:-

for any two vectors $\bar{x}$ and $\bar{y}$ in R^2 or R^3 so

$|\bar{x} + \bar{y}| \leq |\bar{x}| + |\bar{y}|$. This inequality is known as Triangular inequality.

THEOREM : 4

Non zero vectors of R^2 is $\bar{x} = (x_1, x_2)$ અને $\bar{y} = (y_1, y_2)$ are collinear if and only if $x_1 y_2 - x_2 y_1 = 0$

THEOREM : 5

Non zero vectors of R^3 is $\bar{x} = (x_1, x_{2,}, x_{3,})$ and $\bar{y} = (y_1, y_2, y_3)$

are collinear if and only if $\bar{x} \times \bar{y} = \bar{0}$

CO-PLANAR VECTORS:- Let $\bar{x}$, $\bar{y}$ and $\bar{z}$ be vectors in R^3. if we can find α, β, $\gamma \in R$ with atleast one of them non-zero such that $\alpha\bar{x} + \beta\bar{y} + \gamma\bar{z} = \bar{0}$ then $\bar{x}, \bar{y}$ and $\bar{z}$ are called to be co-planer vectors.

LINEARLY INDEPENDENT VECTORS:

If $\bar{x}, \bar{y}, \bar{z}$ are non co-planer vectors they are called non-co-planer vectors or linearly independent vectors.

NOTE:-

If $\bar{x}, \bar{y}, \bar{z}$ are non-co-planer vectors then $\alpha\bar{x} + \beta\bar{y} + \gamma\bar{z} = \bar{0}$

$\Rightarrow \alpha = 0, \beta = 0$ and $\gamma = 0$.

THEOREM : 6

Distinct non-zero vectors $\bar{x},\ \bar{y},\ \bar{z}$ of R^3 *are coplaner if and only if.* $[\bar{x},\bar{y},\bar{z}] = 0$

ANGLE BETWEEN TWO NON –ZERO VECTORS:-

Let $\bar{x}$ and $\bar{y}$ be two non-zero vectors

(1)if $\bar{x} = k\bar{y}, k > 0$ then $\bar{x}$ and $\bar{y}$ have same directions and so the measure of the angle between difined to be zero.

(2) if $\bar{x} = k\bar{y}, k < 0$ then $\bar{x}$ and $\bar{y}$ have oppsite directions and so the measure of the angle between difined to be π.

(3) if $\bar{x}$and $\bar{y}$ are two distict vectors and α is a measure of angle between then $\alpha = (\bar{x} \wedge \bar{y})$ and $\alpha = cos^{-1} \frac{\bar{x}\cdot\bar{y}}{|\bar{x}||\bar{y}|}, \alpha \in (0,\pi)$

ORTHOGONAL VECTORS:- if $\bar{x} \neq \bar{0}$ and $\bar{y} \neq \bar{0}$ and $\left(\bar{x} \wedge \bar{y}\right) = \frac{\pi}{2}$ then $\bar{x}$ and $\bar{y}$ are said to be orthogonal vectors or perpendicular vectors . is denoted by $\bar{x} \perp \bar{y}$.

THEOREM : 7

if $\bar{x},\bar{y} \in R^3, \bar{x} \neq \bar{0}, \bar{y} \neq \bar{0}$ and $\left(\bar{x}, \wedge \bar{y}\right) = \alpha$ હોય તો,

(1) $\bar{x}\cdot\bar{y} = |\bar{x}||\bar{y}|cos\alpha$

(2) $|\bar{x}\times\bar{y}| = |\bar{x}||\bar{y}|\sin\alpha$

(3) $\bar{x} \perp (\bar{x}\times\bar{y})$, $\bar{y} \perp (\bar{x}\times\bar{y})$

NOTE:-

$\bar{x}$ and $\bar{y}$ both are orthogonal and unit vectors is= $\pm\frac{\bar{x}\times\bar{y}}{|\bar{x}\times\bar{y}|}$

PROJECTION OF A VECTOR:- if $\bar{a}$ and $\bar{b}$ are non-zero vectors and they are not orthogonal to each other then the projection of $\bar{a}$ and $\bar{b}$ is difined as the vector $(Projection\ Vector)\left(\frac{\bar{a}\cdot\bar{b}}{|\bar{b}|^2}\right)\bar{b}$ and is dinoted by $Proj_{\bar{b}}{}^{\bar{a}}$

AREA OF TRIANGLE:- : In Δ ABC $\overrightarrow{AB} = \bar{c},\ \overrightarrow{BC} = \bar{a},$ and

$\overrightarrow{CA} = \bar{b}$

$\therefore$ Area of $\Delta\,\mathrm{ABC} = \frac{1}{2}|\bar{b} \times \bar{c}| = \frac{1}{2}|\bar{a} \times \bar{b}| = \frac{1}{2}|\bar{c} \times \bar{a}|$

NOTE:- This formula is applicable only for R^3

$\therefore$ Area of $\Delta\,\mathrm{ABC} = \frac{1}{2}\sqrt{|\bar{b}|^2|\bar{c}|^2 - |\bar{b}\cdot\bar{c}|^2}$

(NOTE):- This formula is applicable for R^2 and R^3

■ in ABCD $\overrightarrow{AC} = \bar{a}$ and $\overrightarrow{BD} = \bar{b}$ then Area of

■^{m}ABCD $= \frac{1}{2}|\bar{a} \times \bar{b}|$

VOLUME OF A PARALLELOPIPED:

a parallelopiped is a solid consisting of six

faces which are parallelograms. Let $\bar{a}, \bar{b}, \bar{c}$ be no-coplaner vectors along the edges of the parallelopiped and having common vectors.

Volume of the parallelopiped $= |[\bar{a}, \bar{b}, \bar{c}]|$

—: list of question :—

1. if $|\bar{a}| = 3.5$ then $|\bar{a} \times \bar{\imath}|^2 + |\bar{a} \times \bar{\jmath}|^2 + |\bar{a} \times \bar{k}|^2$=................

(a) 7 (b) 13.5 (c) 18.5 (d) 24.5

2. $\bar{a}$ is non zero vector which magnitude $|\bar{a}|$, m is scalar .if $m\bar{a}$ is unit vector satiesfied ....

(a) $m = \pm 1$ (b) $m = |\bar{a}|$ (c) $m = \pm \frac{1}{|\bar{a}|}$ (d) $m = \pm 2$

3. if θ is obtuse angle of acute angle between two line segment of

iso scalan right angular triangles then $\cos\theta =$ ....

(a) $-\frac{1}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $-\frac{3}{4}$ (d) $-\frac{4}{5}$

4. $\bar{a}$ and $\bar{b}$ non coplanar. $2\bar{u} - \bar{v} = \bar{w}$, if $\bar{u} = x\bar{a} + 2y\bar{b}$, $\bar{v} = -2y\bar{a} + 3x\bar{b}$ and $\bar{w} = 4\bar{a} - 2\bar{b}$, find x and y = ...

(a) $x = \frac{8}{7}, y = \frac{2}{7}$ (b) $x = 2, y = 3$

(c) $x = \frac{4}{7}, y = \frac{6}{7}$ (d) $x = \frac{10}{7}, y = \frac{4}{7}$

5. two non zero vectors cross product is zero.then vectors are ....

(a) coplanar (b) equal vectors (c) origion at one point (d)same ending point

6. $\bar{x} and\ \bar{y}$ are nonzero vector. if $\bar{x} = k\bar{y}, k < 0$ then

$\bar{x} \cdot \bar{y} =$ ....

(a) $= |\bar{x} + \bar{y}|$ (b) $= |\bar{x}|\ |\bar{y}|$ (c) $> |\bar{x}|\ |\bar{y}|$ (d) $< |\bar{x}|\ |\bar{y}|$

7. $(\bar{x} \cdot \bar{y}) \cdot \bar{z}$ is what?

((a) non of these (b) vector (c) scalar (d) unit vector

8. if $\bar{a} + m\bar{b} + 3\bar{c}$, $-2\bar{a} + 3\bar{b} - 4\bar{c}$ and $\bar{a} - 3\bar{b} - 5\bar{c}$ are coplanar. $m =$ ....

(a) 2 (b) -1 (c) 1 (d) $-9/7$

9. $\bar{x}$ is nonzerovector .find realnumber k suchthat $|\ (5-k)\,\bar{x}\ | < 2|\bar{x}|$ is satisfied ....

(a) $0 < k < 3$ (b) $-7 < k < -3$ (c) $3 < k < 7$ (d) $-7 < k < 3$

10. let $\bar{u}$, $\bar{v}$ and $\bar{w}$ suchthat $|\bar{u}| = 1, |\bar{v}| = 2, |\bar{w}| = 3$. projection of$\bar{v}$ over $\bar{u}$ and projectionof $\bar{w}$ over $\bar{u}$ are same magnitude. $\bar{v}$ and $\bar{w}$ is perpendicular.$|\bar{u} - \bar{v} + \bar{w}| =$ ....

(a) 2 (b) $\sqrt{7}$ (c) $\sqrt{14}$ (d) 14

11. a force $\bar{F} = (2, 1, -1)$ act on a partical and displaces it from the point $A(2, -1, 0)$ to the point $B(2, 1, 0)$ then work done by force is equal to ....

(a) 2 (b) 4 (c) 6

12. if $\bar{a}, \bar{b}$ and $\bar{c}$ are unit vector, $|\bar{a} - \bar{b}|^2 + |\bar{b} - \bar{c}|^2 + |\bar{c} - \bar{a}|^2$ is never greterthen................. .

(a) 4 (b) 9 (c) 8 (d) 6

13. if $\bar{a}$ *and* $\bar{b}$are vector and $\bar{a} \cdot \bar{b} < 0, |\bar{a} \cdot \bar{b}| = |\bar{a} \times \bar{b}|$ then angle between$\bar{a}$ and $\bar{b}$ is $=$ ....

(a) π (b) $\frac{7\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

14. $\bar{x} \times (\bar{y} \cdot \bar{z})$ is what ? where $\bar{x}, \bar{y}, \bar{z} \in R^3$

(a) box product (b) vector (c) scalar (d) non of these

15. $\bar{\imath} \times (\bar{x} \times \bar{\imath}) + \bar{\jmath} \times (\bar{x} \times \bar{\jmath}) + \bar{k} \times (\bar{x} \times \bar{k}) =$ ....

(a) $\bar{x}$ (b) $2\bar{x}$ (c) $3\bar{x}$ (d) 0

16. $\bar{a} = (3, -5, 0), \bar{b} = (6, 3, 0)$and $\bar{c} = \bar{a} \times \bar{b}$ then

$|\bar{a}| : |\bar{b}| : |\bar{c}| =$ ....

(a) $\sqrt{34} : \sqrt{45} : \sqrt{39}$ (b) $\sqrt{34} : \sqrt{45} : 39$

(c) 34 : 39 : 45 (d) 39 : 35 : 34

17. 25 kg box is shifting 10m slope .find the work act on horrizon angle with $\frac{\pi}{2}$.

(a) 125 (b) $125\sqrt{3}$ (c) 250 (d) non of these .

18. $\bar{a}$ and $\bar{b}$ are unit vector .the angle between the vectors is θ.

$|\bar{a} + \bar{b}| > 1$.then

(a) $\theta = \frac{\pi}{2}$ (b) $\theta < \frac{\pi}{3}$ (c) $\theta > \frac{2\pi}{3}$ (d) $\frac{\pi}{2} < \theta < \frac{2\pi}{3}$

19. vectors $\bar{x} + \bar{y}$ and $\bar{x} - \bar{y}$ are equal.satiesfy which condition

(a) non of these (b) જો $\bar{x} = \bar{y}$ (c) જો $\bar{x} = \bar{0}$ (d) જો $\bar{y} = \bar{0}$

20. $\bar{a} \times \left(\bar{a} \times (\bar{a} \times \bar{b})\right) =$.............

$(a)\ |\bar{a}|^2(\bar{a} \times \bar{b})$ $(b)\ |\bar{a}|^2(\bar{b} \times \bar{a})$ $(c)\ |\bar{a}|^2(\bar{a} \times \bar{a})$ $(d)\ 0$

21. $\bar{a} = (1, 0, -1)$, $\bar{b} = (x, 1, 1 - x)$ and $\bar{c} = (y, x, 1 + x - y)$,

$[\bar{a}\ \bar{b}\ \bar{c}]$ is depend on which.

$(a)\ x$ $(b)\ y$ $(c)\ x$ and y (d) non of these

22. $\bar{a}$ and $\bar{b}$ are unit vector . if the vectors $\bar{c} = \bar{a} + 2\bar{b}$ and $\bar{d} = 5\bar{a} - 4\bar{b}$

are perpendicular then angle between $\bar{a}$ and $\bar{b}$ is $=$ ....

$(a)\ \frac{\pi}{3}$ $(b)\ \frac{\pi}{4}$ $(c)\ \frac{\pi}{6}$ $(d)\ \frac{\pi}{2}$

23. the angle between $\bar{y} = (x, -3, 1)$ and $\bar{z} = (2x,\ x, -1)$ is acute , the angle between vector $\bar{z}$ and $y -$ axes is obtuse then $x =$ ....

$(a)\ 1, 2$ $(b)\ -2,\ 3$ $(c)\ \forall x < 0$ $(d)\ \forall x > 0$

24. if $\bar{x}$ and $\bar{y}$ is parallar as well as same magnitude.then satisfying following condition .

$(a)\ \bar{x} = \bar{y}$ $(b)\ \bar{x} \neq \bar{y}$ $(c)\ \bar{x} + \bar{y} = \bar{0}$ $(d)\ \bar{x} = \bar{y}\ or\ \bar{x} + \bar{y} = \bar{0}$

25. $(\bar{A} \times \bar{B}) \cdot [(\bar{B} \times \bar{C}) \times (\bar{C} \times \bar{A})] =$......................

$(a)\ [\bar{A}\ \bar{B}\ \bar{C}]^2$ $(b)\ 2\bar{A} \cdot (\bar{B} \times \bar{C})$

$(c)\ (\bar{B} \times \bar{C}) \cdot [\bar{C} \times \bar{A} + \bar{A} \times \bar{B}]$ (d) non of these

26. if $\bar{v} = (2, 1, -1)$, $\bar{w} = (1, 0, 3)$ and $\bar{u}$ are unit vectors .gretest value of $[\bar{u}\ \bar{v}\ \bar{w}]$ is ....

$(a)\ -1$ $(b)\ \sqrt{10} + \sqrt{6}$ $(c)\ \sqrt{59}$ $(d)\ \sqrt{60}$

27 $\bar{a}$ and $\bar{b}$ are unit vector . if $\left(\bar{a} \wedge \bar{b}\right) = \theta$ and $|\bar{a} - \bar{b}| < 1$ then $\theta \in$ ..... .

$(a)\ \left(0, \frac{\pi}{3}\right)$ $(b)\ \left[\frac{2\pi}{3}, \frac{4\pi}{3}\right]$ $(c)\ \left[\frac{\pi}{8}, \frac{\pi}{2}\right]$ $(d)\ \left[0, \frac{\pi}{3}\right]$

28 if the sum of two unit vectors is unit then angle between two vectors .....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{2\pi}{3}$

29. $\bar{a}$ and $\bar{b}$ are unit vector and θ is angle between them then $\cos\frac{\theta}{2}$ =........... ; $0 < \theta < \pi$

(a) $\frac{1}{2}\,|\bar{a}+\bar{b}|$ (b) $\frac{1}{2}\,|\bar{a}-\bar{b}|$ (c) $\frac{1}{2}\,(\bar{a}\cdot\bar{b})$ (d) $\frac{|\bar{a}\times\bar{b}|}{2|\bar{a}||\bar{b}|}$

30. if $|\bar{A}| = 3, |\bar{B}| = 4, |\bar{C}| = 5, \bar{A} \perp (\bar{B}+\bar{C}), \bar{B} \perp (\bar{C}+\bar{A}), \bar{C} \perp (\bar{A}+\bar{B})$ then

$|\bar{A}+\bar{B}+\bar{C}| =$ ....

(a) $5\sqrt{2}$ (b) $7\sqrt{2}$ (c) $\sqrt{2}$ (d) $3\sqrt{2}$

31. $m\bar{a} = n\bar{b}$; $m, n \in N$ then $\bar{a}\cdot\bar{b} - |\bar{a}||\bar{b}| =$ ....

(a) 0 (b) 1 (c) $m-n$ (d) $m+n$

32. $\bar{a} = (2,-3,6)$ and $\bar{b} = (-2, 2,-1)$.

if λ = projection of $\bar{a}$ over $\bar{b}$ /projection of $\bar{b}$ over $\bar{a}$ .then $\lambda =$ ....

(a) $\frac{3}{7}$ (b) 7 (c) 3 (d) $\frac{7}{3}$

33. $\bar{a} = \bar{u}-\bar{v}, \bar{b} = \bar{u}+\bar{v}, |\bar{u}| = |\bar{u}|^2$ and $|\bar{u}| = |\bar{v}| = 2$ find

$|\bar{a}\times\bar{b}| =$ ....

(a) $2\sqrt{16-(\bar{u}\cdot\bar{v})^2}$ (b) $\sqrt{4-(\bar{u}\cdot\bar{v})^2}$

(c) $\sqrt{16-(\bar{u}\cdot\bar{v})^2}$ (d) $\sqrt{4-(\bar{u}\cdot\bar{v})^2}$

34. if the difference of two unit vectors is unit then angle between two vectors = ...

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{2\pi}{3}$

35. if $\bar{p} = p_1\bar{\imath}+p_2\bar{\jmath}+p_3\bar{k}, \bar{q} = q_1\bar{\imath}+q_2\bar{\jmath}+q_3\bar{k}$ and $\bar{r} = r_1\bar{\imath}+r_2\bar{\jmath}+r_3\bar{k}$ then $[n\bar{p}+\bar{q}\;\; n\bar{q}+\bar{r}\;\; n\bar{r}+\bar{p}] =$ ....

(a) $(n^3+1)[\bar{p}\;\;\bar{q}\;\;\bar{r}]$ (b) $(n^3-1)[\bar{p}\;\;\bar{q}\;\;\bar{r}]$

(c) $2(n^3+1)[2\bar{p}\;\;\bar{q}\;\;\bar{r}]$ (d) $2(n^3+1)[\bar{p}\;\;\bar{q}\;\;\bar{r}]$

36. $\bar{a}$, $\bar{b}$ and $\bar{c}$ are unit vectors, $\bar{a}+\bar{b}+\bar{c}=\bar{0}$ then $\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+$

$\bar{c}\cdot\bar{a}=$ ....

(a) 1 (b) 3 (c) $-\frac{3}{2}$ (d) non of these .

37. if $2\bar{\imath}+4\bar{\jmath}-5\bar{k}$ and $\bar{\imath}+2\bar{\jmath}+3\bar{k}$ is two different sides of rhombus, find the lengh of diagonal = ...

(a) 7, $\sqrt{69}$ (b) 6, $\sqrt{59}$ (c) 5, $\sqrt{65}$ (d) 8, $\sqrt{45}$

38. $\bar{a}=(x,\ y,\ z)$, $\bar{c}$ and $\bar{b}=(0,\ 1,\ 0)$ is satiesfied right hand law then

$\bar{c}=$ ....

(a) $(z,0,-x)$ (b) $\bar{0}$ (c) $(0,\ y,\ 0)$ (d) $(-z,\ 0,\ x)$

39. vector $\bar{b}=(0,3,4)$ is represented by $\bar{b}_1$ and $\bar{b}_2$ where $\bar{b}_1$ is same direction of $\bar{a}=(1,1,0)$ and $\bar{b}_2$ is perpendicular, then $\bar{b}_2=$ ....

(a) $\left(\frac{3}{2},\ \frac{3}{2},\ 0\right)$ (b) $\left(-\frac{3}{2},\ \frac{3}{2},\ 4\right)$ (c) $\left(0,\ \frac{3}{5},\ \frac{4}{5}\right)$ (d) non of these

40. if $\bar{a}\perp\bar{b}$ then $\bar{a}\times\left\{\bar{a}\times\left\{\bar{a}\times\left\{\bar{a}\times\left(\bar{a}\times\bar{b}\right)\right\}\right\}\right\}=$.............

(a) $-|\bar{a}|^2\bar{b}$ (b) $-|\bar{a}|^4\bar{b}$ (c) $-|\bar{a}|^6\bar{b}$ (d) $|a|^6\bar{b}$

41. following which is true?.

(a) $\bar{u}\cdot(\bar{v}\times\bar{w})$ (b) $(\bar{u}\cdot\bar{v})\cdot\bar{w}$ (c) $(\bar{u}\cdot\bar{v})\times\bar{w}$ (d) $(\bar{u}\times\bar{v})\,\bar{w}$

42. $\bar{a}=(2,1,-2)$ and $\bar{b}=(1,1,0)$.are vectors, $\bar{c}$ such a vector that $\bar{a}\cdot\bar{c}=|\,\bar{c}\,|$, $|\,\bar{c}-\bar{a}\,|=2\sqrt{2}$. The angle between $\left(\bar{a}\times\bar{b}\right)$ and $\bar{c}$ is30°,

$\left|(\bar{a}\times\bar{b})\times\bar{c})\right|=$ ......

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 2 (d) 3

43. $\bar{a}=(2,1,1)$, $\bar{b}=(1,2,-1)$ is unit vectors, $\bar{c}$ is coplanar, $\bar{c}$ and $\bar{a}$ are perpendicular then $\bar{c}=$ ....

(a) $\frac{1}{\sqrt{2}}$ $(0,-1,\ 1)$ (b) $\frac{1}{\sqrt{3}}$ $(0,-1,-1)$

(c) $\frac{1}{\sqrt{5}}$ $(1,-2,\ 0)$ (d) $\frac{1}{\sqrt{3}}$ $(1,-1,-\ 1)$

44. $\bar{a}\cdot(2\bar{b}+2\bar{c})\times(3\bar{a}+3\bar{b}+3\bar{c}) =$ ....

(a) $[\bar{a}\ \bar{b}\ \bar{c}]$ (b) $3[\bar{a}\ \bar{b}\ \bar{c}]$ (c) $6[\bar{a}\ \bar{b}\ \bar{c}]$ (d) 0

45. ΔABC $side A, B$ and C position vectors are $\bar{a}$, $\bar{b}$ and $\bar{c}$ find the lengh of line segment from A to $\overline{BC}$.

(a) $\frac{|\bar{b}\times\bar{c}|}{|\bar{b}-\bar{c}|}$ (b) $\frac{|\bar{c}\times\bar{a}|}{|\bar{c}-\bar{a}|}$

(c) $\frac{|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}|}{|\bar{b}-\bar{c}|}$ (d) $\frac{|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}|}{|\bar{b}+\bar{c}|}$

46. if $\bar{a}$ and $\bar{b}$ are per pendicular ,vector $\bar{c}$ and $\bar{d}$ such that $\bar{b}\times\bar{c}=\bar{b}\times\bar{d}$ and $\bar{a}\cdot\bar{d}=0$, then $\bar{d}=$ ....

(a) $\bar{c}+\left(\frac{\bar{a}\cdot\bar{c}}{\bar{a}\cdot\bar{b}}\right)\bar{b}$ (b) $\bar{b}+\left(\frac{\bar{b}\cdot\bar{c}}{\bar{a}\cdot\bar{b}}\right)\bar{c}$

(c) $\bar{c}-\left(\frac{\bar{a}\cdot\bar{c}}{\bar{a}\cdot\bar{b}}\right)\bar{b}$ (d) $\bar{b}-\left(\frac{\bar{b}\cdot\bar{c}}{\bar{a}\cdot\bar{b}}\right)\bar{c}$

47. $\bar{a}$ and $\bar{b}$ are unit vector such as $\bar{a}+2\bar{b}$ and $5\bar{a}-4\bar{b}$ are perpendicular,

find the angle between $\bar{a}$ and $\bar{b}$ = ...

(a) $15°$ (b) $60°$ (c) $cos^{-1}\left(\frac{1}{3}\right)$ (d) $cos^{-1}\left(\frac{2}{7}\right)$

48. $(\bar{a}+2\bar{b}-\bar{c})\cdot\{(\bar{a}-\bar{b})\times(\bar{a}-\bar{b}-\bar{c})\} =$..............

(a) $2[\bar{a}\ \bar{b}\ \bar{c}]$ (b) $3[\bar{a}\ \bar{b}\ \bar{c}]$ (c) $-[\bar{a}\ \bar{b}\ \bar{c}]$ (d) 0

49. $\bar{a}\cdot\left((\bar{b}+\bar{c})\times(\bar{a}+\bar{b}+\bar{c})\right)=0$

(a) 0 (b) $[\bar{a}\ \bar{b}\ \bar{c}]+[\bar{b}\ \bar{c}\ \bar{a}]$ (c) $[\bar{a}\ \bar{b}\ \bar{c}]$ (d) non of these .

50. the angle between $\bar{a}$ and $\bar{b}$ is $\frac{5\pi}{6}$,projectile of $\bar{a}$ over $\bar{b}$ is $\frac{6}{\sqrt{3}}$ then the value $|\bar{a}| =$

.... $(\bar{a}, \bar{b}\neq\bar{0})$

(a) 12 (b) 6 (c) 4 (d) $\frac{\sqrt{3}}{2}$

51. if $\bar{a} = \bar{\imath} - \bar{\jmath} + 2\bar{k}, \bar{b} = 2\bar{\imath} + 4\bar{\jmath} + 4\bar{k}$ and $\bar{c} = \lambda\bar{\imath} + \bar{\jmath} + \mu\bar{k}$

are perpendicular then the value of $(\lambda, \mu) =$ ....

(a) $(-3, 2)$ (b) $(2, -3)$ (c) $(-2, 3)$ (d) $(3, -2)$

52. $\bar{a} = (1,1,1)$, $\bar{c} = (0,1,-1)$. $\bar{a} \cdot \bar{b} = 3$ and $\bar{a} \times \bar{b} = \bar{c}$ find $\bar{b} =$ ....

(a) $(2/3,\ 2/3,\ 5/3)$ (b) $(2/3,\ 5/3,\ 2/3)$

(c) $(5/3,\ 2/3,\ 2/3)$ (b) non of these

53. $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ is dierction cosine of the line then the value of $c =$ ....

(a) $\pm\frac{1}{3}$ (b) ± 3 (c) $\pm\frac{1}{\sqrt{3}}$ (d) $\pm\sqrt{3}$

54. if $\bar{a}$, $\bar{b}$ and $\bar{c}$ such a vectors that $\bar{a} \neq \bar{0}$, $\bar{a} \times \bar{b} = 2\bar{a} \times \bar{c}$,

$|\bar{a}| = |\bar{c}| = 1$, $|\bar{b}| = 4$ and $|\bar{b} \times \bar{c}| = \sqrt{15}$ if $\bar{b} - 2\bar{c} = \lambda\bar{a}$ then $\lambda =$ ....

(a) -1 (b) 1 (c) 2 (d) ± 4

55. $\bar{a} = \frac{1}{\sqrt{10}}(3\,\bar{\imath} + \bar{k})$ $\bar{b} = \frac{1}{7}(2\bar{\imath} + 3\bar{\jmath} - 6\bar{k})$ evalute

$(2\bar{a} - \bar{b}) \cdot [(\bar{a} \times \bar{b}) \times (\bar{a} + 2\bar{b})]$ ....

(a) -3 (b) 5 (c) 3 (d) -5

56. if A, B, C and D any points then $\overrightarrow{AB} \cdot \overrightarrow{CD} + \overrightarrow{BC} \cdot \overrightarrow{AD} + \overrightarrow{CA} \cdot \overrightarrow{BD} =$.......

(a) -1 (b) 0 (c) 1 (d) non of these

57. if $|\,\bar{a}\ \bar{b}\ \bar{c}\,| = 2$ .vectors at origion are $2\bar{a} + \bar{b}$, $2\bar{b} + \bar{c}$ and $2\bar{c} +$

$\bar{a}$ then find the volume of $Parallelopiped$ ....

(a) 9 cube unit (b) 8 cube unit (c) 18 cube unit (d)16 cube unit

58. if $2\bar{\imath} + 4\bar{\jmath} - 5\bar{k}$ and $\bar{r} + 2\bar{\jmath} + 3\bar{k}$ is two different sides of rhombus, find the lengh of diagonals = ...

(a) $7, \sqrt{69}$ (b) $6, \sqrt{59}$ (c) $5, \sqrt{65}$ (d) $8, \sqrt{45}$

59. find the number of vectors inR^3 such the angle between $X-$ axes and vectoers are $\frac{\pi}{3}$.

(a) 1 (b) 2 (c) 4 (d) infinite times

60. $ABCD$ is equadrilateral such as $\overrightarrow{AB} = \alpha, \overrightarrow{AD} = \beta$ and $\overrightarrow{AC} = 2\alpha + 3\beta$,the area of ABCD side of $\overrightarrow{AB}$ and $\overrightarrow{AD}$ is λ times area of rhombus:then the value of $\lambda =$ ....

(a) 5 (b) 1 (c) $\frac{5}{2}$ (d) $\frac{1}{5}$

61. if $\bar{u}$ and $\bar{v}$ are unit vectors and θ is acute angle between them.find θ such that $2\bar{u} \times 3\bar{v}$ becomes unit vectors.

(a) non of these (b) one times

(c) two times (d) more than two

62. find the vector in R^2 such as perpendicular with $\bar{x} = (3,4)$ as well as acute angle with Y–axe.

(a) $\left(\frac{4}{5}, \frac{3}{5}\right)$ (b) $\left(-\frac{4}{5}, \frac{3}{5}\right)$ (c) $\left(-\frac{4}{5}, -\frac{3}{5}\right)$ (d) $\left(\frac{4}{5}, -\frac{3}{5}\right)$

63. $\bar{a}$ and $\bar{b}$ are non coplanar and $5\bar{u} - 3\bar{v} = \bar{w}$. if $\bar{u} = m\bar{a} + 2n\bar{b}$

and $\bar{v} = -2n\bar{a} + 3m\bar{b}$ and $\bar{w} = 4\bar{a} - 2\bar{b}$. find $m =$........... $n =$...........

(a) $\frac{1}{2}, \frac{1}{4}$ (b) $\frac{1}{2}, \frac{1}{3}$ (c) $\frac{1}{3}, \frac{1}{4}$ (d) $-\frac{1}{2}, -\frac{1}{4}$

64. if the angle between unit vectors $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{3}$ then (a) $|\bar{a} + \bar{b}| > 1$ (b) $|\bar{a} + \bar{b}| < 1$ (c) $|\bar{a} - \bar{b}| > 1$ (d) $|\bar{a} - \bar{b}| < 1$

65. if the vectors $\bar{a}$ and $\bar{b}$ such that $|\bar{a} + \bar{b}| < |\bar{a} - \bar{b}|$ the angle between $\bar{a}$ and $\bar{b}$ is.

(a) acute (b) right angle (c) obtuse (d) non of these .

66. if the angle between $\bar{a} = (2, -m, 3m)$ and $\bar{b} = (1 + m, -2m, 1)$ is acute then m=................

(a) $m \in R$ (b) $m \in (-\infty, -2) \cup (-\frac{1}{2}, \infty)$

(c) $m = -\frac{1}{2}$ (d) $m \in \left[-2, -\frac{1}{2}\right]$

67. if the angle between $\bar{a} = (c\bar{x}, -6, -3)$ and $\bar{b} = (x, 2, 2cx)$ is obtuse, thec c such a interval.

(a) $\left(0, \frac{3}{4}\right)$ (b) $\left(0, \frac{4}{3}\right)$ (c) $\left(-\frac{3}{4}, 0\right)$ (d) $\left(-\frac{4}{3}, 0\right)$

68. following is not possible for vectors $\bar{x}$ and $\bar{y}$.

(a) $|\bar{x} \cdot \bar{y}| \leq |\bar{x}| \cdot |\bar{y}|$ (b) $|\bar{x} + \bar{y}| \leq |\bar{x}| + |\bar{y}|$

(c) $|\bar{x} - \bar{y}| \leq |\bar{x}| - |\bar{y}|$ (d) $\left|\overline{|x|} - \overline{|y|}\right| \leq |\bar{x} - \bar{y}|$

69. if the angle between $\bar{a}$ and $\bar{b}$ is θ then $\frac{|\bar{a} \times \bar{b}|}{\bar{a} \cdot \bar{b}} =$ ....

(a) $\tan\theta$ (b) $-\tan\theta$ (c) $\cot\theta$ (d) $-\cot\theta$

70. if the vectors $10\bar{\imath} + 3\bar{\jmath}, 12\bar{\imath} - 5\bar{\jmath}$ and $a\bar{\imath} + 11\bar{\jmath}$ are coplanar then

$a =$..................

(a) -8 (b) 4 (c) 8 (d) 12

71. $\bar{a} = (1, -1, 0), \bar{b} = (0, 1, -1)$ and $\bar{c} = (-1, 0, 1)$ find the unit vector $\bar{d}$ Such that $\bar{a} \cdot \bar{b} = 0 = [\bar{b}\ \bar{c}\ \bar{d}]$.

(a) $\pm\frac{1}{\sqrt{6}}(1, 1, -2)$ (b) $\pm\frac{1}{\sqrt{3}}(1, 1, -1)$

(c) $\pm\frac{1}{\sqrt{3}}(1, 1, 1)$ (d) $\pm(0, 0, 1)$

72. $\bar{a}, \bar{b}$ and $\bar{c}$ are non zero vectors, if $\bar{a} = 8\bar{b}$ and $\bar{c} = -7\bar{b}$ then the angle between $\bar{a}$ and $\bar{c}$ is...

(a) π (b) 0 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

73. $\bar{a} \times \bar{b} = \bar{c}$ and $\bar{b} \times \bar{c} = \bar{a}$ and $\bar{a}$, $\bar{b}$ and $\bar{c}$ magnitude are a, b and c then

(a) $a = 1;\ b = c$ (b) $b = 1;c = a$

(c) $c = 1;a = b$ (d) $b = 2;c = 2a$

74. if $\bar{a}$, $\bar{b}$ and $\bar{c}$ are non coplanar unit vectors such that $\bar{a} \times (\bar{b} \times \bar{c}) = \frac{1}{\sqrt{2}}(\bar{b} + \bar{c})$ and $(\bar{a} \wedge \bar{b}) = \alpha$, $(\bar{a} \wedge \bar{c}) = \beta$ then

$\alpha =$ .... $\beta =$..........,

(a) $\frac{\pi}{4}$; $\frac{3\pi}{4}$ (b) $\frac{3\pi}{4}$; $\frac{\pi}{4}$ (c) $\frac{\pi}{4}$; $\frac{7\pi}{4}$ (d) $\frac{7\pi}{4}$; $\frac{\pi}{4}$

75. $\bar{a}$ is perpendicular with $\bar{b}$ and $\bar{c}$.then........... (a) $\bar{a} \times (\bar{b} \times \bar{c}) = 1$ (b) $\bar{a} \times (\bar{b} \times \bar{c}) = \bar{0}$

(c) $\bar{a} \times (\bar{b} \times \bar{c}) = -1$ (b) non of these.

76. A s position vector is $\bar{a} + 2\bar{b}$ P s position vector is $\bar{a}$. P is division point of $\overline{AB}$ from A in the ratio 2 : 3 .find B s position vector is ......

(a) $2\bar{a} - \bar{b}$ (b) $\bar{b} - 2\bar{a}$ (c) $\bar{a} - 3\bar{b}$ (d) $\bar{b}$

77. if θ is obtuse angle of acute angle between two line segment of equilateral right angular triangles then $cos\ \theta =$ ....

(a) $-\frac{1}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $-\frac{3}{4}$ (d) $-\frac{4}{5}$

HINT

1. $Hint :- \bar{a} = (a_1, a_2, a_3)$

$\therefore\ |\bar{a}| = 3.5$

$\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2} = 3.5$

$\therefore\ {a_1}^2 + {a_2}^2 + {a_3}^2 = (3.5)^2$

2. $Hint :-$ $m\bar{a}$ is unit vector. $\therefore |m\bar{a}| = 1$

3. $Hint :-$ $\Delta\, OAB$ for taking O, A and B position vectors are $\bar{0}, 2\bar{\imath}$

and $2\bar{\jmath}$ then C and D position vectors are $\bar{\imath}$ and $\bar{\jmath}$.

, $\overrightarrow{AD} = -2\bar{\imath} + \bar{\jmath} = (-2, 1)$ $\overrightarrow{BC} = \bar{\imath} - 2\bar{\jmath} = (1, -2)$

4. $Hint :- 2\bar{u} - \bar{v} = \bar{w}$

$\Rightarrow\ 2(x\bar{a} + 2y\bar{b}) - (-2y\bar{a} + 3x\bar{b}) = 4\bar{a} - 2\bar{b}$

5. $Hint :-$ $\bar{a} \times \bar{b} = 0,$ $\bar{a}$ and $\bar{b}$ are parallar

6. $Hint :-$ take $\bar{x}$ and $\bar{y}$ opposite direction.

7. $Hint :-$ its clear.

8. $Hint :-$ vectors are coplanar $\therefore \begin{vmatrix} 1 & m & 3 \\ -2 & 3 & -4 \\ 1 & -3 & -5 \end{vmatrix} = 0$

9. $Hint :-$ $|(5-k)\bar{x}| < 2|\bar{x}|$

$\therefore |\ 5 - k|\ |\bar{x}| < 2\ |\bar{x}|\ , |\bar{x}| \neq 0\ \ \bar{x} \neq \bar{0}$

$\therefore\ |\ 5 - k| < 2$

10. $Hint :-$ $|\bar{u} - \bar{v} + \bar{w}|^2 = (\bar{u} - \bar{v} + \bar{w}) \cdot (\bar{u} - \bar{v} + \bar{w})$

$= |\bar{u}|^2 + |\bar{v}|^2 + |\bar{w}|^2 - 2\,\bar{u} \cdot \bar{v} + 2\,\bar{w} \cdot \bar{u}$ $\quad (\because \bar{v} \cdot \bar{w} = 0)$

$= 1 + 4 + 9 + 2(|\bar{w}||\bar{u}| \cos\alpha - |\bar{u}||\bar{v}| \cos\beta)$

11. $Hint :-$ $\therefore\ w = \bar{F} \cdot \overrightarrow{AB}$

12. $Hint :-$ $|\bar{a} + \bar{b} + \bar{c}|^2 = |\ \bar{a}\ |^2 + |\ \bar{b}\ |^2 + |\ \bar{c}\ |^2 + 2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}) \geq 0$

$\Rightarrow \bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a}\geq -\frac{3}{2}$

13. $Hint:-\ |\bar{a}\cdot\bar{b}| = |\bar{a}\times\bar{b}|$

$\Rightarrow\ |\bar{a}|\ \ |\bar{b}|\ \ |\cos\theta| = |\bar{a}|\ |\bar{b}|\ \sin\theta$

$\Rightarrow\ |\cos\theta| = \sin\theta$

14. $Hint:-$ its clear for any X,Y and Z

15. $Hint:-\ \bar{\imath}\times(\bar{x}\times\bar{\imath})+\bar{\jmath}\times(\bar{x}\times\bar{\jmath})+\bar{k}\times(\bar{x}\times\bar{k})$

$=(\bar{\imath}\cdot i)\,\bar{x}-(\bar{\imath}\cdot\bar{x})\bar{\imath}+(\bar{\jmath}\cdot j)\bar{x}-(\bar{\jmath}\cdot\bar{x})j+(\bar{k}\cdot\bar{x})\bar{x}-(\bar{k}\cdot\bar{x})\bar{k}$

16. $Hint:-\ |\bar{a}|=\sqrt{34}\,,|\bar{b}|=\sqrt{45}\theta.$

$\bar{c}=\bar{a}\times\bar{b}=\left(\begin{vmatrix}-5 & 0\\ 3 & 0\end{vmatrix},-\begin{vmatrix}3 & 0\\ 6 & 0\end{vmatrix},\begin{vmatrix}3 & -5\\ 6 & 3\end{vmatrix}\right)=(0,0,39)$

17. $Hint:-\ w=\vec{F}\cdot\bar{d}=|\vec{F}|\,|\bar{d}|\,cos\frac{\pi}{3}$

18. $Hint:-\ |\bar{a}+\bar{b}|<1\Rightarrow|\bar{a}+\bar{b}|^2<1$

19. $Hint:-\ \bar{x}+\bar{y}=\bar{x}-\bar{y}$

20. $Hint:-\ \bar{a}\times\left(\bar{a}\times(\bar{a}\times\bar{b})\right)$

$=\bar{a}\times[(\bar{a}\cdot\bar{b})\bar{a}-(\bar{a}\cdot\bar{a})b]$

21. $Hint:-\ [\bar{a}\ \bar{b}\ \bar{c}]=\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}$

22. $Hint:-\ \bar{c}=\bar{a}+2\bar{b},\bar{d}=5\bar{a}-4\bar{b}$

$\therefore \bar{c}\cdot\bar{d}=0\ .$

23. $Hint:-$ the angle between $\bar{y}$ and $\bar{z}$ is acute.

$\therefore\ \bar{y}\cdot\bar{z}>0$

the angle between $\bar{z}$ and y axes is obtuse.

$\bar{z}\cdot\hat{\jmath}<0$

24. $Hint:-\ \bar{x}$and $\bar{y}$ are parallar

$\therefore\ \bar{x} = k\,\bar{y}, k \in R - \{0\}$

$|\,\bar{x}\,| = |\,\bar{y}\,|$.

25. *Hint* : $-\quad (\bar{A} \times \bar{B}) \cdot [(\bar{B} \times \bar{C}) \times (\bar{C} \times \bar{A})]$

$= (\bar{A} \times \bar{B})[(\bar{B} \times \bar{C}).\bar{A}]\bar{C} - [(\bar{B} \times \bar{C}).\bar{C}]\bar{A}$

26. *Hint* : $-\quad [\bar{u}\ \ \bar{v}\ \ \bar{w}] = \bar{u} \cdot (\bar{v} \times \bar{w}) \leq |\bar{u}||\bar{v} \times \bar{w}| = |\bar{v} \times \bar{w}|\ \ (\because |\bar{u}| = 1)$

$(\because \bar{x} \cdot \bar{y} \leq |\bar{x}|\ |\bar{y}|)$

$\therefore [\bar{u}\ \ \bar{v}\ \ \bar{w}] \leq |\bar{v} \times \bar{w}|$

27. *Hint* : $-\quad |\bar{a} - \bar{b}| < 1$

$\therefore |\bar{a} - \bar{b}|^2 < 1$

28. *Hint* : $-\ |\,\bar{a} + \bar{b}| = 1$

$\therefore |\bar{a} + \bar{b}|^2 = 1$

29. *Hint* : $-\quad |\bar{a} + \bar{b}|^2 = (\bar{a} + \bar{b}) \cdot (\bar{a} + \bar{b})$

30. *Hint* : $-\quad |\bar{A}| = 3, |\bar{B}| = 4, |\bar{C}| = 5$

$\therefore |\bar{A} + \bar{B} + \bar{C}|^2 = |\bar{A}|^2 + |\bar{B}|^2 + |\bar{C}|^2 + 2(\bar{A} \cdot \bar{B} + \bar{B} \cdot \bar{C} + \bar{C} \cdot \bar{A})$

31. *Hint* : $-\quad m\bar{a} = n\bar{b}$

$\therefore \bar{a} = \frac{n}{m}\,\bar{b}$.

$\bar{a} \cdot \bar{b} = \left(\frac{n}{m}\,\bar{b}\right) \cdot \bar{b}$

32. *Hint* : $-\quad \lambda = \dfrac{\frac{|\bar{a} \cdot \bar{b}|}{|\,\bar{b}\,|}}{\frac{|\bar{a} \cdot \bar{b}|}{|\,\bar{a}\,|}}$

33. *Hint* : $-\quad \bar{a} = \bar{u} - \bar{v}, \bar{b} = \bar{u} + \bar{v}$

$\therefore\ \bar{a} \times \bar{b} = (\bar{u} - \bar{v}) \times (\bar{u} + \bar{v})$

$= \bar{u} \times \bar{u} + \bar{u} \times \bar{v} - \bar{v} \times \bar{u} - \bar{v} \times \bar{v}$

$= 2(\bar{u} \times \bar{v})$

$\therefore \quad |\bar{a} \times \bar{b}| = 2|\bar{u} \times \bar{v}|$

34. $Hint:-\quad |\bar{x} - \bar{y}| = 1$

$\therefore |\bar{x} - \bar{y}|^2 = |\bar{x}|^2 - 2\bar{x} \cdot \bar{y} + |\bar{y}|^2$

35. $Hint:- \therefore [n\bar{p} + \bar{q} \quad n\bar{q} + \bar{r} \quad n\bar{r} + \bar{p}]$

$= (n\bar{p} + \bar{q}) \cdot [(n\bar{q} + \bar{r}) \times (n\bar{r} + \bar{p})]$

$= (n\bar{p} + \bar{q}) \cdot [n^2(\bar{q} \times \bar{r}) + n(\bar{q} \times \bar{p}) + (\bar{r} \times \bar{p})]$

36. $Hint:-\quad \bar{a} + \bar{b} + \bar{c} = \bar{0} \quad \therefore |\bar{a} + \bar{b} + \bar{c}|^2 = 0$

$\therefore (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a} + \bar{b} + \bar{c}) = 0$

37. $Hint:-\quad \overrightarrow{AB} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\overrightarrow{AD} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ sides of rhombus.

$\therefore$ diagonal

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$ અને $\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$

38. $Hint:-\quad \bar{a}, \ \bar{c}$ and $\bar{b}$are followed right hand law.

$\therefore \ \bar{c} = \bar{b} \times \bar{a}$

39. $Hint:-\quad \bar{b}_1$ is projection of $\bar{b}$ over $\bar{a}$.

$\therefore \bar{b}_1 = \left(\frac{\bar{a} \cdot \bar{b}}{|\bar{a}|^2}\right)\bar{a}$

40. $Hint:-\ \bar{a} \cdot \bar{b} = 0$

$\therefore \bar{a} \cdot (\bar{a} \times \bar{b}) = (\bar{a} \cdot \bar{b})\bar{a} - (\bar{a} \cdot \bar{a})\bar{b} = -|\bar{a}|^6\bar{b}$

$\bar{a} \times \left\{\bar{a} \times \left\{\bar{a} \times \left\{\bar{a} \times \{\bar{a} \times (\bar{a} \times \bar{b})\}\right\}\right\}\right\}$

$= \bar{a} \cdot \left\{\bar{a} \times \left\{\bar{a} \times \{\bar{a} \times (\bar{a} \times \bar{b})\}\right\}\right\} \cdot \bar{a} - (\bar{a} \cdot \bar{a})\left\{\bar{a} \times \left\{\bar{a} \times\right.\right.$
$\left.\left.(\bar{a} \times \bar{b})\}\right\}\right\}$

41. $Hint:-\ \bar{u} \cdot \bar{v} \in R, \bar{w} \in R^3 \ \ (\bar{u} \cdot \bar{v}) \cdot \bar{w}$ and $(\bar{u} \cdot \bar{v}) \times \bar{w}$ not possible

$\bar{u} \times \bar{v} \in R^3 \therefore (\bar{u} \times \bar{v})\bar{w}$ not possible $\bar{u} \in R^3, \bar{v} \times \bar{w} \in R^3$

$\therefore \bar{u} \cdot (\bar{v} \times \bar{w})$ clear.

42. $Hint: -\ \bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = (2, -2, 1)$

43. $Hint: -$ $\bar{c}$ is coplanar with $\bar{a}$ and $\bar{b}$

$\bar{c} = \alpha\bar{a} + \beta\bar{b},$ α, β are constant .

$\bar{c} \perp \bar{a} \Rightarrow \bar{c} \cdot \bar{a} = 0$

$\therefore\ \bar{c} \cdot \bar{a} = \alpha\bar{a} \cdot \bar{a} + \beta\bar{a} \cdot \bar{b}$

44. $Hint: -\ \bar{a} \cdot \left(2\bar{b} + 2\bar{c}\right) \times \left(3\bar{a} + 3\bar{b} + 3\bar{c}\right)$

$= \bar{a} \cdot 2(\bar{b} + \bar{c}) \times 3(\bar{a} + \bar{b} + \bar{c})$

$= 6\, \bar{a} \cdot (\bar{b} + \bar{c}) \times (\bar{a} + \bar{b} + \bar{c})$

45. $Hint: -\ \Delta = \frac{1}{2}\ |\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|$

$\Delta = \frac{1}{2}\ AD \cdot BC$

$AD = \frac{2\Delta}{BC} = \frac{2\Delta}{|\overline{BC}|}$

46. $Hint: -\ \bar{b} \times \bar{c} = \bar{b} \times \bar{d}$

$\Rightarrow\ \bar{a} \times \left(\bar{b} \times \bar{c}\right) = \bar{a} \times \left(\bar{b} \times \bar{d}\right)$

47. $Hint: -$ $\bar{a} + 2\bar{b}$ and $5\bar{a} - 4\bar{b}$ are perpendicular,

$\left(\bar{a} + 2\bar{b}\right) \cdot \left(5\bar{a} - 4\bar{b}\right) = 0$

$\therefore\ \bar{a} \cdot \bar{b} = \frac{1}{2}$

48. $Hint: -\ \left(\bar{a} + 2\bar{b} - \bar{c}\right) \cdot \{\bar{a} \times \bar{a} - \bar{a} \times \bar{b} - \bar{a} \times \bar{c} - \bar{b} \times \bar{a} + \bar{b} \times \bar{b} + \bar{b} \times \bar{c}\}$

$= \left(\bar{a} + 2\bar{b} - \bar{c}\right) \cdot \{-\bar{a} \times \bar{b} - \bar{a} \times \bar{c} - \bar{b} \times \bar{a} + \bar{b} \times \bar{c}\}$

49. $Hint: -\ \bar{a} \cdot \left(\left(\bar{b} + \bar{c}\right) \times \left(\bar{a} + \bar{b} + \bar{c}\right)\right) = \left[\bar{a}\quad \bar{b} + \bar{c}\quad \bar{a} + \bar{b} + \bar{c}\right]$

50. $Hint: -\quad cos\frac{5\pi}{6} = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}$

$\therefore\ \bar{a}\cdot\bar{b} = -\frac{\sqrt{3}}{2}\ |\bar{a}|\,|\bar{b}|$

51. $Hint: -\quad \bar{a} = \hat{\imath} - \hat{\jmath} + 2\bar{k}, \bar{b} = 2\bar{\imath} + 4\bar{\jmath} + 4\bar{k},\ \bar{c} = \lambda\bar{\imath} + \bar{\jmath} + \mu\bar{k}$

$\bar{a} \perp \bar{c} \implies \bar{a}\cdot\bar{c} = 0$

52. $Hint: -\quad \bar{a} = (1,1,1)$, take $\bar{b} = (b_1, b_2, b_3), \bar{c} = (0,1,-1)$

$\bar{a}\cdot\bar{b} = 3$

$\therefore b_1 + b_2 + b_3 = 3$

53. $Hint: -$ direction cosine of line $cos\alpha, cos\beta, cos\gamma$ are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$.

$cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

54. $Hint: -$ angle between $\bar{b}$ and $\bar{c}$ is α

$|\bar{b}\times\bar{c}| = \sqrt{15}$

$\Rightarrow |\bar{b}|\,|\bar{c}|\sin\alpha = \sqrt{15}$

55. $Hint: -\quad \bar{a} = \frac{1}{\sqrt{10}}(3,0,1)\ ,\ \bar{b} = \frac{1}{7}(2,3,-6)$

$|\bar{a}| = \sqrt{\frac{9+0+1}{10}} = 1\ ,\qquad |\bar{b}| = \sqrt{\frac{4+9+36}{49}} = 1$

$\bar{a}\cdot\bar{b} = \frac{1}{7\sqrt{10}}(6+0-6) = 0$

56. $Hint: -$ take A, B, C and D position vectors are $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{0}$.

$\overrightarrow{AB}\cdot\overrightarrow{CD} + \overrightarrow{BC}\cdot\overrightarrow{AD} + \overrightarrow{CA}\cdot\overrightarrow{BD}$

57. $Hint: -\ V = \begin{vmatrix} 2\bar{a} & \bar{b} & 0 \\ 0 & 2\bar{b} & \bar{c} \\ \bar{a} & 0 & 2\bar{c} \end{vmatrix}$

58. $Hint: -\quad \overrightarrow{AB} = 2\bar{\imath} + 4\bar{\jmath} - 5\bar{k}$

$\overrightarrow{AD} = \bar{\imath} + 2\bar{\jmath} + 3\bar{k}$ is sides of rhombus.

$\therefore$ diagonals $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$ અને $\overrightarrow{BD} = \overrightarrow{AD} + \overrightarrow{AB}$

59. *Hint* : − the angle between $\bar{x}$ and X−axes is $\alpha = \frac{\pi}{3}$ $\therefore cos\alpha = \frac{1}{2}$

$cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

$\frac{1}{4} + cos^2\beta + cos^2\gamma = 1$

$\therefore cos^2\beta + cos^2\gamma = \frac{3}{4}$

60. *Hint* : − A is origion $B = \alpha$, $C = 2\bar{\alpha} + 3\bar{\beta}$ and $D = \bar{\beta}$

$\therefore$ area of $ABCD$

$= \lambda \; |\overrightarrow{AB} \times \overrightarrow{AD}|$

61. *Hint* : − $|2\bar{u} \times 3\bar{v}| = 1 \Rightarrow 6|\bar{u}||\bar{v}||sin\,\theta| = 1$

62. *Hint* : − $\bar{x} = (3, 4)$is perpendicular with $\bar{y} = (y_1, y_2)$.

$\bar{x} \cdot \bar{y} = 0$ $\therefore 3y_1 + 4y_2 = 0$ $\therefore y_2 = -\frac{3}{4}y_1$

$|\bar{y}| = 1$ $\therefore {y_1}^2 + {y_2}^2 = 1$

63. *Hint* : − $5\bar{u} - 3\bar{v} = \bar{w}$

$5(m\bar{a} + 2n\bar{b}) - 3(-2n\bar{a} + 3m\bar{b}) = 4\bar{a} - 2\bar{b}$

64. *Hint* : − $|\bar{a} + \bar{b}|^2 = (\bar{a} + \bar{b}) \cdot (\bar{a} + \bar{b})$

65. *Hint* : − $|\bar{a} + \bar{b}| < |\bar{a} - \bar{b}|$

$\therefore |\bar{a} + \bar{b}|^2 < |\bar{a} - \bar{b}|^2$ $\therefore \cos\theta < 0$

$\therefore$ the angle between $\bar{a}$ and $\bar{b}$ is obtuse

66. *Hint* : − the angle between $\bar{a}$ and $\bar{b}$ is acute

$\therefore \bar{a} \cdot \bar{b} > 0$

$\therefore 2(1+m)+2m^2+3m>0$

$\therefore 2m^2+5m+2>0$

$\therefore (2m+1)(m+2)>0$

67. $Hint$: – the angle between $\bar{a}$ and $\bar{b}$ is obtuse $\bar{a}\cdot\bar{b}<0$

$\therefore cx^2-6\,cx-12<0$

68. $Hint$: – take any $\bar{x},\bar{y},\bar{z}$.

69. $Hint$: – $\frac{|\bar{a}\times\bar{b}|}{\bar{a}\cdot\bar{b}}=\frac{|\bar{a}|\,|\bar{b}|\sin\theta}{|\bar{a}|\,|\bar{b}|\cos\theta}=\tan\theta$.

70. $Hint$: – $A, B,$ and C position vectors are$(10,3),(12,-5)$

$(a,11)$

$\therefore \overrightarrow{AB}=(2,-8)\ \ \overrightarrow{AC}=(a-10,8)$

$A, B$and C are coplanar then $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are coplanar.

71. $Hint$: – $\bar{a}\cdot\bar{d}=0\ \ [\bar{b}\ \ \bar{c}\ \ \bar{d}]=0$

$\therefore \bar{d}$ is coplanar with $\bar{b}$ and $\bar{c}$ and perpendicular with $\bar{a}$.

$\therefore \bar{d}$ is perpendicular as well as unit with $\bar{b}\times\bar{c}$ and $\bar{a}$.

$\therefore \bar{d}=\pm\frac{\bar{a}\times(\bar{b}\times\bar{c})}{|\,\bar{a}\times(\bar{b}\times\bar{c})\,|}\neq\bar{0}$

72. $Hint$: its clear.

73. $Hint$: – $\bar{a}=\bar{b}\times\bar{c}=\bar{b}\times(\bar{a}\times\bar{b})\ \ (\because\ \bar{c}=\bar{a}\times\bar{b})$

$=(\bar{b}\cdot\bar{b})\bar{a}-(\bar{b}\cdot\bar{a})\bar{b}$

$\bar{b}\cdot\bar{b}=1\,,\bar{b}\cdot\bar{a}=0$

74. $Hint$: – $\bar{a}\times(\bar{b}\times\bar{c})=\frac{1}{\sqrt{2}}\,(\bar{b}+\bar{c})$

$\therefore (\bar{a}\cdot\bar{c})\bar{b}-(\bar{a}\cdot\bar{b})\bar{c}=\frac{1}{\sqrt{2}}\,(\bar{b}+\bar{c})$

$\therefore \bar{a} \cdot \bar{c} = \frac{1}{\sqrt{2}} \quad \bar{a} \cdot \bar{b} = -\frac{1}{\sqrt{2}}$

75. $Hint$: – $\bar{a}$ is $\bar{b}$ and $\bar{c}$ are perpendicular.

$\bar{b}$ and $\bar{c}$ are perpendicular $\bar{b} \times \bar{c}$

76. $Hint$: – B position vector is $\bar{x}$.

$P(\bar{a})$ is division point of $\overline{AB}$ from A in the ratio 2 : 3 .

$\therefore \quad \bar{a} = \frac{2\bar{x} + 3(\bar{a}+2\bar{b})}{2+3}$

77. $Hint$: – OAB for taking O, A and B position vectors are $\bar{0}, 2\bar{\imath}$ and $2\bar{\jmath}$ then C and D position vectors are $\bar{\imath}$ and $\bar{\jmath}$ $\quad \overrightarrow{AB} = -2\bar{\imath} + \bar{\jmath} = (-2, 1) \quad \overrightarrow{BC} = \bar{\imath} - 2\bar{\jmath} = (1, -2)$

ANSWER KEY:		
1 - (d)	31- (a)	61- (b)
2- (c)	32- (d)	62- (b)
3- (d)	33- (a)	63- (a)
4- (d)	34- (b)	64- (a)
5- (a)	35- (a)	65- (c)
6- (d)	36- (c)	66- (b)
7- (a)	37- (a)	67- (d)
8- (d)	38- (a)	68- (c)
9- (c)	39- (b)	69- (a)
10- (c)	40- (c)	70- (c)
11- (a)	41- (a)	71- (a)
12- (b)	42- (b)	72- (a)
13- (d)	43- (a)	73- (b)
14- (d)	44- (d)	74- (b)
15- (b)	45- (c)	75- (b)
16- (b)	46- (c)	76- (b)
17- (a)	47- (b)	77- (d)
18- (c)	48- (b)	
19- (d)	49- (a)	
20- (b)	50- (c)	
21- (d)	51- (a)	
22- (a)	52- (c)	
23- (c)	53- (d)	
24- (d)	54- (d)	
2?- (a)	55- (d)	
26- (c)	56- (b)	
27- (a)	57- (c)	
28- (d)	58- (a)	
29- (a)	59- (d)	
30- (a)	60- (c)	

Unit No. - 14

Statistics and Probability

Important Points

(1) Mean : Ungrouped Data :

(i) $\bar{x} = \frac{\sum xi}{n}$ (Direct method)

(ii) $\bar{x} = A + \frac{\sum di}{n}$ where $di = xi - A$ (Short cut Method)

- **Discrete data :**

(i) $\bar{x} = \frac{\sum fixi}{n}$ (Direct method)

(ii) $\bar{x} = A + \frac{\sum fidi}{n}$ where $di = xi - A$ (Short cut Method)

- **Continuous data :**

(i) $\bar{x} = \frac{\sum fixi}{n}$ (Direct method)

(ii) $\bar{x} = A + \frac{\sum fidi}{n} \times C$ where $di = \frac{xi - A}{C}$ (Short cut Method)

(2) Median : Ungrouped data :

(i) $M = \frac{n+1}{2}$th observation (n odd)

(ii) $M = \frac{\left(\frac{n}{2}\text{ th observation}\right) + \left(\left(\frac{n}{2}+1\right)\text{th observation}\right)}{2}$ (n even)

- **Discrete data :**

(i) $M = \frac{n+1}{2}$th observation

- **Continuous data :**

(ii) $M = L + \frac{\left(\frac{n}{2} - F\right)}{f} \times C$

Where L = Lower boundary point of median class

f = frequency of median class

F = c.f. of class preceeding to median class

C = class length of median class

(3) Range :

Range R = Maximum value of observation – Minimum value of observation

(4) Average deviation from mean :

(i) Ungrouped data $\delta\bar{x} = \frac{\sum |x_i - \bar{x}|}{n}$

(ii) Discrete (continuous) data $\delta\bar{x} = \frac{\sum f_i |x_i - M|}{n}$

(5) Average deviation from $\delta M = \frac{\sum |x_i - M|}{n}$

9.2

(1) Standard deviation :

- **Ungrouped Data :**

(i) $S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$ (Direct method)

(ii) $S = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$

(iii) $S = \sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2}$

(iv) $= \sqrt{\frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n}\right)^2}$ where $di = xi$ – A (Short cut Method)

- **Discrete Data :**

(i) $S = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n}}$ (Direct method)

(ii) $S = \sqrt{\frac{\sum f_i d_i^2}{n} - \left(\frac{\sum f_i d_i}{n}\right)^2}$ where $di = xi$ – A (Short cut Method)

- **Continuous data :**

(i) $S = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n}}$ (Direct method)

(ii) $= \sqrt{\frac{\sum f_i d_i^{\,2}}{n} - \left(\frac{\sum f_i d_i}{n}\right)^2} \times c$ where $di = \frac{xi - A}{c}$ (Short cut Method)

(7) **Variance :** Variance = s^2

(8) **Coefficient of Variation** (C.V.)

(9) Smaller value of C.V. That group is stable (consistent)

Larger value of C.V. That group shows more variation

9.3

(10) $y = ax + b \quad \bar{y} \quad a\bar{x} \quad b$

(11) $y = ax + b \quad S_y = |\,a\,|S_x$

(12) Correct $\sum x_i = n\bar{x}$ – (sum of incorrect observations) + (sum of correct observation)

(13) Correct $\sum x_i^{\,2} = n(S^2 + \bar{x}^2)$ – (sum of squares of incorrect observation) + (sum of squares of correct observation)

(14) For 1, 2, 3, ... n variance $S^2 = \frac{n^2 \quad 1}{12}$

(15) For 2, 4, 6, ... $2n$ variance $S^2 = \frac{n^2 \quad 1}{3}$

(16) For 1, 3, 5, ... $(2n - 1)$ variance $S = \frac{n^2 \quad 1}{3}$

C.V. of 1, 2, 3, ... n is $S^2 = \sqrt{\frac{1}{3}\left(\frac{n \quad 1}{n \quad 1}\right)}$

Question Bank

(1) The median of a set of 7 distinct observations is 10.5 If each of the last 3 observation of the set is increased by 3 then the median of the new set = ________

(a) in decreased by 2 (b) is two times the original median

(c) remain the same as that of the original set (d) is increased by 2

(2) The value of the variable of the given data for which the number of observations with values less than it and grater than it are equal is ________

(a) mean (b) median (c) mode (d) range

(3) The daily pocket expanses of 8 student are ` 20, 17, 8, 15, 22, 9, 10, 14 the median of the data is ` ________

(a) 14.5 (b) 14 (c) 15 (d) 15.5

(4) The median of first $n + 3$ natural number is ________

(a) $\frac{n \quad 4}{2}$ (b) $\frac{n \quad 4}{2}$ (c) $\frac{n-1}{2}$ (d) $\frac{n \quad 3}{4}$

(5) The marks for passing in the examination in a subject is 33.4 out of 9 students who appeared at the examination have failed and the marks of remaining student are 78, 40, 97, 65, 50. The median of the data is ________ marks.

(a) 40 (b) 64.5 (c) 50 (d) 55.5

(6) If the median of the observations $x, \frac{x}{5}, \frac{x}{2}, \frac{x}{3}, \frac{x}{7}, \frac{x}{4}, \frac{x}{8}$ $(x > 0)$ is 10 value of x is ________

(a) 30 (b) 20 (c) 50 (d) 40

(7) For the data 3, 3, 3, 3, 3 which of the following is true ?

(a) $\delta M > \delta \overline{x}$ (b) $\delta M = \delta \overline{x}$ (c) $\delta M < \delta \overline{x}$ (d) $\delta M + \delta \overline{x} = 6$

(8) The sum of 10 observation is 150 and the sum of their squares is 2700. The standared deviation is ________

(a) $3\sqrt{5}$ (b) $5\sqrt{3}$ (c) 15 (d) 5

(9) The standard deviation and coefficient of variation of 7, 7, 7, 7, 7 is ________

(a) 0, 7 (b) 7, 0 (c) 7, 7 (d) 0, 0

(10) If $n = 10$, $\overline{x} = 12$ and $xi^2 = 1530$ the value of coefficient of variation is ________

(a) 25% (b) 20% (c) 30% (d) 40%

(11) If x and y are related as $4x - 3y = 10$ and the mean deviation of x is 10 then the mean deviation of y is ________

(a) 13 (b) 12.3 (c) 13.3 (d) 13.5

(12) For a data $\overset{10}{\underset{i \quad 1}{}}(xi \quad 15) \quad 0$ and $\overset{10}{\underset{i \quad 1}{}}(xi \quad 15)^2 \quad 160$. Then coefficient of variation = ________

(a) 26.6 (b) 25.6 (c) 26.5 (d) 25.6

(13) The range of set of 15 observations is 0 then its variance is ________

(a) 8.25 (b) $\sqrt{15}$ (c) 2.85 (d) 0

(14) Observations for variable x are 2, 5, 14 and the observations for variable y are 7, 5, 9 then which of the following is true ?

(a) $CVx > CVy$ (b) $CVx < CVy$ (c) $CVx = CVy$ (d) $CVx \quad CVy$

(15) If $n = 100$, $\bar{x} = 3$ and $S^2 = 11$ then $\dfrac{xi^2}{xi}$ is ________

(a) 10 (b) 22 (c) 6.66 (d) 2000

(16) The median of the following incomplete frequency distribution is 4

xi	1	2	3	4	5
f	2	3	4	1	–

The frequency of 5 is ________

(a) 9 (b) 10 (c) 5 (d) 8

(17) Let x_1, x_2, x_3 .... x_n be n observations such that $xi^2 = 200$ and $xi = 60$ then a possible value of n among the following is ________

(a) 16 (b) 19 (c) 18 (d) 10

(18) The standard deviation for the scores 1, 2, 3, 4, 5, 6 and 7 is 2 then the standard deviation of 13, 24, 35, 46, 57, 68 and 79 is________

(a) 2 (b) 22 (c) 11 (d) 23

(19) The sum of the squares of deviation for 10 observations taken from their mean 30 is 90. The coefficient of variation is ________

(a) 20% (b) 10% (c) 11% (d) 12%

(20) If the mean and standard deviation of x is b and a respectively then the standard deviation of $\dfrac{x \quad b}{a}$ is ________

(a) 1 (b) $\dfrac{a}{b}$ (c) $\dfrac{b}{a}$ (d) ab

(21) The mean and standard deviation of x is 40 and 4 respectively the mean and standard deviation of $\dfrac{x \quad 40}{4}$ is ________

(a) 1, 0 (b) 1, 1 (c) 0, 1 (d) 0, – 1

(22) If x and y are related as $2x + 5y = 15$ and mean deviation of y about mean is 10 then the mean deviation of x about mean is ________

(a) 25 (b) 50 (c) 20 (d) 25

(23) If the variance of x is 4 then the variance of $3 + 5x$ is ________

(a) 100 (b) 103 (c) 20 (d) 23

(24) Given the observation 5, 9, 13, 17, 25 the mean deviation about the median is ________

(a) 5.5 (b) 5.8 (c) 13 (d) 5.6

(25) If coefficient of variation = 70 and mean =10 then variance is ________

(a) 49 (b) 7 (c) 100 (d) 80

(26) The avarage of n numbers y_1, y_2 .... y_n is M. If y_n is replaced by y' then the new avarage is ________

(a) $\frac{M \quad y_n \quad y'}{n}$ (b) $\frac{(n \quad 1)\,M \quad y'}{n}$ (c) $\frac{nM \quad y_n \quad y'}{n}$ (d) $M - y_n + y'$

(27) The mean of the series a, $a + d$, $a + 2d$ .... $a + (2n + 1)d$ is ________

(a) $a \left(\frac{2n \quad 1}{2}\right) d$ (b) $a + (n + 1)d$ (c) $a + (2n + 1)d$ (d) $a + \frac{(2n-1)}{2}d$

(28) If a variable takes discrete values $x + 2$, $x \quad \frac{5}{2}$, $x \quad \frac{3}{2}$, $x - 3$, $x - 2$, $x + 3$, $x + 5$, $x + 4$, $(x > 0)$ then the median is ________

(a) $x \quad \frac{3}{2}$ (b) $x + 2$ (c) $x \quad \frac{1}{4}$ (d) $x \quad \frac{1}{8}$

(29) If the mean of numbers $20 + x$, $24 + x$, $82 + x$, $100 + x$, $149 + x$ is 75 then the mean of $130 + x$, $126 + x$, $68 + x$, $50 + x$ and $1 + x$ is ________

(a) 75 (b) 76 (c) 73 (d) 70

(30) The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80 then which one of following gives possible values of a and b ?

(a) $a = 3$, $b = 4$ (b) $a = 0$, $b = 7$ (c) $a = 5$, $b = 2$ (d) $a = 1$, $b = 6$

(31) The A.M. of a 50 set of numbers is 38. If two numbers of the set namely 55 and 45 are discarded the A.M. of the remaining set of numbers is ________

(a) 38.5 (b) 37.5 (c) 36.5 (d) 38

(32) The average weight of students in a class of 35 students is 40 kg If the weight of the teacher be included the average rises by $\frac{1}{2}$ kg the weight of the teacher is ________

(a) 40.5 kg (b) 50 kg (c) 41 kg (d) 58 kg

(33) If the mean of the distribution is 2.6 then the value of y is ________

Variable xi	1	2	3	4	5
Frequency fi of x	4	5	y	1	2

(a) 24 (b) 13 (c) 8 (d) 3

(34) If the mean of the set of numbers $x_1, x_2, \ldots\ldots\ldots x_n$ is $\bar{x}$ then the mean of the numbers $xi + 2i$, $1 \quad i \quad n$ is ________

(a) $\bar{x} \quad 2n$ (b) $\bar{x} \quad n \quad 1$ (c) $\bar{x} \quad 2$ (d) $\bar{x} \quad n$

(35) The arithmetic mean of 7 consecutive integers starting with a is m then the arithmetic mean of 11 consecutive integers starting with $a + 2$ is ________

(a) $2a$ (b) $2m$ (c) $a + 4$ (d) $m + 4$

(36) The A.M. of 9 terms is 15. If one more term is added to this series then the A.M. becomes 16. The value of added term is ________

(a) 30 (b) 27 (c) 25 (d) 23

(37) If the mean deviation about the median of the observations a, $2a$, ..., 50a is 50 then $|a|$ = ________

(a) 2 (b) 3 (c) 4 (d) 5

(38) If standard deviation of $3xi - 2$ is 8 then variance of $\frac{2}{3}xi$ is ________

(a) $\frac{144}{81}$ (b) $\frac{81}{144}$ (c) $\frac{16}{9}$ (d) $\frac{4}{3}$

(39) If mean of $\log x$, $\log 2x$, $\log 8$, $\log 4x$, $\log 4$, $\log x$ is log 8 then x = ________ (where $x > 0$)

(a) 4 (b) 2 (c) 8 (d) 16

(40) Mean of x, y, z and y, z, r is equal then which of following is true ?

(a) $x = y = z$ (b) $y = z = r$ (c) $y = z$ (d) $x = r$

(41) If the mean of x and $\frac{1}{x}$ is m then mean of x^3 and $\frac{1}{x^3}$ is ________

(a) $\frac{8m^3-6m}{2}$ (b) $\frac{8m^3 \quad 3m}{2}$ (c) $\frac{3m^2 \quad 8m}{2}$ (d) $\frac{3m^2 \quad 8m}{2}$

(42) If mean of first n odd natural Integer is n then n is ________

(a) 2 (b) 3

(c) 1 (d) any natural integer

(43) If L = 44.5, N = 50, F = 15, f = 5 and C = 10 then median of data is ________

(a) 84.5 (b) 74.5 (c) 64.5 (d) 54.5

(44) If xi^2 = 10000, xi = 400 and C.V. = 50% then value of n is ________

(a) 5 (b) 40 (c) 20 (d) 25

(45) Mean of n observations is m and sum of n – 3 observations is b then mean of remaining 3 observations is ________

(a) $nm + b$ (b) $\frac{nm \quad b}{3}$ (c) $\frac{nm \quad b}{3}$ (d) $nm - b$

(46) Mean deviation of 12, 3, 18, 17, 4, 9, 17, 19, 20, 15, 8, 17, 2, 3, 16, 11, 3, 1, 0, 5 is ________

(a) 10 (b) 6.2 (c) 5.6 (d) None of the above

(47) Standard deviation of – 1, – 2, – 3, – 4, – 5, – 6, – 7 is ________

(a) – 4 (b) 4 (c) 2 (d) – 2

(48) If $\overset{10}{\underset{i \quad 1}{}}(xi \quad 8) \quad 9$ and $\overset{10}{\underset{i \quad 1}{}}(xi \quad 8)^2 \quad 45$ then standard deviation of x_1, x_2, x_3 .... x_{10} is ________

(a) 19.2 (b) 12.92 (c) 1.82 (d) 1.92

(49) Mean of following frequency distribution is 9.3 then K is ________

xi	4	6	7	K	12	14
fi	5	6	8	10	2	9

(a) 11 (b) 12 (c) 10 (d) 13

(50) Mean of sequence 1, 2, 4, 8, 16 .... 2^{n-1} is ________

(a) $\frac{2^n \quad 1}{n}$ (b) $\frac{2^{n \quad 1} \quad 1}{n \quad 1}$ (c) $\frac{2^n \quad 1}{n}$ (d) $\frac{2^n \quad 1}{n \quad 1}$

(51) Standard deviation of two observations is 3.5, one observation is 3 then second observation is ________

(a) 9 (b) 10 (c) 7 (d) 3

(52) For observations x_1, x_2, ... x_n, $\overset{n}{\underset{i \quad 1}{}}(xi \quad 4) \quad 100$ and $\overset{n}{\underset{i \quad 1}{}}(xi \quad 6) \quad 140$ then n = ________ and $\bar{x}$ = ________

(a) 3, 20 (b) 20, 3 (c) 1, 20 (d) 20, 1

(53) If frequencys nC_1, nC_2 .... nC_n are respectively of 1, 2, 3 .... n then mean of 1, 2, 3, ... n is ________

(a) $\frac{n \cdot 2^{n \quad 1}}{2^n \quad 1}$ (b) $\frac{3n(n \quad 1)}{2(2n \quad 1)}$ (c) $\frac{n \cdot 2^n}{2^n \quad 1}$ (d) $\frac{(n \quad 1)(2n \quad 1)}{6}$

(54) Variance of 1, 3, 5, 7 .... $(4n + 1)$ is ________

(a) $\frac{2n(2n \quad 1)}{3}$ (b) $\sqrt{\frac{(n \quad 1)}{3(n \quad 1)}}$ (c) $\frac{1}{n}\sqrt{\frac{n^2 \quad 1}{3}}$ 100 (d) $\frac{4n(n \quad 1)}{3}$

(55) In an experiment with 10 observations on x the following results were available $xi^2 = 2830$, $xi = 170$ on observation that was 20 way found to be wrong and was replaced by the correct value of 30 then the corrected variance is

(a) 7 (b) 10 (c) 9 (d) 8

(56) In a series of $2m$ observations half of them equal to b and remaining half equal to $-b$. If the standard deviation of the observations is 3 then $|b| =$ ________

(a) 3 (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{n}$ (d) $\frac{1}{n}$

(57) If for a slightly assymetric distribution, mean and median are 20 and 21 respectively. What is its mode ________

(a) 24.5 (b) 23.5 (c) 24 (d) 23

(58) Suppose a population A has 50 observations 101, 102, ... 150 and another population B has 50 observations 201, 202, ... 250. If V_A and V_B represent the variance of the two populations respectively then $\frac{V_A}{V_B}$ is ________

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{9}{4}$

(59) The average marks of boys in a class is 50 and that of girls is 40. The average marks of boys and girls combined is 48. The percentage of boys in the class is ________

(a) 75 (b) 80 (c) 60 (d) 55

(60) The median of following distribution is ________

Class	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24
Frequency f	8	12	3	25	13	7

(a) 11 (b) 13.76 (c) 12 (d) 9.5

(61) The mean of five observations is 4.4 and variance is 8.24 among five three observations are 1, 2, 6 then remaining observatiosn are ________

(a) 5, 10 (b) 4, 9 (c) 3, 10 (d) 5, 8

(62) The mean and S.D. of 100 observations were found to be 20 and 3 respectively. Later it was discovered that three observations 21, 21, 18 was wrongly taken. Then the mean and S.D. of remaining observations are ________

(a) 20, 3.036 (b) 20, 2.964 (C) 19, 3.036 (c) 19, 2.964

(63) Find mean and S.D. from given data

Class	33 – 35	36 – 38	39 – 41	42 – 44	45 – 47
Frequency f	17	19	23	21	20

(a) 40.24, 4.20 (b) 40.24, 4.10 (c) 4.5, 40.20 (d) 40.24, 4.30

(64) Find average deviation from median for given frequency distributions ________

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency f	6	7	15	16	4	2

(a) 10.16 (b) 16.10 (c) 10.10 (d) 16.16

(65) For observations $x_1, x_2, \dots\dots\dots\dots x_n$. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$ then standard deviation of the data is ________

(a) $\sqrt{3}$ (b) $\sqrt{5}$ (d) $\sqrt{2}$ (d) $\sqrt{10}$

(66) Let r be the range and $S^2 \quad \frac{1}{n \quad 1} \sum_{i \quad 1}^{n} (xi \quad \bar{x})^2$ be the variance of a set of observations $x_1, x_2, \dots\dots\dots\dots x_n$ then ________

(a) $S \quad r\sqrt{\frac{n}{n \quad 1}}$ (b) $S \quad r\sqrt{\frac{n}{n \quad 1}}$ (c) $S \quad r\sqrt{\frac{n}{n \quad 1}}$ (d) $S \quad r\sqrt{\frac{n}{n \quad 1}}$

(67) If the mean deviation of the number 1, 1 + d, 1 + 2d, ...1 + 50d. from their mean is 260 then d is ________

(a) 20.5 (b) 20.3 (c) 20.4 (d) 10.4

(68) Suppose value taken by a variable y are such that $p \quad yi \quad q$ where yi denotes the value of y in the i^{th} case for i = 1, 2, ... n then ________

(a) $\frac{p^2}{4} \quad var(y)$ (b) $(q-p)^2 \quad var(y)$ (c) $p \quad var(y) \quad q$ (d) $p^2 \quad var\ (y) \quad q^2$

(69) In any discrete series (when all values are not same) the relationship between M.D. about mean and S.D. is ________

(a) M.D. = S.D. (b) M.D. S.D. (c) M.D. < S.D. (d) M.D. S.D.

(70) A student obtain 75%, 80% and 85% in three subjects. If the marks of another subject are added then his average cannot be less then

(a) 60% (b) 65% (c) 80% (d) 90%

(71) The weighted mean of first n. natural numbers whose weights are equal to the squares of corresponding numbers is ________

(a) $\frac{n \quad 1}{2}$ (b) $\frac{3n(n \quad 1)}{2(2n \quad 1)}$ (c) $\frac{(n \quad 1)(2n \quad 1)}{6}$ (d) $\frac{n(n \quad 1)}{2}$

(72) For a data there are 34 observations in which first n observations are $a - d$, second n observation are a and last n observations are $a + d$ and there variance is $\frac{4}{3}$ then $|d|$ = ________

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{3}{2}}$

(73) If a is any real number then $(xi - \bar{x})^2$ ________ $(xi - a)^2$

(a) > (b) (c) = (d)

(74) Standard deviation of four consecutive numbers which are in arithmetic series is $\sqrt{5}$ then common disfference of this series is ________

(a) $\sqrt{5}$ (b) $2\sqrt{5}$ (c) ± 2 (d) $\sqrt{2}$

(75) Observations for a group, sum of square of observations form mean is 521 and variance is 52.1 then number of observations are ________

(a) 10 (b) 100 (c) 101 (d) 11

(76) If mean of observations x_1, x_2, x_3 and x_4 is $\bar{x}$ and difference of first three observations with respect to $\bar{x}$ is respectively – 1, – 3, – 5 then difference of fourth observation with respect to $\bar{x}$ is ________

(a) 8 (b) 9 (c) 10 (d) 11

(77) For 100 observations $(xi - 30) = 0$ and $(xi - 30)^2 = 10000$ then C.V. (coefficient of variance) is ________ %

(a) 10 (b) 100 (c) 20 (d) 30

Hints

1. **(c)** M = $\frac{n+1}{2}$ = 4^{th} observation

2. **(b)** By definition

3. **(a)** $M = \frac{\left(\frac{n}{2} th\, obeservation\right) + \left(\left(\frac{n}{2}+1\right)^{th} obeservation\right)}{2}$

4. **(a)** M = $\left(\frac{n+4}{2}\right)^{th}$ observation

5. (a) M = 5^{th} observation

6. **(d)** Marks of four students 40, 50, 64, 78, 97.

7. **(b)** Assending order $\frac{x}{8},\frac{x}{7},\frac{x}{5},\frac{x}{4},\frac{x}{3},\frac{x}{2},x$ median = $\left(\frac{n+1}{2}\right)^{th}$ observation = 5^{th}

observation = 40

8. (a) by defination

9. **(d)** All observations are equal

So that $\bar{x}$ = M = 3 δm = $\delta \bar{x}$ = 0

10. **(a)** s = $\sqrt{\frac{\sum xi^2}{n} - \left(\frac{\sum xi}{n}\right)^2}$

11. **(c)** All observations are equal so that s = 0

C. V. = $\frac{s}{\bar{x}}$ × 100 = 0

12. (a) by defination

13. **(d)** Range = R = 0 ∴ ∴ S = 0 S^2 = 0

14. **(a)** s = $\sqrt{\frac{\Sigma xi^2}{n} - (\bar{x})^2}$ and C. V. = $\frac{s}{\bar{x}}$ × 100

15. **(c)** $\Sigma xi = n\bar{x}$ and $\Sigma(x_i)^2$ = n ($s^2 + \bar{x}^2$)

$\therefore \frac{\Sigma x_i^2}{\Sigma xi} = \frac{20}{3}$ = 6.66

16. **(a)** M = $\frac{n+1}{2}$ = 10^{th} observation

$\frac{10+x+1}{2}$ = 10 ⇒ 11 + x = 20 ⇒ 3x = 9

17. **(b)** $\frac{\Sigma xi^2}{n} \quad \frac{\sum xi^2}{n} - \left(\frac{\sum xi}{n}\right)^2 \geq 0 \Rightarrow \frac{200}{n} \geq \frac{3600}{n^2}$

$\Rightarrow n \geq 18 \therefore$ n possible is 19

18. **(b)** $yi = 11x_i + 2$

$s_y = 11S_x$

19. **(b)** $n = 10,\ \bar{x} = 30\ \Sigma\ (xi - \bar{x}) = 90$

$\therefore s \Rightarrow \sqrt{\frac{\Sigma\ (xi - \bar{x})^2}{n}}$ & C.V. $= \frac{s}{x} \times 100$

20. **(a)** $y = \frac{x}{a} - \frac{b}{a} \Rightarrow Sy = \frac{1}{a}\ Sx = \frac{9}{a} = 1$

21. **(c)** $y = \frac{x}{4} - 10 \Rightarrow Sy = \frac{1}{4}\ Sx = \frac{4}{4} = 1$

22. **(a)** $2x + 5y = 15 \Rightarrow x = \frac{15}{2} - \frac{5}{2}y$

$\delta\bar{x} = \left|\frac{-5}{2}\right|\ \delta\bar{y}$

23. **(a)** $y = 3 + 5x \Rightarrow 1\ S^2y = (5)^2\ S^2x = 25 \times 4 = 100$

24. **(d)** $M = \frac{n+1}{2}$ = 3rd observation = 13

$\delta M = \frac{\Sigma\ |xi - M|}{n}$

25. **(a)** C.V. = 70 $\bar{x} = 10$

$\frac{s}{\bar{x}} \times 100 = 70 \Rightarrow \frac{s}{10} \times 100 = 70$

$\Rightarrow s^2 = 49$

26. **(c)** New $\Sigma yi = n\bar{y}$ – (deleted observation) + (added observation)

$= nm - yn + y'$

New Mean $= \frac{New\ \Sigma yi}{n} = \frac{nM - y_n + y'}{n}$

27. **(a)** Number of terms = 2n + 2

$\Sigma xi = \frac{(2n+2)}{2}\ [2a + (2n + 2 - 1)\ d] = (n + 1)\ [2a + (2n + 1)\ d]$

$\bar{x} = \frac{\Sigma xi}{n} = a + \frac{(2n+1)\ d}{2}$

28. **(d)** Arrange observations as assending order

$x-3$ $x-\frac{5}{2}$ $x-2$ $x-\frac{3}{2}$ $x+2$ x $+3$ $x+4$ $x+5$ n =8

$$M = \frac{4^{th}\ observation + 5^{th}\ observation}{2} = \frac{x-\frac{3}{2}+x+2}{2} = x + \frac{1}{8}$$

29. (a) $75 = \frac{20+x+24+x+82+x+100+x+149+x}{5}$

$375 = 375 + 5x \Rightarrow x = 0$

New Mean = $\frac{130+126+68+50+1}{5} = 75\ (x = 0)$

30. (a) $\frac{a+b+8+5+10}{5} = 6,\quad a + b = 7\quad (1)$

$\Sigma xi^2 = 2a^2 - 4a + 238$ $\qquad$ $s^2 = \frac{\Sigma xi^2}{n} - (\overline{x})^2$

$6.8 = \frac{2a^2 - 4a + 238}{5} + 36$

$\Rightarrow a^2 - 7a + 12 = 0\ a = 3\ b = 4$

31. (b) $\frac{\Sigma xi}{50} = 38 = \sum xi = 1900$

New $\Sigma xi = 1900 - 55 - 44 = 1800\ n = 48$

New Mean = $\frac{New\ \Sigma xi}{n} = \frac{1800}{48} = 37.5$

32. (d) Suppose weight of teacher is w

$\therefore\ 40 + \frac{1}{2} = \frac{35 \times 40 + 40}{35+1} \Rightarrow w = 58$

33. (c) Mean = $\frac{\sum_{i=1}^{n} fixi}{\sum_{i=1}^{n} fi} \Rightarrow 2.6 = \frac{4+10+3y+4+10}{12+y}$

$\Rightarrow 0.4y = 3.2 \Rightarrow y = 8$

34. (b) $\Sigma xi = n\overline{x}$

Mean of $xi + 2i = \frac{\sum_{i=1}^{n}(xi+2i)}{n} = \frac{\sum_{i=1}^{n} xi + 2\sum_{i=1}^{n} i}{n}$

$$= \frac{n\overline{x} + 2\frac{n(n+1)}{2}}{n} = \overline{x} + (n + 1)$$

35. (d) $m = \frac{a + (a+1) + ... + (a+6)}{7} = a + 3$

New Mean $= \frac{(a+2) + (a+3) + ... (a+12)}{11} = \frac{11a + 77}{11} = 9 + 7$

$= (a + 3) + 4 = m + 4$

36. (c) Sum of first 9 terms = 15 × 9 = 135

Sum of first 10 terms = 16 × 10 = 160

Added term = 160 – 135 = 25

37. (a) $5xi + 2 = yi$

$= xi = \frac{yi}{2} - \frac{2}{5}$

$s^2y = 20,\ s^2x = \left(\frac{1}{5}\right)^2,\ s^2y = \frac{4}{5}$

$sx = \sqrt{\frac{4}{5}}$

38. (a) $yi = 3xi - 2 \Rightarrow xi = \frac{yi}{3} + \frac{2}{3} = \frac{2}{3}xi \Rightarrow \frac{2yi}{9} + \frac{4}{9}$

$sy = 8$ S.D. of $\frac{2}{3}\ xi = 8 \times \frac{2}{9} = \frac{16}{9}$

Variance of $\frac{2}{3}xi = s^2x = \frac{144}{81}$

39. (b) Mean $= \frac{\log x + \log 2x + \log 8 + \log 4x + \log x + \log 4}{6}$

$\log 4 = \frac{\log 256x^4}{6}$ $\Rightarrow \log_4{}^6 = \log\ 256x^4$

$\Rightarrow 2^4 = x^4 \Rightarrow x = 2$

40. (d) Mean of x y z $= \frac{x+y+z}{3}$

Mena of y z r $= \frac{y+z+r}{3}$ $\Rightarrow x = r$

41. (a) $\frac{1}{2}\left(x+\frac{1}{x}\right) = m \Rightarrow x + \frac{1}{x} = 2m$

Mean of x^3 & $\frac{1}{x^3} = \frac{1}{2}\left[x^3+\frac{1}{x^3}\right]$

$= \frac{1}{2}\left[\left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)\right] = \frac{1}{2}\ [8m^3 - 6m]$

42. (d) $\Sigma xi = n\bar{x}$ then $n = \bar{x}$

$1 + 3 + 5 + ... + (2n - 1) = n.n = n^2,$

true for all $n \in$ N

43. (c) $M = L + \frac{\left(\frac{N}{2}-F\right)}{f}\times c$

44. (c) $\bar{x} = \frac{\Sigma xi}{n} = \frac{400}{n}$

$s = \sqrt{\frac{\Sigma xi^2}{n} - (\bar{x})^2} \qquad = \frac{\sqrt{10000n - 160000}}{n}$

$\frac{s}{x} \times 100 = 50 \qquad \Rightarrow n = 20$

45. (b) $\sum_{i=1}^{n} xi = nm \ \sum_{i=1}^{n-3} xi = b$

Mean of remaining 3 observation

$= \frac{\sum_{i=1}^{n} xi - \sum_{i=1}^{n-3} xi}{3} = \frac{mn - b}{3}$

46. (b) $\bar{x} = \frac{\Sigma xi}{n} = \frac{200}{20} = 10$

$\Sigma\ |xi - \bar{x}| = 124 \ \therefore\ \delta M = \frac{\Sigma\ |xi - \bar{x}|}{n}$

47. (c) $s = \sqrt{\frac{n^2 - 1}{12}}$

48. (d) Suppose $xi - 8 = yi$

$\sum_{i=1}^{10} yi = 9\ \&\ \sum_{i=1}^{10} yi^2 = 45$

S.D. of $y_1, y_2, \ldots\ldots, y_{10} = \sqrt{\frac{45}{10} - \left(\frac{9}{10}\right)^2} = \sqrt{\frac{369}{100}} = \sqrt{3.69} = 1.92$

$\therefore$ S.D. of $x_1 - 8, x_2 - 8, \ldots\ldots x_{10} - 8$ is 1.92

$\therefore$ S.D. of $x_1, x_2 \ldots\ldots\ldots x_{10}$ is 1.92

49. (a) $\bar{x} = \frac{\Sigma fixi}{n}$ $(n = 40 \ \Sigma fixi = 262 + 10k)$

50. (a) $\bar{x} = \frac{1 + 2 + 2^2 + \ldots + 2^{n-1}}{n} = \frac{2^n - 1}{n}$

51. (b) $s^2 = \frac{x_1 + x_2}{2} \Rightarrow S = \left|\frac{x_1 - x_2}{2}\right|$

52. (d) $\sum_{i=1}^{n} (xi + 4) = 100 \Rightarrow \sum_{i=1}^{n} xi + 4n = 100 \quad (1)$

$\sum_{i=1}^{n} (xi + 6) = 140 \Rightarrow \sum_{i=1}^{n} xi + 6n = 140 \quad (2)$

by equting eq (1) & (2) n = 20 $\therefore \ \bar{x} = \frac{\Sigma xi}{n} = \frac{20}{20} = 1$

53. (a) $\bar{x} = \frac{{}^nC_1 + 2.{}^nC_2 + 3{}^nC_3 + \ldots\ldots + n.{}^nC_n}{{}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n} = \frac{n.2^{n-1}}{2^n - 1}$

54. (d) $yi = 2xi - 1$

Variance of 1 . 2 . 3 .... m is = $\frac{m^2 - 1}{12}$

$\therefore \ s^2y = 4s^2x \Rightarrow s^2y = 4. \ \frac{m^2 - 1}{12}$ take $m = 2n + 1$

$\therefore$ variance of 1 3 5 .... (4n + 1) = $s^2_y = \frac{(2n+1)^2 - 1}{3} = \frac{4n\,(n+1)}{3}$

55. (c) $\Sigma x_i^2 = 2830 \ \Sigma xi = 170$

addition of Σxi is $\Sigma x_i' = 170 + 10 = 180$

addition of Σxi^2 is $\Sigma x_i'^2 = 900 - 400 = 500$

$S^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i^1}{n}\right)^2 = \frac{3330}{10} - \left(\frac{180}{10}\right)^2 = 9$

56. (a) Mean $\bar{x} = \frac{bm - bm}{2m} = 0$

S. D = $\sqrt{\frac{\Sigma(xi-\bar{x})^2}{n}}$ = 161

57. **(d)** Mode + 2 (Mean) = 3 (Median)

58. **(a)** definitoaion $V_A = V_B \Rightarrow \frac{V_A}{V_B} = 1$

59. **(b)** suppose no of boys is x and no of girls is y then $50x + 40y = 48(x + y) \Rightarrow \frac{x}{y} = 4$

$\Rightarrow \frac{x}{x+y} = \frac{4}{5}$

Percentage of girls = $\frac{x}{x+y} \times 100 = \frac{4}{5} \times 100 = 80\%$

60. **(b)** $M = L + \frac{\frac{n}{2} - F}{f} \times c$

61. (b) $s^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$ $\bar{x} = 4s^2 = 5.20$

$\therefore 5.20 = \frac{1+4+36+a^2+b^2}{5} - 4^2 \Rightarrow a^2 + b^2 = 65$

$\frac{1+2+6+a+b}{5} = 4 \Rightarrow b = 11 - a$

$\therefore a = 4$ $b = 7$

62. **(c)** $\Sigma x_i = n\bar{x} = 2000$ $(n = 100)$

$\Sigma x_i = 2000 - 21 - 21 - 18 = 1940$

$\bar{x}' = \frac{\sum xi}{n'} = \frac{1940}{97} = 20$ (nc = 97)

$\Sigma x_i^2 = n(s^2 + \bar{x}^2)$ 100 (9 + 400) = 40900

$\Sigma x_i^2 = 40900 - 441 - 441 - 324 = 39694$

$s'^2 = \frac{\sum x_i^{2'}}{n'} - (\bar{x}')^2 = \frac{39694}{97} - (20)^2 = 9.2$

63. **(b)** $n = 100$, $\Sigma fidi = 8$, $\Sigma fidi^2 = 188$

$\bar{x} = A + \frac{\Sigma fidi}{n} \times c$

$= 40 + \frac{8}{100} \times 3 = 40 - 24$

$$s = \sqrt{\frac{\sum fidi^2}{n} - \left(\frac{\sum fidi}{n}\right)^2} \times c = 4.10$$

64. **(a)** $M = L + \dfrac{\left(\dfrac{n}{2} - F\right)}{f} \times c$ and $\delta M = \dfrac{\Sigma fi\,|xi - m|}{n}$

$\dfrac{n}{2} = 25$, L = 20, f = 15, F = 13, c = 10

65. **(b)** $\Sigma x_i^2 = 6n$ & $\Sigma x_i = n$

$$s = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

66. **(a)** $x_1 - \bar{x} \le r, x_2 - \bar{x} \le r .... x_n - \bar{x} \le r$

$(x_1 - \bar{x})^2 \le r^2$, $(x_2 - \bar{x})^2 \le r^2$ ..... $(x_n - \bar{x})^2 \le r^2$

$\Sigma (xi - \bar{x})^2 = (x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ... + (x_n - \bar{x})^2$

$\le r^2 + r^2 + ... + r^2$ (n times)

$\therefore \dfrac{1}{n-1} \Sigma (x_i - \bar{x})^2 \le \dfrac{n}{n-1} r^2 \Rightarrow s \le r\sqrt{\dfrac{n}{n-1}}$

67. **(c)** $\bar{x} = \dfrac{\sum x_i}{n} = 1 + 25d$, $\bar{x} = T_{26} = 1 + 25d$

$$\delta_x = \frac{\sum |x_i - \bar{x}|}{n} = \sum_{r=0}^{50} \frac{[(1+rd)-(1+25d)]}{n}$$

$$= \frac{d}{51} \sum_{r=0}^{50} | r - 50 | \Rightarrow d = 20.4$$

68. **(b)** $p \le yi \le q \Rightarrow \sum_{i=1}^{n} p \le \sum_{i=1}^{n} yi \le \sum_{i=1}^{n} q$

$\therefore np \le n\bar{y} \le nq \Rightarrow p \le \bar{y} \le q$

similarly $-q \le -\bar{y} \le -p \Rightarrow (p - q) \le y_i - \bar{y} \le (q - p)$

$| yi - \bar{y}| \le (q - p) \Rightarrow (y_i - \bar{y})^2 \le (q - p)^2$

$\dfrac{\Sigma (yi - \bar{y})^2}{n} \le (q - p)^2 = (q - p)^2 \le$ (variance $\bar{y}$)

69. **(d)** By definiation $(SD)^2 - (MD)^2 = \sigma^2 \geq 0$

$\Rightarrow$ S. D. $\geq$ M.D.

70. **(a)** total marks of 3 subject = 240

at least average marks of $\geq$ 240 marks out of 400

$\therefore$ at least average marks = $\frac{240}{40}$ = 60% (Marks of fourth sub. = 0)

71. **(b)** Mix Mean = $\frac{\Sigma n^3}{\Sigma n^2} = \frac{3n\,(n+1)}{2\,(2n+1)}$

72. **(b)** $\sum x_i = 3na,\ \sum x_i^2 = n\,[3a^2 + 2d^2]$

$$s^2 = \frac{\Sigma x_i^2}{3n} - \left(\frac{\sum x_i^2}{n}\right)^2 = \frac{n(3a^2 + 2d^2)}{3n} - \left(\frac{3na}{3n}\right)^2$$

$d^2 = 2 \Rightarrow |\,d\,| = \sqrt{2}$

73. **(d)** Here $\Sigma(x_i - \bar{x})^2 - \sum(x_i - a)^2 - n\,(\bar{x}^2 - 2a\bar{x} + a^2)$

$= -\,n\,(\bar{x} - a)^2 \leq 0$

$\therefore \Sigma\,(xi - \bar{x})^2 \leq \sum(xi - a)^2$

74. **(c)** $\sum x_i = 4a,\ \sum x_i^2 = 4a + 20d^2$

$$s^2 = \frac{\Sigma xi^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

75. **(a)** $s^2 = \frac{\Sigma\left(x_i - \bar{x}\right)^2}{n} \Rightarrow n = 10$

76. **(b)** Here $x_1 - \bar{x} = -1\ x^2 - \bar{x} = -3\ x_1 - \bar{x} = -5$

now $\Sigma\left(x_i - \bar{x}\right) = 0$

$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 - \bar{x}) + (x_4 - \bar{x}) = 0$

$\therefore x_4 - \bar{x} = 9$

77. **(c)** $\sum(x_i - 30) = 0,\ \bar{x} = 30$

$$s = \sqrt{\frac{\Sigma\,(xi - \bar{x})^2}{n}} = \sqrt{\frac{10000}{100}} = \sqrt{100} = 10$$

$$C.V \frac{s}{x} \times 100 = \frac{10}{50} \times 100 = 20\%$$

Answer

(1)	(c)	(2)	(b)	(3)	(a)	(4)	(a)	(5)	(a)	(6)	(d)	(7)	(b)
(8)	(a)	(9)	(d)	(10)	(a)	(11)	(c)	(12)	(a)	(13)	(d)	(14)	(a)
(15)	(c)	(16)	(a)	(17)	(b)	(18)	(b)	(19)	(b)	(20)	(a)	(21)	(c)
(22)	(a)	(23)	(a)	(24)	(d)	(25)	(a)	(26)	(c)	(27)	(a)	(28)	(d)
(29)	(a)	(30)	(a)	(31)	(b)	(32)	(d)	(33)	(c)	(34)	(b)	(35)	(d)
(36)	(c)	(37)	(a)	(38)	(a)	(39)	(b)	(40)	(d)	(41)	(a)	(42)	(d)
(43)	(c)	(44)	(c)	(45)	(b)	(46)	(b)	(47)	(c)	(48)	(d)	(49)	(a)
(50)	(a)	(51)	(b)	(52)	(d)	(53)	(a)	(54)	(d)	(55)	(c)	(56)	(a)
(57)	(d)	(58)	(a)	(59)	(b)	(60)	(b)	(61)	(b)	(62)	(c)	(63)	(b)
(64)	(a)	(65)	(b)	(66)	(a)	(67)	(c)	(68)	(b)	(69)	(d)	(70)	(a)
(71)	(b)	(72)	(b)	(73)	(d)	(74)	(c)	(75)	(a)	(76)	(b)	(77)	(c)

QUESTION BANK

1. 3 dice are tossed. Find the probability that the sum of the integers is 9.

   (a) $\frac{27}{6^3}$ (b) $\frac{25}{6^3}$ (c) $\frac{21}{6^3}$ (d) $\frac{15}{6^3}$

2. There are 4 addressed covers and 4 letters. If 4 letters are put in 4 covers randomly then the probability that not more than one letter is put in proper cover is___

   (a) $\frac{15}{24}$ (b) $\frac{7}{24}$ (c) $\frac{17}{24}$ (d) $\frac{7}{17}$

3. A box contains 4 Red and 3 White balls. Every time one ball is drawn randomly and is placed back along with two more balls of opposite colour. What is the probability that among first 3 trials in first two one get red colour ball and in 3rd he get white ball.

   (a) $\frac{8}{27}$ (b) $\frac{16}{99}$ (c) $\frac{16}{231}$ (d) None

4. A, B and C can solve 50%, 60% and 70% of the sums from a book. If one sum from that book is given them to solve then probability that the sum will be solved is--

   (a) 0.94 (b) 0.06 (c) 0.47 (d) None

5. A 2 x 2 determinant is such that all its enteries are 1, -1 or 0. If one determinant is chosen from such determinants what is the probability that the value of the determinant is zero ?

   (a) $\frac{3}{8}$ (b) $\frac{11}{27}$ (c) $\frac{2}{9}$ (d) $\frac{25}{81}$

6. Three unbiased dice are tossed. Probability that the sum of digits is more than 15 is ____

   (a) $\frac{1}{12}$ (b) $\frac{1}{36}$ (c) $\frac{1}{72}$ (d) $\frac{5}{108}$

7. 3 dice are tossed. Find the probability that sum of digits is 14.

   (a) $\frac{21}{6^3}$ (b) $\frac{15}{6^3}$ (c) $\frac{27}{6^3}$ (d) $\frac{16}{6^3}$

8. A random variable takes values 0, 1, 2, 3 ..... with probability proportional to $(x+1)\left(\frac{1}{5}\right)^x$. Then

   (a) $P(x=0)=\frac{16}{25}$ (b) $P(x \geq 1)=\frac{16}{25}$

   (c) $P(x \geq 1)=\frac{7}{25}$ (d) None

9. Using 1, 2, 3, 4, 5, 6 four digit numbers without repetation of any digit are formed. If one number is taken from these what is the probability that the selected number is divisible by 4 ?

(a) $\frac{96}{6!}$ (b) $\frac{96}{6P_4}$ (c) $\frac{84}{6P_4}$ (d) None

10. A team of five person is formed from 8 boys and 5 girls. The probability that the team contains at least 3 girls is ___________

(a) $\frac{321}{13P_5}$ (b) $\frac{321}{13C_5}$ (c) $\frac{123}{13C_5}$ (d) $\frac{213}{13C_5}$

11. A and B throws a dice. The probability that A wing, if he throws a number heigher than B is ______

(a) $\frac{1}{2}$ (b) $\frac{15}{36}$ (c) $\frac{1}{36}$ (d) None

12. A, B, C can hit the target with probability $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that exactly two of them can hit the target ?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

13. If $\left(\frac{a+1}{3}\right)$ and $\left(\frac{1-a}{4}\right)$ are probabilities of two mutually exclusive events then the set of values of a is

(a) $-1 \le a \le 1$ (b) $-7 \le a \le 5$ (c) $-1 \le a \le 2$ (d) $-4 \le a \le 1$

14. There are two boxes. Box I contains 4 Red and 3 white balls. Box II contains 5 red and 2 white balls. Two balls are transferred from Box I to Box II. One ball is then drawn from box II randomly. What is the probability for that ball to be red ?

(a) $\frac{43}{63}$ (b) $\frac{23}{73}$ (c) $\frac{34}{63}$ (d) None

15. Two numbers *a* and *b* are chosen from a set of first 30 natural numbers. The probability that a^2-b^2 is divisible by 3 is ____

(a) $\frac{9}{87}$ (b) $\frac{12}{87}$ (c) $\frac{15}{87}$ (d) $\frac{47}{87}$

16. The probability that a leap year will have 53 Sunday or 53 Monday is ____

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{1}{7}$

17. Three identical dice are rolled. The probability that the same number will appear on each of them is _____

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$ (c) $\frac{1}{216}$ (d) $\frac{1}{18}$

18. The probability of having atleast one tail in 4 throws with a coin is ____

(a) $\frac{15}{16}$ (b) $\frac{1}{16}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

19. A five digit number is chosen at random. The probability that all digits are distinct and digits at odd places are odd and digits at even places are even is _____

(a) $\frac{1}{60}$ (b) $\frac{2}{75}$ (c) $\frac{1}{50}$ (d) $\frac{1}{75}$

20. A three digit number which is a multiple of 11 is chosen at random. The probability the number so chosen is also a multiple of 9 is _____

(a) $1/9$ (b) $2/9$ (c) $1/100$ (d) $9/100$

21. If *p* and *q* are chosen from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} with replacement determine the probability that the roots of $x^2 + px + q = 0$ are real.

(a) 0.62 (b) 0.61 (c) 0.60 (d) None

22. 10 balls are distributed among three boxes. Probability that the first box will contain 3 balls is______

(a) $\frac{10C_3 \times 2^7}{3^{10}}$ (b) $\frac{10C_3 \times 2^7}{10^3}$ (c) $\frac{10C_3 \ . \ 7C_2}{3^{10}}$ (d) $\frac{10P_3 \ . \ 2^7}{3^{10}}$

23. Four numbers are multiplyed together. Probability that the product is divisible by 5 or 10 is _____

(a) $\frac{369}{625}$ (b) $\frac{324}{625}$ (c) $\frac{16}{625}$ (d) $\frac{369}{1000}$

24. There are 100 tickets in a box numbered 00, 01, ... 99. One ticket is drawn at random. If A is the event that sum of the digits of the number is 7 and B is the event that product of digit is 0.
Then P(A/B) = _____

(a) $\frac{2}{13}$ (b) $\frac{2}{19}$ (c) $\frac{1}{50}$ (d) None

25. A dice is rolled three times, the probability of getting a larger number than the previous number is ______

(a) $\frac{6}{216}$ (b) $\frac{5}{54}$ (c) $\frac{1}{6}$ (d) $\frac{7}{36}$

26. Two dice are rolled one after the other. The probability that the number on the first is smaller than the number on the second is _____

(a) $1/2$ (b) $7/18$ (c) $3/4$ (d) $5/12$

27. A and B are events of same experiments with P(A) = 0.02, P(B) = 0.5
Maximum value of $P(A^1 \cap B)$ = ____

(a) 0.2 (b) 0.5 (c) 0.1 (d) 0.4

28. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangles is equilateral is _________

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{20}$

29. Probability of India winning the one day match against Pakistan is $\frac{1}{2}$. In a 5 match series probability of second win of India in 3rd one day match is___

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{16}$

30. From a set of numbers {1, 2, 3, 4, 5, 6, 7, 8, 9}. Three numbers are selected at a time without repetation. Find the probability that the sum of numbers is equal to 10.

(a) $\frac{1}{180}$ (b) $\frac{1}{21}$ (c) $\frac{7}{30}$ (d) None

31. If $P(B) = \frac{3}{4}$, $P(A \cap B \cap C^1) = \frac{1}{3}$, $P(A^1 \cap B \cap C^1) = \frac{1}{3}$ then $P(B \cap C) =$ ______

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{15}$ (d) $\frac{1}{9}$

32. A box contain 4 red and 3 black ball. One ball is taken away from the box. After that two balls are drawn at random and both found red, what is the probability that the first ball taken aways was also red ?

(a) $\frac{2}{5}$ (b) $\frac{4}{7}$ (c) $\frac{24}{105}$ (d) None

33. A, B, C are mutually exclusive events such that

$P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$, $P(C) = \frac{1-2x}{2}$

Then $x \in$ ____

(a) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (b) $\left[\frac{1}{3}, 4\right]$ (c) $[0,1]$ (d) $\left[\frac{1}{3}, \frac{1}{2}\right]$

34. A die is thrown 3 times and the sum of the thrown numbers is 15. The probability for which the number 5 appears in first throw is _______

(a) $\frac{3}{10}$ (b) $\frac{1}{36}$

(c) $\frac{1}{9}$ (d) $\frac{1}{3}$

35. A dice is loaded so that the probability of face *i* is proportional to *i*. *i* = 1, 2, ...6. Then the probability of an even number occupy when the dice is rolled is ____

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

36. 12 balls are distributed among three boxes. The probability that the first box contain 3 balls is ______

(a) $\frac{110}{9}\left(\frac{2}{3}\right)^{10}$ (b) $\frac{9}{110}\left(\frac{2}{3}\right)^{10}$ (c) $\frac{\binom{12}{3}}{12^3} \cdot 2^9$ (d) $\frac{\binom{12}{3}}{3^{12}}$

37. A and B are two independent events. Such that $P(A^1 \cap B) = \frac{2}{15}$ and $P(A \cap B^1) = \frac{1}{6}$. then P (B) = ________

(a) $\frac{1}{5}$ (b) $\frac{1}{2}$ (c) $4/5$ (d) $5/6$

38. Probability that a bomb hitting a bridge is $\frac{1}{2}$ and 2 direct hits are needed to destroy it. The least number of bombs required so that the probability of the bridge being destroyed is greater than 0.9 is ____

(a) 8 (b) 6 (c) 5 (d) 9

39. An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white it is not replaced in to urn, otherwise it is replaced along with another ball of the same colour. The process is repeated. The probability that the Third ball is black is ________

(a) $\frac{2}{3}$ (b) $\frac{17}{20}$

(c) $19/20$ (d) None

40. $P(A) = 0.6$, $P(B) = 0.4$, $P(C) = 0.5$, $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$

Then range of $P(B \cap C)$ is _____

(a) [0.3, 0.4] (b) [0.1, 0.25] (c) [0.2, 0.35] (d) None

41. If n integers taken at random are multiplied together, then the probability that the last digit of the product is 1, 3, 7 or 9 is _____

(a) $\frac{2^n}{5^n}$ (b) $\frac{4^n - 2^n}{5^n}$ (c) $\frac{4^n}{5^n}$ (d) $\frac{2}{5}$

42. A fair dice is thrown 20 Times. The probability that on the tenth throw the fourth six appear is _____

(a) $\frac{\binom{20}{10} \cdot 5^6}{6^{20}}$ (b) $\frac{120 \times 5^7}{6^{10}}$ (c) $\frac{84 \times 5^6}{6^{10}}$ (d) None

43. A coin is tossed 2n times. The probability that the number of times one get head is not equal to number of times one gats tail is _____

(a) $1-\frac{2}{4^n}$ (b) $1-\frac{(2n)!}{(n!)^2}\cdot\frac{1}{4^n}$

(c) $1-\frac{(2n)!}{(n!)^2}$ (d) $\frac{(2n)!}{(n!)^2}\cdot\frac{1}{4^n}$

44. There are 20 cards in a box. 10 of which are printed 'I' and 10 printed 'T'. One by one three cards are drawn without replacement and kept in the same order, the probability of making the word IIT is ______

(a) $\frac{5}{38}$ (b) $\frac{1}{8}$ (c) $\frac{9}{40}$ (d) $\frac{9}{80}$

45. For three events A, B, C

P (exactly one of A or B occur) = p

P (exactly one of B or C occur) = p

P (exactly one of C or D occur) = p

And P (all three occur) = p^2. Where $0 < P < \frac{1}{2}$. Then probability of atleast one of the three occur is ______

(a) $\frac{3p+2p^2}{2}$ (b) $\frac{p+3p^2}{4}$

(c) $\frac{p+3p^2}{2}$ (d) $\frac{3p+2p^2}{4}$

46. Two numbers from S = {1, 2, 3, 4, 5, 6} are selected one by one without replacement. The probability that minimum of the two numbers is less than 4 is _____

(a) $\frac{1}{15}$ (b) $\frac{14}{15}$ (c) $1/5$ (d) $\frac{4}{5}$

47. A dice is tossal untill 1 comes. Then the probability that 1 comes in even number of trials is _________

(a) $\frac{5}{11}$ (b) $\frac{5}{6}$ (c) $\frac{6}{11}$ (d) $\frac{1}{6}$

48. Out of 3n consecutive integers three are selected at random the probability that there sum is divisible by 3 is ______

(a) $\frac{3n^2-n-2}{(3n-1)\ (3n-2)}$ (b) $\frac{n^2-3n+2}{(3n-1)\ (3n-2)}$

(c) $\frac{3n^2-3n+2}{(3n-2)\ (3n-3)}$ (d) $\frac{3n^2-3n+2}{(3n-1)\ (3n-2)}$

49. A and B are independent events. Probability that both A and B occur is $\frac{1}{8}$. Probability that neither of them occur is $\frac{3}{8}$. Probability of occurence of A is ____

(a) $\frac{3}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

50. Out of 20 consecutive whole numbers two are chosen at random. Then the probability that their sum is odd is ______

(a) $\frac{5}{19}$ (b) $\frac{10}{19}$ (c) $\frac{9}{19}$ (d) $\frac{11}{19}$

Hint

(1) Dice i) tossed thrice

$\therefore n = 6^3$

A = sum of digit is 9

Total no. of triplets = 25

$\therefore$ Probability = $\frac{25}{6^3}$

(2) 4 letters are inserted in 4 addressed covers that can be done in 4! ways

So n = 4!

let A = 0 letter is in proper cover

B = 1 letter is in proper cover

C = 2 letters are in proper cover

D = 3 ie 4 letters are in proper cover

$A \cup B \cup C \cup D = \cup$

So P (A$\cup$B) = 1 - P (C$\cup$D)

$\therefore$ P (A$\cup$B) $= 1 - [(PCC) + P(D)]$

$$= 1 - \left[\frac{\binom{4}{2} + \binom{4}{4}}{4!} \right]$$

$$= 1 - \frac{7}{24}$$

$$= \frac{17}{24}$$

(3) Use P (A$\cap$B$\cap$C) = P (A) . P(B/C) . P (C/A$\cap$B)

$$= \frac{16}{99}$$

(4) P(A) = $\frac{5}{10}$, P (B) = $\frac{6}{10}$, P (C) = $\frac{7}{10}$

$\therefore$ P ($\cup$A$\cup$B$\cup$C) $= 1 - P(A \cup B \cup C)'$ (A, B, C, are independent events)

$= 1 - P(A' \cap B' \cap C')$

$= 1 - P(A').P(B').P(C')$

$$=1-\left(\tfrac{5}{10}\right)\left(\tfrac{4}{10}\right)\left(\tfrac{3}{10}\right)$$

$$=1-\frac{6}{100} \qquad = 0.94$$

(5) Let $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

a, b, c, d are selected from {-1, 0, 1}
So each can be selected in 3 way
So Total no. of different determinants.

$= 3\times3\times3\times3$
$= 81$

Now D = 0

$\therefore ad - bc = 0$

So ad = 0 [5 ways] and bc = 0 [5 ways]
or ad = 1 [2 ways] and bc = 1 [2 ways]
or ad = -1 [2 way] and bc = -1 [2 way]

So Total ways $= (5\times5)+(2\times2)+(2\times2)$
$= 25 + 4 + 4$
$= 33$

$\therefore$ Required probability $= \frac{33}{81} = \frac{11}{27}$

(6) Die is tossed 3 times

So n = 6^3

A = Sum is more than 15.
Sum can be 16, 17 or 18

Sum	Triplet	Total permutation
16	(6, 6, 4)	3
	(5, 5, 6)	3
17	(6, 6, 5)	3
18	(6, 6, 6)	1

		10

Probability $= \frac{10}{6\times6\times6}$

$= \frac{5}{108}$

(7) $n = 6^3$

Sum = 14

Triplet	Permutation
(6, 6, 2)	3
(6, 5, 3)	6
(6, 4, 4)	3
(5, 5, 4)	3

Total =	15

$\therefore$ Probability $= \dfrac{15}{6^3}$

Note: If three dice are tossed $\times$ is the random variable showing sum of digits the $\times$ carries 3, 4, 5, ... 16, 17, 18 values.

$\therefore$ Prob. dist of $\times$ is.

$\times$	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$P(\times = x)$	$\frac{1}{6^3}$	$\frac{3}{6^3}$	$\frac{6}{6^3}$	$\frac{10}{6^3}$	$\frac{15}{6^3}$	$\frac{21}{6^3}$	$\frac{25}{6^3}$	$\frac{27}{6^3}$	$\frac{27}{6^3}$	$\frac{25}{6^3}$	$\frac{21}{6^3}$	$\frac{15}{6^3}$	$\frac{10}{6^3}$	$\frac{6}{6^3}$	$\frac{3}{6^3}$	$\frac{1}{6^3}$

(8) Given $P(\times = x) = K(x+1)\left(\dfrac{1}{5}\right)^x$

$\sum P(\times) = 1$

$\Rightarrow K\left[1 + \dfrac{2}{5} + \dfrac{3}{25} + ...\right] = 1$

$\Rightarrow K[S] = 1$

Where $S = 1 + 2.\dfrac{1}{5} + 3.\left(\dfrac{1}{5}\right)^2 + ...\alpha$

$\therefore$ S = $\dfrac{25}{16}$

= $K \times \dfrac{25}{16} = 1$

$\therefore$ $K = \dfrac{16}{25}$

$P(\times = 0) = \dfrac{16}{25}\ [0+1]\ \left(\dfrac{1}{5}\right)^6 = \dfrac{16}{25}$

$\left[P(\times \geq 1) = 1 - P(\times = 1) = 1 - \dfrac{16}{25} = \dfrac{9}{25}\right]$

$$p(x=0)=\frac{16}{25}$$ i) the correct option

(9) 1, 2, 3, 4, 5, 6 are 6 digits, using these without repeating any, total ${}_6P_4$ four digited numbers can be formed .

So n = ${}_6P_4$

For a number to be divisible by 4 last two digits must be divisible by 4

Such numbers are 12, 16, 24, 32, 36, 52 and 56, 64

in all they are 8. So such nos. [div. by 4]

= 8 $\times$ 4 $\times$ 3 = 96

$$\therefore \quad Prob=\frac{96}{{}^6P_4}=\frac{4}{15}$$

(10)

(boys) (8)		girls (5)
2	3	= 5
1	4	= 5
0	5	= 5

$$Prob=\frac{\binom{8}{2}.\binom{5}{3}+\binom{8}{1}\binom{5}{4}+\binom{8}{0}\binom{5}{5}}{\binom{13}{5}}$$

$$=\frac{321}{\binom{13}{5}}$$

(11) Favourable outcome are

{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2) (4, 3)
(5, 1), (5, 2) (5, 3) (5, 4), (6,1), (6,2), (6,3) (6,4) (6,5)}

$$\therefore \quad Probability=\frac{15}{36}$$

(12) Probability that exactly 2 can hit the target

$= P(A\cap B\cap C^1)+P(A\cap B^1\cap C)+P(A^1\cap B\cap C)$

$= P(A) \cdot P(B) \cdot P(C^1) + P(A) \cdot P(B^1) \cdot P(C) + P(A^1) \cdot P(B) \cdot P(C)$

$$=\frac{1}{2}.\frac{1}{3}.\frac{3}{4}+\frac{1}{2}.\frac{2}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{1}{4}$$

$$=\frac{1}{8}+\frac{1}{12}+\frac{1}{24}$$

$$=\frac{3+2+1}{24} \qquad =\frac{1}{4}$$

(13) $0 \le \dfrac{a+1}{3} \le 1 \Rightarrow -1 \le a \le 2$ (1)

$0 \le \dfrac{a-9}{4} \le 1 \Rightarrow -3 \le a \le 1$ (2)

$0 \le \dfrac{a+1}{3} + \dfrac{a-9}{4} \le 1 \Rightarrow -1 \le a \le 1$ (3)

From (1), (2) and (3) $-1 \le a \le 1$

(14)

	Red	White	Total
Box I	4	3	7
Box II	5	2	7

A = Both balls from box I are Red
B = 1 ball is Red and 1 is white from Box I
C = Both balls from box I are white
D = 1 ball from box II is Red.

$$P(D) = P(A) \cdot P\left(D/A\right) + P(B) \cdot P\left(D/B\right) + P(C) \cdot \left(D/C\right)$$

$$= \frac{4C_2}{7C_2} \cdot \frac{7}{9} + \frac{3C_2}{7C_2} \cdot \frac{5}{9} + \frac{4.3}{7C_2} \cdot \frac{6}{4}$$

$$= \frac{43}{63}$$

(15) 2 number from (1, 2, ... 30) can be chosen

in $\binom{30}{2}$ = 435 ways.

$a^2 - b^2$ is divisible by 3 iff.

(i) a and b both are divisible by 3

or (ii) a and b both are not divisible by 3.

Among {1, 2, ... 30} there are 10 numbers which are divisible by 3 and 20 are not.

So $r = \binom{10}{2} + \binom{20}{2}$

= 45 + 190
= 235

$\therefore$ Probability $= \dfrac{235}{435}$

$= \dfrac{47 \times 5}{87 \times 5}$

$= \dfrac{47}{87}$

(16) Number of days in a leap year = 366.

$$= (52\times7)+2$$

So there are 52 weeks and 2 more days.

2 extra days can be (MT), (TW), (WT), (TF), (FS), (S Sun), (Sun, M)

P (53 Sunday) = $2/7$, P (53 Mon.) = $2/7$

P (53 Sun. and 53 Mon.) $= \frac{1}{7}$

P (53 Sun. or 53 Mon.) $= \frac{2}{7}+\frac{2}{7}-\frac{1}{7}$

$= \frac{3}{7}$

(17) A = {(1, 1, 1), (2, 2, 2), (3, 3, 3) .... (6, 6, 6)}

$\therefore P(A) = \frac{6}{6^3}$

$= \frac{1}{3^6}$

(18) Considering the event of getting a coin as success $p=\frac{1}{2},\ q=\frac{1}{2}$ and n = 4

$\therefore\ p(\times\geq1) = 1-P(\times=0)$

$= 1-\binom{4}{0}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$

$= 1-\frac{1}{16}$

$= 15/16$

(19) Total no. = $9\times10\times10\times10\times10$

$\therefore$ 9 × 10000

3 odd places contains 1, 3, 5, 7, 9

2 even places contains 0, 2, 4, 6, 8

Which can be done in $[5\times4\times3].[5\times4]$

Probability $= \frac{5\times4\times3\times5\times4}{9\times10\times10\times10\times10} = \frac{1}{75}$

(20) 3 digited numbers which are multiple of 11 are {121 .... 990)

$\therefore$ n = 81

Among these, nos. divisible by 9 i.e. by 99 are

{198, 297....... 990}

They are 9

$\therefore$ Probability = $\frac{9}{81} = \frac{1}{9}$

(21) Root of $x^2 + px + q = 0$ are real

i.e $p^2 - 4q \geq 0$

i.e. $p^2 \geq 4q$

p and q are chosen from {1, 2, 3.......... 10}

Favourables values are 62.

(22) There are 10 balls and 3 boxes each ball has 3 chances

So Total no. of chances = $3 \times 3 \times .... \times 3$ [10 times]

$= 3^{10}$

$\therefore$ $n = 3^{10}$

Selecting any three and keeping them in first box, first box can be filled in $\binom{10}{3}$ ways. 7 balls are left. They are to keep in 2 boxes which can be done in 2^7 ways

So Prob = $\dfrac{\binom{10}{3} \times 2^7}{3^{10}}$

(23) Last digits in four numbers can be $10 \times 10 \times 10 \times 10 = 10^4$

Numbers not divisible by 5 or 10 = 8^4

So Probability that the product divisible by 5 or 10 = $1 - \dfrac{8^4}{10^4}$

$$= 1 - \left(\frac{4}{5}\right)^4$$

$$= \frac{5^4 - 4^4}{5^4}$$

$$= \frac{369}{625}$$

(24) U = {00, 01, ... 99}

A : Sum of digit = 7

A = {07, 16, 25, 34, 43, 52, 61, 70}

B : product = 0

B = {01 to 09, 10, 20 .... 90}, {n(B) = 19}

$\therefore$ A$\cap$B = {07, 70}

$$\therefore P(A/B) = \frac{P(A\cap B)}{P(B)}$$

$$= \frac{2}{19}$$

(25) n = 6^3

= 216

A = Event getting a larger no. then previous

a	<	b	<	c	Ways
1		2		{3, 4, 5, 6}	4
1		3		{4, 5, 6}	3
1		4		{5, 6}	2
1		5		{6}	1
2		3		{4, 5, 6}	3
2		4		{5, 6}	2
2		5		{6}	1
3		4		{5, 6}	2
3		5		{6}	1
4		5		{6}	1

					20

Probability $= \frac{20}{6\times6\times6} \qquad = \frac{5}{54}$

(26) Pro.= $\frac{15}{36} \qquad = \frac{5}{12}$

(27) P(A^1) = 0.8, P (B) = 0.5, P(B) < P(A^1)

P($A^1 \cap$B) is Maximum when B = A^1

Maximum Value = P(B) = 0.5

(28) $n = \binom{6}{3} = 20$

r = 2

(29) $P\left(A_1^{1}.A_2.A_3\right) + P\left(A_1^{1}.A_2.A_3\right)$

$=\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 = \frac{1}{4}$

(30)

Faviouable pairs are {1, 2, 7},{1, 3, 6},{1, 4, 5},{2, 3, 5}

$\therefore r = 4$ and $n = \binom{9}{3}$

(31) $(A^1 \cap B \cap C^1) \cap (A \cap B \cap C^1) = \varnothing$

$P[A^1 \cap B \cap C^1) \cup (A \cap B \cap C^1] = P[B \cap C^1]$

$P(A^1 \cap B \cap C^1) + P(A \cap B \cap C^1) = P(B) - P(A \cap B)$

$\frac{1}{3} + \frac{1}{3} = \frac{3}{4} - P(A \cap B)$

$P(A \cap B) = \frac{3}{4} - \frac{2}{3} = \frac{9-8}{12} = \frac{1}{12}$

(32) Use Baye's rule

(33) $O \leq P(A) \leq 1 \Rightarrow -\frac{1}{3} \leq \times \leq \frac{2}{3}$ (1)

$O \leq P(B) \leq 1 \Rightarrow -3 \leq \times \leq 1$ (2)

$O \leq P(C) \leq 1 \Rightarrow -\frac{1}{2} \leq \times \leq \frac{1}{2}$ (3)

$O \leq P(A) + P(B) + P(C) \leq 1 \Rightarrow \frac{1}{3} \leq \times \leq 4$ (4)

So from (1), (2), (3) and (4)

$\frac{1}{3} \leq \times \leq \frac{1}{2}$

(34) Sum of the numbers on three dice = 15

Such triplets are

(3, 6, 6)

(4, 5, 6) (4, 6, 5)

(5, 5, 5), (5, 6, 4), (5, 4, 6)

(6, 3, 6), (6, 6, 3), (6, 4, 5), (6, 5, 4)

Among them 5 are at first place = 3

Probability = $\frac{3}{10}$

(35) U = {1, 2, 3, 4, 5, 6}

P(i) α i ; i = 1 to 6

$\sum P(i) = 1$

$\therefore \sum ki = 1$

$\therefore k\sum i = 1$

= k (21) = 1

$= k = \frac{1}{21}$

A = {2, 4, 6}

P(A) = P{2} + P{4} + P{6}

= 2k + 4k + 6k

= 12k

$= \frac{12}{21} = \frac{4}{7}$

(36) □ □ □

a_1, a_2 .... a_{12} are balls.

each ball can be placed in any one of 3 boxes.

So n $= 3 \times 3 \times$ .... $\times\ 3$

$= 3^{12}$

no. of ways that 3 ball out of 12 can be put on 1st box

$= \binom{12}{3}$

Remaining 9 balls can be distributed in remaining 2 boxes in

$2 \times 2 \times .... \times 2^{9T} = 2^9$ way

So that can be done in

$$r = \binom{12}{3} \cdot 2^9 \text{ ways}$$

$$\therefore \text{ Prob.} = \frac{110}{9}\left(\begin{matrix}2\\3\end{matrix}\right)^{10}$$

(37) Let P (A) = x and P (B) = y

$(1-x)\,y = \frac{2}{15} \Rightarrow y - xy = \frac{2}{15}$

$x\,(1-y) = \frac{1}{6} \Rightarrow x - xy = \frac{1}{6}$

$$\therefore\ y-x=\frac{2}{15}-\frac{1}{6}$$

$$=\frac{4-5}{30}$$

$$\therefore x-y=\frac{1}{30}$$

$$\therefore x=\frac{1}{30}+y$$

$$x(1-y)=\frac{1}{6}\Rightarrow\left(\frac{1}{30}+y\right)(1-y)=\frac{1}{6}$$

$$\therefore (1+3y)\ (1-y)\ =5$$

$$\therefore 1+29y-30y^2=5$$

$$\therefore 30y^2-29y+4=0$$

$$\therefore 30y^2-24y-5y+4=0$$

$$\therefore 6y(5y-4)-1(5y-4)=0$$

$$\therefore y=5/4\ \ or\ \ \frac{1}{6}$$

(38) Let n be the number of bombs required.

x be the no. of bombs that hit the bridge.

X follows the Binomial distribution with parameters n and $r=\frac{1}{2}$

$$P(x\geq 2)>0.9$$

$$=1\quad 1-P(x<2)>0.9$$

= P (X < 2) < 0.1

= P (X = 0) + P(X = 1) < 0.1

$$\left(\frac{1}{2}\right)^n+n\left(\frac{1}{2}\right)^n<0.1$$

$$\therefore\ \frac{1+n}{2^n}<\frac{1}{10}$$

$\therefore$ 10(n+1) < 2^n

for n = 8 it is true

(39) use $P(E) = P(E_1)\ .\ P\left(E/E_1 \right) + P(E_2)\ .\ P\left(E/E_2 \right) + P(E_3)\ .\ P\left(E/E_3 \right)$

$$+ P(E_4)\ .\ P\left(E/E_4 \right).$$

(40) $P(A \cup B) = 0.8$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.6 + 0.4 - 0.8$

$= 0.2$

$P(A \cup B \cup C) = 0.6 + 0.4 + 0.5 - 0.2 + P(B \cap C) - .3 + 2$

$= 1.5 - .3 - x$

$= 1.2 - x$

$P(A \cup B \cup C) \geq .85$

i.e. $0.85 \leq 1.2 - x \leq 1$

$0.20 \leq x \leq 0.35$

(41) Last digit can be 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9. So last digit of each number can be chosen in 10 ways. Thus exhaustive number of ways = 10^n.

If last digit be 1, 3, 7 or 9

[non of the numbers is even or 0 or 5]

We have a choice of 4 digits

viz 1, 3, 7, 9 with each n numbers should end.

So favourable number of way = 4^n

$$= \text{Probability} = \frac{4^n}{10^n} = \frac{2^n}{5^n}$$

(42) 10^{th} throw should get 4^{th} six

i.e. in first 9 throws 3 sixes & 6 non sixes and six in the 10^{th} throw will be the 4^{th} Six. No matter what face then after

$$\therefore \text{Probability} = \binom{9}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^6 \times \frac{1}{6}$$

$$= \frac{84 \times 5^6}{6^{10}}$$

(43) Prob. = 1 - [Prob. that No. of H = No. of tail = n]

$$= 1 - \binom{2n}{n} \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^n$$

$$= 1 - \frac{(2n)!}{(n!)\,(n!)} \cdot \frac{1}{4^n}$$

(45) P (exactly one of A or B)

= $P(A) + P(B) - 2P(A \cap B) = P$

$P(B) + P(C) - 2P(B \cap C) = P$

$P(C) + P(A) - 2P(A \cap C) = P$

Adding

$2 [P(A) + P(B) + P(C) - P(A \cap B - P(B \cap C) - P(C \cap B)] = 3p$

$\therefore P(A) + P(B) + P(C) - P(A \cap B - P(B \cap C) - P(C \cap B)] = \frac{3p}{2}$

$\therefore P(A \cup B \cup C) - P(A \cap B \cap C) = \frac{3p}{2}$

$\therefore P(A \cup B \cup C) - p^2 = \frac{3p}{2}$

$\therefore P(A \cup B \cup C) = \frac{3p}{2} + p^2 = \frac{3p+2p^2}{2}$

(46) S = {1, 2, 3, 4, 5, 6}

A = Minimum no. is < 4

A^1 = Min. no. is $\geq$ 4 i.e. 4, 5, 6

$P(A) = 1 - P\left(A^1\right)$

$= 1 - \frac{3}{6} \times \frac{2}{5}$

$= 1 - \frac{1}{5}$

$= \frac{4}{5}$

(47) Prob. of getting 1 in each trial = $\frac{1}{6}$

Not getting 1 = $\frac{5}{6}$

P [getting 1 in even chances]

= P [getting 1 in 2^{nd} or 4^{th} or 6^{th} ....]

$= \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \ldots\ldots \infty$

$= \frac{1}{6} \left[\left(\frac{5}{6}\right) + \left(\frac{5}{6}\right)^3 + \ldots\ldots \infty \right]$

$$= \frac{1}{6}\left[\frac{5/6}{1-25/36}\right] \qquad \left[Sn = \frac{a}{1-r}\right] as\ r < 1$$

$$= \frac{5}{36} \times \frac{36}{36-25}$$

$$= \frac{5}{11}$$

(48) Let $x,\ x+1,\ x+2 .... x+3n-1$ be 3n consecutive numbers

Let us divide them in 3 groups

$S_1 = x,\ x+3,\ x+6 .... x+(3n-3)$

$S_2 = x+1,\ x+4,\ x+7 .... x+(3n-2)$

$S_3 = x+2,\ x+5,\ x+8 .... x+(3n-1)$

No. is div. by 3 if [all Three are from S_1 or S_2 or S_3] OR [1 from each]

$$\text{Pr} = \frac{\binom{n}{3}\times 3 + n.n.n}{\binom{3n}{3}}$$

(49) $P(A \cap B) = \frac{1}{8}$ $\qquad$ $P(A^1 \cap B^1) = \frac{3}{8}$

$P(A)\ .P(B) = \frac{1}{8}$ $\qquad$ $P(A^1)\ .P(B^1) = \frac{3}{8}$

$x - y = \frac{1}{8}$ $\qquad$ $(1-x)\ (1-y) = \frac{3}{8}$

Solving $x = \frac{1}{2}\ or\ \frac{1}{4}$

(50) Out of 20 consecutive whole numbers 10 are even and 10 are odd

Sum is odd if one is even and other is odd.

$$\text{So Probability} = \frac{\binom{10}{1}\binom{10}{1}}{\binom{20}{2}}$$

$$= \frac{10}{19}$$

Answers

1	b	11	b	21	a	31	a	41	a
2	c	12	c	22	a	32	a	42	c
3	b	13	a	23	a	33	d	43	b
4	a	14	a	24	b	34	a	44	a
5	b	15	d	25	b	35	c	45	a
6	d	16	b	26	d	36	a	46	d
7	b	17	b	27	b	37	c	47	a
8	a	18	a	28	c	38	a	48	b
9	b	19	d	29	b	39	a	49	c
10	b	20	a	30	b	40	c	50	b

Unit - 15

(Trigonometry)

Important Points

(1) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

(2) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

(3) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

(4) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

(5) $\cos\left(\pi/2-\theta\right)=\sin\theta,\quad \sin\left(\pi/2-\theta\right)=\cos\theta$

(6) $\cos\frac{\pi}{12}=\frac{\sqrt{6}+\sqrt{2}}{4} \qquad \sin\frac{\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$

(7) $\sin(\alpha+\beta).\sin(\alpha-\beta)=\sin^2\alpha-\sin^2\beta$

(8) $\sin(\alpha+\beta).\sin(\alpha-\beta)=\cos^2\beta-\cos^2\alpha$

(9) $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta$

(10) $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\beta-\sin^2\alpha$

(11) $f(\alpha)=a\cos\alpha+b\sin\alpha,\ \alpha\in R,\ a,b\in R$

Range of $f(\alpha)$ $\left[-\sqrt{a^2+b^2},\ \sqrt{a^2+b^2}\right]$ where $a^2+b^2\neq 0$

(12) $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

(13) $\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

(14) $\cot(\alpha+\beta) = \frac{\cot\alpha.\cot\beta-1}{\cot\beta+\cot\alpha}$

(15) $\cot(\alpha-\beta)= \frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$

(16) $\tan\frac{\pi}{12}=2-\sqrt{3} \qquad \cot\frac{\pi}{12}=2+\sqrt{3}$

(17) $2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$

(18) $2\cos\alpha\sin\beta=\sin(\alpha+\beta)-\sin(\alpha-\beta),\ \alpha>\beta$

(19) $2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$

(20) $2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$

(21) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(22) $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(23) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(24) $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(25) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

(26) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$

(27) $1 + \cos 2\alpha = 2\cos^2\alpha, \quad 1 - \cos 2\alpha = 2\sin^2\alpha$

(28) $\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}, \ \cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}, \ \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

(29) $\cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}, \ \alpha \in R - \left\{\frac{k\pi}{2} \mid k \in Z\right\}$

(30) $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

(31) $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

(32) $\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}, \quad \cot 3\alpha = \frac{\cot^3\alpha - 3\cot\alpha}{3\cot^2\alpha - 1}$

(33) $\sin^2 \alpha/2 = \frac{1-\cos\alpha}{2}, \ \cos^2 \alpha/2 = \frac{1+\cos\alpha}{2}, \ \tan^2 \alpha/2 = \frac{1-\cos\alpha}{1+\cos\alpha}$

(34) $\sin 18^0 = \frac{\sqrt{5}-1}{4}, \quad \cos 18^0 = \sqrt{\frac{10+2\sqrt{5}}{16}}$

(35) $\sin 36^0 = \sqrt{\frac{10-2\sqrt{5}}{16}}, \quad \cos 36^0 = \frac{\sqrt{5}+1}{4}$

(36) $\sin 22\,{1/2}^0 = \sqrt{\frac{2-\sqrt{2}}{2}}, \quad \cos 22\,{1/2}^0 = \sqrt{\frac{2+\sqrt{2}}{2}}, \quad \tan 22\,{1/2}^0 = \sqrt{2} - 1,$

$\cot\ 22\,{1/2}^0 = \sqrt{2} + 1$

(37) $\sin\theta = 0 \Leftrightarrow \theta = k\pi,\ k \in z$

(38) $\cos\theta = 0 \Leftrightarrow \theta = (2k+1)\pi/2,\ k \in z$

(39) $\tan\theta = 0 \Leftrightarrow \theta = k\pi,\ k \in z$

(40) $\sin\theta = a,\ -1 \le a \le 1,$ Set of solution $\left\{k\pi + (-1)^k \alpha \mid k \in z\right\}$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and $\sin\theta = a = \sin\alpha$

(41) $\cos\theta = a,\ -1 \le a \le 1,$ Set of solution $\{2k\pi \pm \alpha \mid k \in z\}$

where $\alpha \in [0, \pi]$ and $\cos\theta = a = \cos\alpha$

(42) $\tan\theta$ = a, a $\in$ R Set of solution $\{k\pi + \alpha \mid k \in z\}$

where $\alpha \in (-\pi/2, \pi/2)$ and $\tan\theta = a = \tan\alpha$

(43) sin formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

(44) cos formula, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, $\cos B = \frac{c^2 + a^2 - b^2}{2ac}$, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

(45) Projection formula,

$a = b\cos C + c\cos B,\ b = a\cos C + c\cos A,\ c = a\cos B + b\cos A,$

(46) (a) $\sin^{-1}(-x) = -\sin^{-1}x \quad |x| \le 1$

(b) $\cos^{-1}(-x) = \pi - \cos^{-1}x \ |x| \le 1$

(c) $\tan^{-1}(-x) = -\tan^{-1}x,\ x \in R$

(d) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x,\ |x| \ge 1$

(e) $\sec^{-1}(-x) = \pi - \sec^{-1}x,\ |x| \ge 1$

(f) $\cot^{-1}(-x) = \pi - \cot^{-1}x,\ x \in R$

(47) (a) $\operatorname{cosec}^{-1}x = \sin^{-1}\frac{1}{x},\ |x| \ge 1$

(b) $\sec^{-1}x = \cot^{-1}\frac{1}{x},\ |x| \ge 1$

(c) $\cot^{-1}x = \tan^{-1}\frac{1}{x},\ x > 0$

$= \pi + \tan^{-1}\frac{1}{x},\ x < 0$

(48) (a) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2},\ |x| \le 1$

(b) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2},\ |x| \ge 1$

(c) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2},\ x \in R$

(49) If $x > 0 \quad y > 0$,

(a) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \quad xy < 1$

(b) $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ .. $xy < 1$

(c) $\tan^{-1} x + \tan^{-1} y = \pi/2$ ... $xy = 1$

(d) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(50) (a) $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$, where $0 < x < 1$

(b) $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$, where $0 < x < 1$

(c) $\tan^{-1} x = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$, where $x > 0$

(51) $\sin A/2 = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin B/2 = \sqrt{\frac{(s-c)(s-a)}{ac}}$

$\sin C/2 = \sqrt{\frac{(s-a)(s-b)}{ab}}$, $\cos A/2 = \sqrt{\frac{s(s-a)}{bc}}$

$\cos B/2 = \sqrt{\frac{s(s-b)}{ac}}$, $\cos C/2 = \sqrt{\frac{s(s-c)}{ab}}$

$\tan A/2 = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

(52) $\Delta = \frac{1}{2} bc \sin A, \quad \Delta = \frac{abc}{4R}$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \frac{(b^2 + c^2 - a^2)}{4\cot A} = \frac{a^2 + b^2 + c^2}{4\cot C} = \frac{a^2 + c^2 + b^2}{4\cot B}$

(53) $r = \frac{\triangle}{s}$ $r = (s-a)\ \tan A/2$

$r = (s - b)\ \tan \frac{B}{2} = (s - c)\ \tan \frac{C}{2}$

$r = 4R \sin A/2 \sin B/2 \sin C/2$

QUESTION BANK

(1) If $2\sec^2\alpha - \sec^4\alpha - 2\operatorname{cosec}^2\alpha + \operatorname{cosec}^4\alpha = \frac{15}{4}$, then $\tan^2\alpha$ = _____

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{4}$

(2) If the roots of the quadratic equation

$x^2 + Ax + B = 0$ are $\tan 30^0$ and $\tan 15^0$ then the value of A-B = ______

(a) 1 (b) -1 (c) 2 (d) 3

(3) If $A = \frac{6\pi}{7}$ and $x = \tan A + \cot(-A)$ then

(a) $x > 0$ (b) $x < 0$ (c) $x = 0$ (d) $x \geq 0$

(4) $0 < A,\ B < \frac{\pi}{2}$ If $\tan A = 7/8$, $\tan B = \frac{1}{15}$

then the value of A + B =

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

(5) $x + y = \pi/2$, then range of $\cos x . \cos y$ is

(a) $[-1,\ 1]$ (b) $[0,\ 1]$ (c) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ (d) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(6) If $\triangle$ABC, sinA + cosB = 0 then range of angle A is

(a) $\left(0, \frac{\pi}{4}\right)$ (b) $\left(0, \frac{\pi}{6}\right)$ (c) $\left(0, \frac{\pi}{3}\right)$ (d) $\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$

(7) $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos\frac{4\pi}{3}}}}$ = ___________

(a) $\frac{1}{\sqrt{2}}$ (b) 1 (c) $\frac{1}{2}$ (d) $\sqrt{3}$

(8) $Cot\left(52\frac{1}{2}\right)^0$ = _____

(a) $\sqrt{6} + \sqrt{3} - \sqrt{2} - 2$ (b) $2 + \sqrt{2} - \sqrt{6} - \sqrt{3}$

(c) $\sqrt{6} + \sqrt{2} - \sqrt{3} - 2$ (d) $\sqrt{6} - 2 + \sqrt{3} - \sqrt{2}$

(9) The number of solutions of

$\cos x + \cos 2x + \cos 3x = 0,\ x \in [0, 2\pi]\ is$

(a) 4 (b) 5 (c) 6 (d) 7

(10) If K [sin18^0 + cos36^0) = 5 then K = __________

(a) $2\sqrt{5}$ (b) $\frac{\sqrt{5}}{2}$ (c) 4 (d) 5

(11) If $\frac{\sin x}{a} = \frac{\cos x}{b} = \frac{\tan x}{c} = K$ then

$bc + \frac{1}{ck} + \frac{ak}{1+bk} =$ ______

(a) $k\left(a + \frac{1}{a}\right)$ (b) $\frac{1}{k}\left(a + \frac{1}{a}\right)$ (c) $\frac{1}{k^2}$ (d) $\frac{a}{k}$

(12) If $\cos x = 1 - 2sim^2 32^0$, α, β are the value of x between 0^0 and 360^0 with $\alpha < \beta$ then α = ______

(a) $180^0 - \beta$ (b) $200^0 - \beta$ (c) $\frac{\beta}{4} - 10^0$ (d) $\frac{\beta}{5} - 4^0$

(13) The minimum value of $125\tan^2\theta + 5\cot^2\theta$ is

(a) 5 (b) 25 (c) 125 (d) 50

(14) If $A = \cos^4\theta\ +\ \sin^2\theta,\ \forall\theta \in R$

then A lies in the interval

(a) [1, 2] (b) $\left[\frac{3}{4}, 1\right]$ (c) $\left[\frac{13}{16}, 1\right]$ (d) $\left[\frac{3}{4}, \frac{13}{16}\right]$

(15) If $A = \begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix}$ $then\ A =$ __________

(a) 0 (b) $10\sin^2 x$ (c) $12\cos^2 x - 10\sin^2 x$ (d) $12\cos^2 x$

(16) If $\frac{cosA}{3} = \frac{\cos B}{4} = \frac{1}{5},\ -\pi/2\ <\ A,\ B\ <\ 0$

then 3sinA + 6sinB = __________

(a) 0 (b) 3 (c) -4 (d) -6

(17) If tan (A + B) + 2 tanB = 0, angle B is acute and A is obtuse : then

(a) $\tan B = \frac{1}{\sqrt{2}}$ (b) $\tan B > \frac{1}{\sqrt{2}}$ (c) $\tan B < \frac{1}{\sqrt{2}}$ (d) $0 < \tan B < \frac{1}{\sqrt{2}}$

(18) $\sin^2\left(\frac{4\pi}{3}\right) + \sin^2\left(\frac{\pi}{6}\right)$ *then* $A =$ ______

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $5/2$ (d) $4/5$

(19) If $x = \cos^4\frac{\pi}{24} - \sin^4\frac{\pi}{24}$ then $x =$ ____________

(a) $\frac{\sqrt{5}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{5}-1}{4}$ (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\frac{\sqrt{2+\sqrt{2}}}{4}$

(20) The roots of equation $6x - 8x^3 = \sqrt{3}$ is ____________

(a) $\sin 10^0$ (b) $\sin 30^0$ (c) $\sin 20^0$ (d) $\cos 10^0$

(21) If $\sin\alpha - \sin\beta = m$ and $\cos\alpha - \cos\beta = n$ then $\cos(\alpha - \beta)$ =

(a) $\frac{2+m^2+n^2}{2}$ (b) $\frac{2-m^2-n^2}{2}$ (c) $\frac{m^2+n^2}{2}$ (d) $-\left(\frac{m^2+n^2}{2}\right)$

(22) $\cos 12^0 + \cos 84^0 + \cos 156^0 + \cos 132^0 =$ _____

(a) $\frac{1}{8}$ (b) $-\frac{1}{2}$ (c) 1 (d) $\frac{1}{2}$

(23) If A = $\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$ then A lies in interval ________

(a) [2, 4] (b) [3, 4] (c) [1, 4] (d) [0, 4]

(24) If $\sin(120^0 - \alpha) = \sin(120^0 - \beta)$ and $0 < \alpha,\ \beta < \pi$

then all values of $\alpha,\ \beta$ are given by

(a) $\alpha + \beta = \frac{\pi}{3}$ (b) $\alpha = \beta$ (c) $\alpha = \beta$ *or* $\alpha + \beta = \frac{\pi}{3}$ (d) $\alpha + \beta = 0$

(25) If $\cos\theta + \sec\theta = 2$ then $\cos^{2012}\theta + \sec^{2012}\theta =$ _______

(a) 2^{2012} (b) 2^{2013} (c) 2 (d) 0

(26) If $\cos x = \cos y \cos z$ then $\tan\left(\frac{x+y}{2}\right)\tan\left(\frac{x-y}{2}\right)$ = ___________

(a) $\tan^2 x/2$ (b) $\tan^2 y/2$ (c) $\tan^2 z/2$ (d) $\cot^2 z/2$

(27) If $4\cot^2\alpha - 16\cot\alpha + 15 < 0$ and $\alpha \in R$ then $\cot\alpha$ lies in the interval

(a) $\left(\frac{3}{2}, 5/2\right)$ (b) $\left(0, 3/2\right)$ (c) $\left(0, 5/2\right)$ (d) $\left(5/2, \infty\right)$

(28) $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{7\pi}{7}$ = _______

(a) 1 (b) -1 (c) $1/2$ (d) $-3/2$

(29) If $x = a\cos^3\theta\sin^2\theta,\ y = a\sin^3\theta\cos^2\theta$ and $\frac{\left(x^2+y^2\right)^m}{(xy)^n}$ (m, n$\in$N, Q $\in[0, 2\pi]$) is independent of $\theta \in [0, 2\pi]$ then .....

(a) 4m=5n (b) 4n=5m (c) m+n=9 (d) mn=20

(30) If $tanA - tanB = m,\ \cot B - \cot A = n$ then $tan(A+B)$ = ___________

(a) $\frac{m+n}{mn}$ (b) $\frac{mn}{m+n}$ (c) $\frac{m-n}{mn}$ (d) $\frac{mn}{n-m}$

(31) If $\sin x \cos y = \frac{1}{8}$ and $2\cot x = 3\cot y$ then $\sin(x+y)$ =

(a) $\frac{1}{16}$ (b) $\frac{5}{16}$ (c) $\frac{1}{8}$ (d) $5/8$

(32) If $x = \tan 10^0$, then $\tan 70^0$ = ______

(a) $\frac{2x}{1-x^2}$ (b) $\frac{1-x^2}{2x}$ (c) $7x$ (d) $2x$

(33) If $A = 3\sin^2\theta + 3\sin\theta\cos\theta + 7\cos^2\theta$, then A lies in the interval

(a) $\left[-\sqrt{2}, \sqrt{2}\right]$ (b) $\left[5/2\ \frac{15}{2}\right]$ (c) [0, 10] (d) $\left[-5/2, 5/2\right]$

(34) If $\cos(\alpha+\beta) = \frac{4}{5}$, $\sin(\alpha-\beta) = 5/13$, $0 < \alpha, \beta < \frac{\pi}{4}$ then $\cot 2\alpha$ = __________

(a) $\frac{12}{19}$ (b) $\frac{7}{20}$ (c) $\frac{16}{25}$ (d) $\frac{33}{56}$

(35) The root of the equation $2\sin^2\theta+\sin^2 2\theta=2\left(0\le\theta\le\frac{\pi}{2}\right)$ is α and β $\alpha<\beta$ then $\beta-\alpha$ = __________

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

(36) If $\tan\alpha=\frac{m}{m+1}$ and $\tan\beta=\frac{1}{2m+1}$ then $\alpha+\beta$ = __________

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$ (c) $\frac{3\pi}{4}$ (d) $-\frac{3\pi}{4}$

(37) $\operatorname{cosec}\left[\tan^{-1}\left(\cos\left(\cot^{-1}\frac{4}{\sqrt{15}}\right)\right)\right]=$ ____________

(a) $\sqrt{3}$ (b) $\frac{\sqrt{11}}{2}$ (c) $\frac{\sqrt{47}}{4}$ (d) $\frac{\sqrt{47}}{2}$

(38) $\sec^2(\tan^{-1}3)+\operatorname{cosec}^2(\tan^{-1}5)=$ ____________

(a) 276 (b) $\frac{276}{25}$ (c) 36 (d) 6

(39) If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=2\pi/3$ then $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z$ = ____________

(a) $\frac{\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

(40) $\left(3+|5-7\sin^2 x|\right)^2$ lies in the interval

(a) [9, 64] (b) [3, 8] (c) [0, 25] (d) [9, 25]

(41) The value of $\operatorname{cosec}^{-1}\sqrt{5}+\operatorname{cosec}^{-1}\sqrt{65}+\operatorname{cosec}^{-1}\sqrt{325}+.........+\infty$ is ___________

(a) π (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(42) If the side of a triangle are in the ratio 3:7:8 then R:r: is equal to

(a) 2:7 (b) 7:2 (c) 3:7 (d) 7:3

(43) If $\cos x+\cos y=0$ and $\sin x+\sin y=0$ then $\cos(x-y)$ = ___________

(a) 1 (b) $\frac{1}{2}$ (c) -1 (d) $-\frac{1}{2}$

(44) If $\cos A=\frac{1}{7}$ and $\cos B=\frac{13}{14}$, $0<A, B<\frac{\pi}{2}$, then A - B = ________

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(45) If $\cos\alpha = \frac{3}{5}$, $\cos\beta = \frac{5}{13}$, $0<\alpha,\beta<\frac{\pi}{2}$, then $\sin^2\left(\frac{\alpha-\beta}{2}\right)=$ _______

(a) $\frac{64}{65}$ (b) $\frac{1}{65}$ (c) $\frac{63}{65}$ (d) $\frac{2}{65}$

(46) If the roots of the quadratic equation $4x^2-4x+1=\cos^2\theta$ is α and β then $\alpha+\beta=$ _____

(a) $\cos^2\theta/2$ (b) $\sin^2\theta/2$ (c) 1 (d) $2\cos^2\theta/2$

(47) $\cot^{-1}1+\cot^{-1}3+\cot^{-1}5+\cot^{-1}7+\cot^{-1}+8=$ _______

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{3}$

(48) $\tan\left(\frac{\pi}{4}+\frac{1}{2}\sin^{-1}\frac{a}{b}\right)-\tan\left(\frac{\pi}{4}-\frac{1}{2}\sin^{-1}\frac{a}{b}\right)=$ _________

(a) $\frac{2a}{\sqrt{b^2-a^2}}$ (b) $\frac{2b}{\sqrt{b^2-a^2}}$ (c) $\frac{2b}{a}$ (d) $\frac{a}{2b}$

(49) $\tan 20^0+4\sin 20^0=$ _______

(a) $\sqrt{3}/2$ (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

(50) The number of values of θ in the interval [0, 2π] satisfying the equation $\tan 2\theta\tan\theta=1$ is

(a) 4 (b) 5 (c) 6 (d) 7

(51) The solution of the equation $\tan3\theta+\cot\theta=0$ is

(a) $\left\{(2k+1)\frac{\pi}{2}, k\in z\right\}$ (b) $\{k\pi, k\in z\}$

(c) $\left\{(2k+1)\frac{\pi}{4}, k\in z\right\}$ (d) $\left\{(2k+1)\frac{\pi}{6}, k\in z\right\}$

(52) If $\tan\theta + ab\cot\theta = a+b$ then $\tan\theta$ = _________

(a) a (b) b (c) a or b (d) $\frac{\pi}{4}$

(53) The number of values of θ in the interval [0, 5π] satisfying the equation $\sin^2\theta - \cos\theta - \frac{1}{4} = 0$ is _______

(a) 3 (b) 4 (c) 5 (d) 6

(54) If $\triangle$ ABC, $A = \frac{\pi}{3}$ and $\overline{AD}$ is Median of $\triangle$ ABC then AD^2 =

(a) $\frac{a^2+b^2+c^2}{4}$ (b) $\frac{b^2+bc+c^2}{4}$ (c) $\frac{a^2+ab+b^2}{4}$ (d) $\frac{a^2+ac+c^2}{4}$

(55) Right circular cone has a height 40 cm and its semi vertical angle is 45^0 then radias of its base circle is

(a) 40 cm (b) 80 cm (c) $\frac{40\sqrt{3}}{2}$ cm (d) 20 cm

(56) The angle of depression for two consecutive km stones on a horizontal road observed from a plane are α and β respectively and if the height of the plane is h then h = ____________

(a) $\frac{\tan\alpha - \tan\beta}{\tan\alpha\tan\beta}$ (b) $\frac{\tan\alpha\tan\beta}{\tan\alpha - \tan\beta}$ (c) $\frac{\tan\alpha + \tan\beta}{\tan\alpha\tan\beta}$ (d) $\frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$

(57) The angle of elevation and angle of depression of top of the flag observing from the top and bottom of tower of 100 m height are $\tan^{-1}$ and $\tan^{-1} \frac{1}{2}$ respectively then the height of flag = _________

(a) 50 m (b) 40 m (c) 20 m (d) 30 m

(58) There is a bridge of the length h on a valley. The angle of depression of a temple lying in a valley from two ends of a bridge are α and β, then the height of the bridge from top of the temple = _________

(a) $\frac{h\tan\alpha\tan\beta}{\tan\alpha - \tan\beta}$ (b) $\frac{h\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$ (c) $\frac{\tan\alpha\tan\beta}{h(\tan\alpha - \tan\beta)}$ (d) $\frac{h(\tan\alpha + \tan\beta)}{\tan\alpha\tan\beta}$

(59) The house of height h covers an angle 90^0 at the window of an opposite side house. If the height of the window is b then distance between two houses is ________ b<h

(a) $\sqrt{h(h-b)}$ (b) $\sqrt{b(h-b)}$ (c) $\sqrt{h(h+b)}$ (d) $\sqrt{b(h+b)}$

(60) $15\sin^4 x + 10\cos^4 x = 6$ *then* $\tan^2 x =$ ________

(a) $2/5$ (b) $\frac{1}{3}$ (c) $3/5$ (d) $2/3$

(61) If $\tan\frac{x}{2} = \cos ecx - \sin x$ then $\tan^2 x/2 =$ ___________

(a) $\sqrt{5}+1$ (b) $\sqrt{5}-1$ (c) $\sqrt{5}-2$ (d) $\sqrt{5}+2$

(62) If $2\tan\alpha + \cot\beta = \tan\beta$ then $\tan(\beta-\alpha) =$ __________

(a) $\tan\alpha$ (b) $\cot\alpha$ (c) $\tan\beta$ (d) $\cot\beta$

(63) $\cos(x-y) = a \quad \cos(x+y) \Rightarrow \cot x \cot y =$

(a) $\frac{a-1}{a+1}$ (b) $\frac{a+1}{a-1}$ (c) $a-1$ (d) $a+1$

(64) If $\frac{3\sin 2\theta}{5+4\cos 2\theta} = 1$ then $\tan\theta =$ ___________

(a) 1 (b) $\frac{1}{3}$ (c) 3 (d) $\frac{1}{4}$

(65) If a, b, c the sides of $\triangle$ ABC are in A.P. and *a* is the smallest side then cosA equals

(a) $\frac{3c-4b}{2c}$ (b) $\frac{3c-4b}{2b}$ (c) $\frac{4c-3b}{2c}$ (d) None of these

(66) $\sin^{-1}(\sin 4) =$

(a) 4 (b) $4-2\pi$ (c) $\pi-4$ (d) $4-\pi$

(67) If $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$ then its solution is

(a) $\left\{1, \frac{1}{6}\right\}$ (b) $\left\{\pm\frac{1}{6}\right\}$ (c) $\left\{-1, \frac{1}{6}\right\}$ (d) $\left\{\frac{1}{6}\right\}$

(68) $\sin^{-1}(\sin 2) + \sin^{-1}(\sin 4) + \sin^{-1}(\sin 6) =$ __________

(a) $\pi-12$ (b) 0 (c) 12 (d) $12-\pi$

(69) If $4\sin^{-1}x + 3\cos^{-1}x = 2\pi$, then x = ___________

(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(70) $\cot\left(\cos^{-1}\frac{3}{4} + \sin^{-1}\frac{3}{4} - \sec^{-1}3\right) =$ __________

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $2\sqrt{3}$ (d) $2\sqrt{2}$

(71) $\sum_{r=0}^{n} \tan^{-1}\left(\frac{1}{r^2+3r+3}\right) =$ ______________

(a) $\tan^{-1}(n+1)-\frac{\pi}{4}$ (b) $\tan^{-1}(n+2)-\frac{\pi}{4}$

(c) $\tan^{-1}(n+2)+\tan^{-1}(n+1)-\frac{\pi}{4}$ (d) $\tan^{-1}n-\frac{\pi}{4}$

(72) $\sin^{-1}(\sin 10) =$ ______

(a) 10 (b) $3\pi-10$ (c) $10-3\pi$ (d) $2\pi-10$

(73) If the lengths of the sides are $1, \sin x, \cos x$ in a triangle ABC then the greatest value of the angle in $\triangle$ABC is $\left(0 < x < \pi/2\right)$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $x-\frac{\pi}{2}$ (d) $\frac{\pi}{2}-x$

(74) The number of solution of the equation $\sqrt{3}\sin x+\cos x=4$ is ________ $x \in [0, 2\pi]$

(a) 1 (b) 2 (c) 0 (d) 3

(75) If $3\cos x+4\sin x=K$ has a possible solution then number of values of integral K is ________

(a) 3 (b) 5 (c) 10 (d) 11

(76) Which of the following equation has no solution

(a) $4\sin\theta+3\cos\theta=1$ (b) $\operatorname{cosec}\theta \,.\, \sec\theta=1$

(c) $\sin\theta\cos\theta=1/2$ (d) $\operatorname{cosec}\theta-\sec\theta=\operatorname{cosec}\theta\sec\theta$

(77) The number of values of θ in the interval [0, 4π] satisfying the equation

$2\sin^2\theta-\cos 2\theta=0$

(a) 4 (b) 8 (c) 2 (d) 6

(78) If $\tan(\cot x) = \cot(\tan x)$ *then* $\operatorname{cosec} 2x =$

(a) $(2n+1)\frac{\pi}{2},\ n\in z$ (b) $(2n+1)\frac{\pi}{4},\ n\in z$

(c) $\frac{n(n+1)\pi}{2}, n\in z$ (d) $\frac{n\pi}{4}, n\in z$

(79) If $\triangle$ABC, a=2, b=3 and sinA = $\frac{1}{3}$, then B = __________

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(80) If $\sqrt{3}\sin\alpha+\cos\alpha=r\cos(\alpha+\theta), -\frac{\pi}{2}, <\theta<0$, then $\theta=$________

(a) $-\pi/3$ (b) $-\pi/6$ (c) $-\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(81) $\log\cot 1^0+\log\cot 2^0+\log\cot 3^0+........=$__________

(a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(82) $\sqrt{3}\cos ec20^0-\sec 20^0=$_______

(a) -4 (b) 1 (c) 2 (d) 4

(83) $\cos^2\left(727\frac{1}{2}^0\right)-\cos^2\left(397\frac{1}{2}^0\right)=$__________

(a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $\frac{1}{2\sqrt{2}}$

(84) If $2+12\cos\theta-16\cos^3\theta - = A$, then A lies in the interval is _________

(a) [-2, -1] (b) [-2, 1] (c) [-6, 2] (d) [-2, 6]

(85) $\cos^{-1}(\cos 8)=$__________

(a) 8 (b) $8-2\pi$ (c) $\pi-8$ (d) $2\pi-8$

(86) If $\cos^{-1}x-\sin^{-1}x=\frac{\pi}{4}$ then x =_________

(a) $\frac{\sqrt{2-\sqrt{2}}}{2}$ (b) $\frac{\sqrt{2+\sqrt{2}}}{2}$ (c) $\sqrt{2}-1$ (d) $\sqrt{2}+1$

(87) If $\sin^{-1}x-\cos^{-1}x<0$ then _________

(a) $-1\le x<\frac{1}{\sqrt{2}}$ (b) $-1<x<0$ (c) $-1\le x<\frac{1}{2}$ (d) $-1\le x<\sqrt{3}/2$

(88) $A=\sin^{-1}x+\tan^{-1}x+\sec^{-1}x$, then A lies in the interval set ________

(a) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$ (b) $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$ (c) $\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$ (d) $\left\{\frac{-3\pi}{4},\frac{3\pi}{4}\right\}$

(89) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$ then $xy+yz+zx$ = ____________

(a) 1 (b) 0 (c) -3 (d) 3

(90) If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=3\pi/2$ then

$$x^{10}+y^{10}+z^{10}+\frac{3}{x^{10}+y^{10}+z^{10}}=\_\_\_\_\_\_\_\_\_\_$$

(a) 0 (b) 2 (c) 4 (d) 3

(91) If $\sum_{i=1}^{20}\cos^{-1} xi=20\pi$ then $\sum_{i=1}^{20} x_i$ = __________

(a) -20 (b) 20 (c) 0 (d) 10

(92) The number of values x satisfying the equation

$$\cot^{-1}\left(\sqrt{x(x+1)}\right)+\cos^{-1}\left(\sqrt{x^2+x+1)}\right)=\frac{\pi}{2}$$ is __________

(a) 0 (b) 1 (c) 2 (d) 3

(93) If $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ *then* $x=$ _________

(a) 0, $\frac{1}{2}$ (b) 1, $\frac{1}{2}$ (c) 0 (d) $\frac{1}{2}$

(94) $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=$ __________

(a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$ (c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{8}{9}\right)$

(95) If $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)=3/10$ then x = __________

(a) $\tan\left(\frac{3}{10}\right)$ (b) $\tan\left(\frac{4}{10}\right)$ (c) $\tan\left(\frac{10}{3}\right)$ (d) $\tan\left(\frac{6}{10}\right)$

(96) $\tan^{-1}(\tan 4)-\tan^{-1}(\tan(-6))+\cos^{-1}(\cos 10)=$ ________

(a) 16 (b) π (c) $-\pi$ (d) $5\pi-12$

(97) $\sin\left[\cot^{-1}(\cos(\tan^{-1}x))\right]$ = ________

(a) $\sqrt{\frac{x^2+2}{x^2+1}}$ (b) $\sqrt{\frac{x^2+1}{x^2+2}}$ (c) $\frac{x}{\sqrt{x^2+2}}$ (d) $\frac{1}{\sqrt{x^2+2}}$

(98) If ΔABC, $\overline{AM} \perp \overline{BC}$ and AB = 8 cm BC = 11 cm and $m\angle B = 50^0$ then area of ΔABC is =______________

(a) $28\,(cm)^2$ (b) $33.70\,(cm)^2$ (c) $38\,(cm)^2$ (d) $43.70\,cm^2$

(99) The angle of depression of the top and bottom of a tower observed from top of a lighthouse of 300 meter height are 30^0 and 60^0 respectively then the height of the tower is _____________

(a) 300 meter (b) 100 m (c) 200 m (d) 50 m

(100) The angle of elevation of a parachute measured from a point at a height 60 m from the surface of a lake is 30^0 and the angle of depression of reflection of parachute seen in the lake from the same point is 60^0. Then height of the parachute from the surface of a lake is __________

(a) 120m (b) 60m (c) 90m (d) 150m

(101) If A = sin2 sin3 sin5 then

(a) a > 0 (b) A = 0 (c) A < 0 (d) A $\geq$ 0

(102) $\sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{2r^2}\right)$ = ____________

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\tan^{-1}(n)-\frac{\pi}{4}$ (d) $\tan^{-1}(n+1)-\frac{\pi}{4}$

Answers

1	a	21	b	41	c	61	c	81	a
2	b	22	b	42	b	62	d	82	d
3	a	23	a	43	c	63	b	83	d
4	b	24	c	44	b	64	c	84	d
5	d	25	c	45	b	65	d	85	b
6	a	26	c	46	c	66	c	86	a
7	d	27	a	47	b	67	d	87	a
8	a	28	d	48	b	68	b	88	c
9	c	29	a	49	c	69	a	89	d
10	a	30	b	50	c	70	d	90	c
11	a	31	b	51	c	71	b	91	a
12	c	32	b	52	c	72	b	92	c
13	d	33	b	53	c	73	a	93	c
14	b	34	d	54	b	74	c	94	b
15	a	35	a	55	a	75	d	95	d
16	d	36	a	56	b	76	b	96	b
17	d	37	c	57	c	77	b	97	b
18	b	38	b	58	b	78	b	98	b
19	c	39	b	59	b	79	b	99	c
20	c	40	a	60	d	80	a	100	a
								101	c
								102	a

Unit - 16
Mathematical Reasoning
Summary

1. $\sim(\sim p) = p$

2. $\sim(p \wedge q) = (\sim p) \vee (\sim q)$

   $\sim(p \vee q) = (\sim p) \wedge (\sim q)$

3. $p \Rightarrow q = (\sim p) \vee q$

   $= \sim q \Rightarrow \sim p$

4. $p \Leftrightarrow q = q \Leftrightarrow p$

   $= (p \Rightarrow q) \wedge (q \Rightarrow p)$

   $= (\sim p \vee q) \wedge (\sim q \vee p)$

5. $\sim(p \Leftrightarrow q) = (p \wedge \sim q) \vee (q \wedge \sim p)$

   $= p \Leftrightarrow \sim q$

   $= \sim p \Leftrightarrow q$

6. $\left.\begin{array}{l} p \vee q = q \vee p \\ p \wedge q = q \wedge p \end{array}\right\}$

   $\left.\begin{array}{l} (p \vee q) \vee r = p \vee (q \vee r) \\ (p \wedge q) \wedge r = p \wedge (q \wedge r) \end{array}\right\}$

7. Tautology : The statement which is always true is called tautology is denoted by t.

   $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

   $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

8. Cantradiction or fallacy

   The statement which is always false is called contradiction.

   is denoted by 'c' or 'f '.

   (i) $p \vee t = t$

   (ii) $p \wedge t = p$

   (iii) $p \vee (\sim p) = t$

9. Contrapositive of $p \Rightarrow q$ is $q \Rightarrow p$

10. $p \Rightarrow q$ is false only when p is true and q is false

QUESTION BANK

1. $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is

   (a) a contradiction (b) a tautology

   (c) neither a tautology nor a contradiction (d) both tautology and contradiction

2. Which of the following a tautology ?

   (a) $p \wedge (\sim p)$ (b) $p \wedge c$ (c) $p \vee t$ (d) $p \wedge p$

3. Which of the following is true ?

   (a) $p \wedge (\sim p) = t$ (b) $p \vee (\sim p) = f$

   (c) $p \Rightarrow q = q \Rightarrow p$ (d) $P \Rightarrow q = (\sim q) \Rightarrow (\sim p)$

4. If both p and q are false then

   (a) $p \wedge q$ is true (b) $p \vee q$ is false

   (c) $p \Rightarrow q$ is false (d) $(\sim p) \vee q$ is false

5. If both p and q are true

   (a) $p \wedge q$ is true (b) $p \vee q$ is false

   (c) $p \Rightarrow q$ is false (d) None of them

6. $p \Rightarrow q \vee r$ is false then the true values of p, q and r are respectively.

   (a) F, T, T (b) T, T, F (c) T, F, F (d) F, F, F

7. The logically equivalent proposition of $p \Rightarrow \sim q$ is

   (a) $\sim q \Rightarrow p$ (b) $\sim p \Rightarrow q$

   (c) $\sim q \Rightarrow \sim p$ (d) $\sim p \Rightarrow \sim q$

8. The contrapositive of the converse of $p \Leftrightarrow q$ is

   (a) $(p \wedge q) \vee p$ (b) $(p \Rightarrow q) \wedge (q \Rightarrow p)$

   (c) $(p \wedge q) \wedge (q \Rightarrow p)$ (d) $(p \wedge q) \Rightarrow (p \vee q)$

9. $\sim(p \vee q) \vee (\sim p \wedge q) =$ ....

   (a) q (b) p (c) $\sim p$ (d) $\sim q$

10. The proposition of $(p \Rightarrow \sim p) \wedge (\sim p \Rightarrow p)$ is

(a) neither tautology nor contradiction

(b) contradiction

(c) Tautology and contradiction

(d) tautology

11. Each of the following statement is true

$p \Rightarrow \sim q$

$p \Rightarrow \sim r$

$\sim r$ Then

(a) p is false

(b) p is true

(c) q is true

(d) None of these

12. If each of the statement $p \Rightarrow \sim q$ and $\sim r \Rightarrow q$, p is true than

(a) r is false

(b) r is true

(c) q is true

(d) None of these

13. The negation of compound proposition $p \vee (\sim p \vee q)$ is ....

(a) $(p \wedge \sim q) \wedge \sim p$

(b) $(p \wedge \sim p) \vee \sim q$

(c) $(p \wedge \sim q) \vee (\sim p)$

(d) $(p \wedge \sim q) \vee \sim p$

14. $p \Rightarrow (q \Rightarrow p) \Rightarrow r$ is

(a) Contradiction

(b) tautology

(c) Neither contradiction Nor tautology

(d) Both contradiction & tautology

15. Negation of 'for all x , p' is ....

(a) there exists x , ~ p

(b) for all x; ~ p

(c) ~P

(d) P

16. $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a

(a) contradiction

(b) tautology

(c) both tautology & contradiction

(d) None of above

17. $p \Rightarrow (q \Rightarrow p)$ is equivalent to

(a) $p \Rightarrow (p \Leftrightarrow q)$

(b) $p \Rightarrow (p \Rightarrow q)$

(c) $p \Rightarrow (p \vee q)$

(d) $p \Rightarrow (p \wedge q)$

18. If p = He is intelligent

q = He is strong

Then symolic form of the statemant

" It is wrong that he is intelligent or strong " is

(a) $\sim p \vee \sim q$ (b) $\sim(p \wedge q)$

(c) $\sim(p \vee q)$ (d) $p \vee \sim q$

19. If statement p and r are false and q is true then truth value of $\sim p \Rightarrow (q \wedge r) \vee r$ is ....

(a) T (b) F

(c) T or F (d) Can not say

20. Which one of the following is false

(a) $p \wedge (\sim p)$ is a contradiction (b) $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a contradiction

(c) $\sim(\sim p) \Leftrightarrow p$ is a tautology (d) $p \vee (\sim p)$ is a tautology

21. $p \Rightarrow (\sim p \vee q)$ is false then value of p and q are respectively

(a) F, F (b) F,T (c) T, T (d) T, F

ANSWERS

1	a	11	a	21	d
2	c	12	b		
3	d	13	a		
4	b	14	a		
5	a	15	a		
6	c	16	d		
7	b	17	c		
8	b	18	c		
9	c	19	b		
10	b	20	b		

Gujarat Secondary and Higher Secondary Education Board

Sector 10-B, Near Old Sachivalaya, Gandhinagar-382 010.