Gujarat Secondary and Higher Secondary Education Board, Gandhinagar

MATHEMATICS

PART-1

Guiding Question Bank (MCQ) for Preparation of JEE Examination

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Contribution

1	Dr. Hasmukh Adhiya (IAS)	Principal Secretary , Education Department Gandhinagar
2	Shri R. R. Varsani (IAS)	Chairman , G.S&H.S.E. Bord, Gandhinagar
3	Shri H. K. Patel (G.A.S)	Dy. Chairman, G.S&H.S.E. Bord, Gandhinagar
4	Shri M. I. Joshi (G.E.S)	Secretary , G.S&H.S.E. Bord, Gandhinagar

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PREFACE

Uptil now , the Students had to appear in various entrance examinations for engineering and medical courses after std-12. The burden of examinations on the side of the students was increasing day-by-day. For alleviating this difficulty faced by the students, from the current year, the Ministry of Human Resource Development , Government of India, has Introduced a system of examination covering whole country. For entrance to engineering colleges, JEE(Main) and JEE(Advanced) examinations will be held by the CBSE. The Government of Gujarat has except the new system and has decided to follow the examinations to be held by the CBSE.

Necessary information pertaining to the proposed JEE (Main) and JEE(Advanced) examination is available on CBSE website www.cbse.nic.in and it is requested that the parents and students may visit this website and obtain latest information – guidance and prepare for the proposed examination accordingly. The detailed information about the syllabus of the proposed examination, method of entrances in the examination /centers/ places/cities of the examinations etc. is available on the said website. You are requested to go through the same carefully. The information booklet in Gujarati for JEE(Main) examination booklet has been brought out by the Board for Students and the beneficieries and a copy of this has been already sent to all the schools of the state. You are requested to take full advantage of the same also However, it is very essential to visit the above CBSE website from time to time for the latest information – guidance . An humble effort has been made by the Gujarat secondary and Higher Secondary Education Boards, Gandhinagar for JEE and NEET examinations considering the demands of the students and parents , a question bank has been prepared by the expert teachers of the science stream in the state. The MCQ type Objective questions in this Question Bank will provide best guidance to the students and we hope that it will be helpful for the JEE and NEET examinations.

It may please be noted that this "Question Bank" is only for the guidance of the Students and it is not a necessary to believe that questions given in it will be asked in the examinations. This Question Bank is only for the guidance and practice of the Students. We hope that this Question Bank will be useful and guiding for the Students appearing in JEE and NEET entrance examinations. We have taken all the care to make this Question Bank error free, however, if any error or omission is found, you are requested to refer to the text – books.

	M.I. Joshi	**R.R. Varsani (IAS)**
Date: 02/01/2013	**Secretary**	**Chairman**

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PART - I

Unit - 1

Sets, Relation and Function

Important Points

1. **Sets :** set is an undefined terms in mathemetics. set means a well-defined collection of objects.

Set is denoted by A, B, C, X, Y, Z, ... etc.

The objects in a set are called elements of the sets are denoted by *a, b, c, x, y, z* etc.

If x is a member of set A, then we write $x \in$ A, which is read as x belongs to A

2. **Methods of expressing a set**

There are two methods of expressing a set.

(1) **Listing Method (Roster From) :** In this method elements of the set are explicity written (listed) separated by commas

(2) **Property Method (set Builder Form) :** In this method a set is expressed by some common characteristic property $p(x)$ of elemernts x of the set. We have the notation $\{x \mid p(x) = \{x|$The property of $x\}$ which is read as the set of all x possesing given propety $p(x)$.

3. **Types of sets :**

(1) **Singleton set :** A set consisting of only one elements is called a singleton set.

(2) **Empty set :** A set which does not contain any element is called an empty set A set which is not empty is called a non-empty set.

(3) **Universal set :** Generally when we consider many sets of similar nature the elements in the sets are selected from a definite set. This set is called the universal and it is denoted by U.

Sub set : A set A is said to be subset of a set B if every element of A is also an element set B

If a set A is a subset of a set B then B is called super set of A

(4) **Power set :** For any set A, the set consisting of all the subsets of A is called the power set of A and it is denoted by P(A)

(5) **Equal sets :** Two sets A and B are said to be equal sets, if they have the same elements Thus if for all x if $x \in$ A, then $x \in$ B and if for all x, if $x \in$ B then $x \in$ A, then A = B, In other words if $A \subset B$ and $B \subset A$, then A = B

(6) **Finite and infinite sets :**

(1) Finite set : A set is said to be finite if it has finite number of elements

(2) Infinite set : A set is said to be infinite if it has an infinite number of elements.

Operations on sets :

(7) **Union of sets :** Let A, B $\in$ P(U) The set consisting of all elements of U which are in A or in B is called the union of sets A and B and it is denoted by $A \cup B$. The operation of taking the union of two sets is called the union operation.

Thus, $A \cup B = \{x | x \in A \text{ or } x \in B\}$

(8) Intersection of Sets :

Let A, B $\in$ P(U) Then the set consisting of all elements of U which are in both A and

B is called the intersection set of sets A and B and is denoted by $A \cap B$. The operation of fining the intersection of two sets is called the intersection operation. Thus, $A \cap B = \{x | x \in A \text{ and } x \in B\}$

(9) An Important Resut For union :

(1) $A \cup B \in P(U)$

(2) $A \subset A \cup B$, $B \subset A \cup B$

(3) $A \cup A = A$

(4) If $A \subset B$ and $C \subset D$ then $(A \cup B) \subset (B \cup D)$

(5) Commutative law $A \cup B = B \cup A$

(6) $(A \cup B) \cup C = A \cup (B \cup C)$ Associative law

(7) $A \cup \phi = A$

(8) $A \cup U = U$

(10) For Intersection

(1) $A \cap B \in P(U)$

(2) $(A \cap B) \subset A$, $(A \cap B) \subset B$

(3) $A \cap A = A$

(4) If $A \subset B$, $C \subset D$ then $(A \cap C) \subset (B \cap D)$

(5) $A \cap B = B \cap A$ Commutatinve law

(6) $A \cap (B \cap C) = (A \cap B) \cap C$ Associatative law

(7) $A \cap \phi = \phi$

(8) $A \cap U = A$

(11) Distrbutive laws :

(1) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(2) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(12) Disjoints sets : Non-empty sets A and B are said to be disjoint if their intersection is the empty set.

$\rightarrow$ If A and B are disjoint sets then $A \cap B = \phi$

(13) Complementation : For A $\in$ P(U) the set consting of all those elements of U which are not in A, is called complement of A and is denoted by A' The operation of finding the complement of a set is called complementaion operation.

Here. $A' = \{x | x \in U \text{ and } x \notin A\}$

(14) Difference set : For the sets A, B $\in$ P(U) the set consisting of all elements of A which are not in B, is called the difference set A and B, This set is denoted by A – B. The operation of taking the difference of two sets is called the difference operation.

(15) Symmetric Difference set : For sets, A, B $\in$ P(U) the set consisting of all elements which are in the set A or in the set B, but not in both is called symmetric difference of the set A and B. Symmetric difference of two sets is denoted by A Δ B.

(16) Cartesian Product of Sets : Let A and B be two non-empty sets. Then the set of all ordered pairs (x, y), where $x \in$ A, $y \in$ B is called cartesian product of A and B and cartesian product of A and B is denoted byA $\times$ B (read : 'A crooss B')

Thus, $A \times B = \{(x, y) | x \in A, y \in B\}$

If A or B or both are empty sets then we take $A \times B = \phi$ Also $A \times A = A^2$

4. **An Important Result :**

(1) If $A \subset B$ then $A \cup B = B$ and $A \cap B = A$

(2) $A \cap A' = \phi$, $A \cup A' = U$, $\phi' = U$ $U' = \phi$, $(A')' = A$

(3) $A - B = A \cap B'$ and $B - A = A' \cap B$

(4) $A - B \subset A$ and $B - A \subset B$

(5) $A' = U - A$

(6) If $A \subseteq B$ then $B' \subseteq A'$

(7) $A \Delta B = (A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

(17) Number of Elements of a Finite set

n(A) denotes the number of elements in a finite set A

5. **An Importantan result :**

(1) $n(A \cup B) = n(A) + n(B)$ A, B are disjoints sets

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$

A, B, C are disjoint sets

(2) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(3) $n(A \cap B') = n(A) - n(A \cap B)$

(4) $n(A' \cap B) = n(B) - n(A \cap B)$

(5) $n(A \cup B) = n(A \cap B') + n(A' \cap B) + n(A \cap B)$

(6) $n(A \times B) = n(B \times A) = n(A) \cdot n(B)$

(7) $n(A \times B \times C) = n(A) \cdot n(B) \cdot n(C)$

(8) $n(A \times A) = (n(A))^2$

Relations

Relation : For any non-empty sets A and B, a subset of $A \times B$ is called a relation from A to B.

If S is a relation in A i.e. $S \subset A \times A$ and $(x, y) \in S$, we say x is related to y by S or xSy

Some verious types of relation.

(1) Void or Empty relation : A relation in the set A with no elements is called an empty relation. $\phi \subset A \times A$, ϕ is a relation called empty relation.

(2) Universal Relation : A relation in the set A which is $A \times A$ itself is called a universal relation.

(3) Reflexive Relation : If S is a relation in the set A and aSa, $\forall\ a \in A$ i.e. $(a, a) \in S$, $a \in A$, we say S is a reflexive relation.

(4) Symmetric Relation : If S is a relation in a set A and if $aSb \Rightarrow bSa$ i.e. $(a, b) \in S \Rightarrow (b, a) \in S\ \forall\ a, b \in A$. We say S is a symmetric realtion in A.

(5) Transitive Relation : If S is a relation in the set A and if aSb and $bSc \Rightarrow aSc$ $\forall\ a, b, c \in A$ i.e. $(a, b) \in S$ and $(b, c) \in S \Rightarrow (a, c) \in S\ \forall\ a, b, c \in A$, thus we say that S is a transitive relation in A.

Equivalence Relation : If a relation S in a set A is reflexive, symmetric and transitive is called an equivalence relation in A.

If S is equivalence relation and $(x, y) \in S$ then $x \sim y$.

Antisymmetric Relation : If S ia a relation in A and if $(a, b) \in S$ and $(b, a) \in S$ $\Rightarrow a = b\ \forall\ a, b, \in A$ then S is said be an antisymmetric relation.

Equivalence Classes : Let A be an equivalence relation in A. let $a \in A$, then the subset $\{x \in A, xSa\}$ is said to be equivalence class corresponding to a.

Remamber :

→ If A has m and B has n elements then $A \times B$ has mn odered pairs

$\therefore$ No of subsets of $A \times B$ is 2^{mn}

$\therefore$ Total no. of relations from A to B = 2^{mn}

→ A relation R is a set is said to be identify relation if $R = \{(a, a); a \in A\}$

→ Indentify relation on a non-empty set is an equivalence relation.

→ Universal relation on a non-empty set is an universal relation.

→ Indentify relation on a non-empty y set is anti-symmetric.

Function

Function : Let A and B be two non-empty sets and $f \subset (A \times B)$ and $f \neq \phi$. Then $f : A \to B$ is said to be a function. if $\forall\, x \in A$. there corresponds a unique ordered pair $(x, y) \in f$. The set A is called the domain and B is called the codomain of the function.

→ The domain and range of a function $f : A \to B$ are denoted by D_f and R_f respectively.

Equal Functions : Two function are said to be equal if their domains, codomains and graphs (set of ordered pairs) or formula (if any) are equal.

→ For a function f : A → B, $f(x)$ is said to be value of f at x or image of x under f. and x is called pre image of $f(x)$

Some Special Functions :

(1) Indentity Function : Let A be a non-empty set. The function $f : A \to A$ defined by $f(x) = x \;\forall\; x \in A$ is called the identity function on A.

(2) Constant Function : A function whose range is singleton is called a constant function.

(3) Modulus Function : The function $f : R \to R$ defined by $f(x) = |x|$ is called modulus function where $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

(4) Signum Function : The function $f : R \to R$ defined by $f(x) = \begin{cases} 1 & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x = 0 \\ -1 & \text{if} \quad x < 0 \end{cases}$

is called signum function.

(5) Polynominal Function : Let a function g be defined as $g : R \to R$ $g(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$, $a_n \neq 0$ Then g is called polynominal function.

(6) Rational Function : A function $h(x)$, which can be expressed as $h(x) = \frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynominal function of x defined in a domain where $g(x) \neq 0$ is called a rational function.

(7) Greatest Integer Function : The function $f : R \to R$ defined by $f(x) = [x]$, assumes the value of the greatest integer, less than or equal to x, $[x]$ is also the greatest integer not exceeding x this function is called the greatests integet function.

This function is also called 'Floor' function.

(8) Ceiling Function : $g : R \to R$ given by $g(x) = \lceil x \rceil$ = least integer not less than x. This function is called 'ceiling' function.

(9) One-One Function : If $f : A \to B$ is a function and if $\forall\; x_1, x_2 \in A$, $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$, we say $f : A \to B$ is a one-one function, also called injective function.

(10) Many-one function : If $f : A \to B$ is a function and if $\exists x_1 x_2 \in A$ such that $x_1 \neq x_2$ and $f(x_1) = f(x_2)$, then $f : A \to B$ is said to be a many-one function.

(11) On to function : If the range of the function $f : A \to B$ is B we say that f is on onto function or surjective function or more

If $R_f = f(A) = B$ then, f is on to.

(12) Composite Function : If $f : A \to B$ and $g : B \to C$ are two function their composite function gof : $A \to C$ is defined by $gof(x) = g(f(x))$

If $f : A \to B$ and $g : C \to D$ are functions anf $R_f \subset D_g$ gof : $A \to D$ is defined by $gof(x) = g(f(x))$

Inverse Function : If $f : A \to B$ is a function and if there exists a function $g : B \to A$ such that gof $= I_A$ and fog $= I_B$ we say $g : B \to A$ is the inverse function of $f : A \to B$ and denoted by f^{-1}

Some Important Formula

→ If $f : A \to B$ has inverse $g : B \to A$ then $f : A \to B$ is one-one and onto.

→ If f is one-one and onto it has an iverse $g : B \to A$

→ $f : A \to B$ is one-one and onto if and only if f^{-1} exists.

→ If $f : A \to B$ and $g : B \to C$ are one-one and onto, gof : $A \to C$ is one-one and onto and $(gof)^{-1} = f^{-1} o\, g^{-1}$

Inverse -Triogonometric Function.

Function	Domain	Range
(1) $f(x) = \sin^{-1} x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
(2) $f(x) = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
(3) $f(x) = \tan^{-1} x$	R	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
(4) $f(x) = \cot^{-1} x$	R	$(0, \pi)$
(5) $f(x) = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
(6) $f(x) = \text{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[\frac{-\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

Formula for domain of function.

(1) $D(f \pm g) = D(f) \cap D(g)$

(2) $D(fg) = D(f) \cap D(g)$

(3) $D\left(\frac{f}{g}\right) = D(f) \cap D/g \cap \{x : g(x) \neq 0\}$

(4) $D\left(\sqrt{f}\right) = D(f) \cap \{x : f(x) \geq 0\}$

(5) $D(fog) = D(g)$ where $(fog)(x) = f(g(x))$

Questions Bank

1. If aN = $\{ax/x \in N\}$ and $bN \cap cN = dN$ Where $b,c \in N$ are relatively prime then

   (a) d=bc (b) c=bd (c) b=cd (d) a=bd

2. Two finite sets have m and n element respectively The total number of subsets of first set is 112 more than the total number of sub sets of the second set The value of m and n respectively are

   (a) 5,2 (b) 4,7 (c) 7,4 (d) 2,5

3. A survey shows that 70% of the Indian like mango wheres 82% like apple. If x% of Indian like both mango and apples then

   (a) $x = 52$ (b) $52 \le x \le 70$ (c) $x = 70$ (d) $70 \le x \le 82$

4. If $X \cup \{3,4\} = \{1,2,3,4,5,6\}$ the which of the following is true

   (a) Smallest set X = {1,2,5,6} (b) Smalest set X= {1,2,3,5,6}

   (c) Smallest set X= {1,2,3,4} (d) Greatest set X= {1,2,3,4}

5 If two sets A and B are having 43 elements in comman, then the number of elements common to each of the sets $A \times B$ and $B \times A$ is

   (a) 43^2 (b) 2^{43} (c) 43^{43} (d) 2^{86}

6. Let $X = \{x, y, z\} / x, y, z \in N, x + y + z = 10, x < y < z \}$ and $Y = \{(x, y, z) / x, y, z \in N, y = |x - z|\}$ then $X \cap Y$ is equal to

   (a) {(2,3,5)} (b) {(1,4,5)} (c) {(5,1,4)} (d) {(2,3,5), (1,4,5)}

7. In a certain town 30% families own a scooter and 40% on a car 50% own neither a scooter nor a car 2000 families own both a scooter and car consider the following statements in this regard

   (1) 20% families own both scooter and car

   (2) 35% families own either a car or a scooter

   (3) 10000 families live in town Which of the above statement are correct ?

   (a) 2 and 3 (b) 1,2 and 3 (c) 1 and 2 (d) 1 and 3

8. Let A={θ : $\tan\theta + \sec\theta = \sqrt{2}\sec\theta$} and B = {$\theta$: $\sec\theta - \tan\theta = \sqrt{2}\tan\theta$} be two sets then.

   (a) A=B (b) $A \subset B$ (c) $A \neq B$ (d) $B \subset A$

9. Suppose sets A_i (i = 1,2, ... 60) each set having 12 elements and set B_j (j = 1,2,3.....n) each set having 4 elements let $\bigcup_{i=1}^{60} A_i = \bigcup_{j=1}^{n} B_j = C$ and each element of C

belongs to exactly 20 of A_i's exactly 18 of B_j's then n is equal to

(a) 162 (b) 36 (c) 60 (d) 120

10. If set A is empty set then $n\left[P\left[P\left[P(A)\right]\right]\right] = \ldots$

(a) 6(b) 16 (c) 2 (d) 4

11. A and B are two sets n(A-B) = 8 + 2x, n(B-A) = 6x and n(A ∩ B)=x If n(A) = n(B) then n(A ∩ B) = ...

(a) 26 (b) 50 (c) 24 (d) none of these

12. A={(a,b) / b = 2a - 5} If (m,5) and (6,n) are the member of set A then m and n are respectively

(a) 5,7 (b) 7,5 (c) 2,3 (d) 5,3

13. There are three-three sets are given in column-A and column-B

(1) {L,A,T} (A) $\{x/x \in z,\ x^2 < 5\}$

(2) $\{x \in z \ /\ x^3 - x = 0\}$ (B) {x/x is a letter of the word LATA}

(3) {-2,-1,0,1,2} (C) $\{\sin 0, \sin\frac{3\pi}{2}, \tan\frac{5\pi}{4}\}$

which one of the following matches is correct?

(a) 1-A 2-B, 3-C (b) 1-B, 2-A, 3-C (c) 1-B, 2-C, 3-A (d) 1-A, 2-C, 3-B

14 If S_1 = {1,2,3,....,20}, S_2 ={a,b,c,d}, S_3 = {b,d,e,f}. The number of elements of $(S_1 \times S_2) \cup (S_1 \times S_3)$ is

(a) 100 (b) 120 (c) 140 (d) 40

15. For two events A and B which of the following is simple expresion of (A ∩ B) ∪ (A ∩ B') ∪ (A' ∩ B) ?

(a) A ∩ B (b) A ∪ B (c) A' ∩ B' (d) A ∩ B'

16. In a collage of 400 students every students read 5 newspapers and every news paper is read by 80 students. The number of news paper is

(a) 25 (b) at the most 20 (c) at the most 25 (d) at least 25

17. If $A=\{x/x^2=1\}$ and $B = \{x/x^4 =1\}$ then A Δ B is equal to ($x \in$ C)

(a) {-1,1,i,-i} (b) {-1,1} (c) {i, -i}(d) {-1,1,i}

18. If $A=\{(x,y) \ /\ |x - 3| < 1, |y - 3| < 1, x, y \in R\}$ and $B = \{(x,y) \ /\ 4x^2 + 9y^2 - 32x - 54y + 109 \le 0, x, y \in R\}$ then which of the following is true

(a) A is a proper sub set of B (b) B is a proper sub set of A

(c) A = B (d) A' = B

19. If A={$n^3 + (n+1)^3 + (n+z)^3$; n ∈ N} and B = {9n, n ∈ N} then

(a) A ⊂ B (b) B ⊂ A (c) A=B (d) A'=B

20. If U={1,2,3} and A={1,2} then [P(A)]' = ...

(a) { {3}, {2,3}, {1,3}, {1,2}, φ }

(b) { {3}, {2,3}, {1,3}, {1,2,3} }

(c) { {3}, {2,3}, {1,3}, {1,2,3}, φ }

(d) { {3}, {2,3}, {1,3}, {1,2}}

21. If n(A)=6 and n (B)=4 then minimum value of n(A-B) is

(a) 2 (b) 7 (c) 6 (d) 4

22. If A={x ∈ R / f(x)=0} and B={x ∈ R / g(x) = 0} then A ∩ B = ...

(a) ϕ (b) {x ∈ R / f(x)=0}

(c) {x ∈ R / f(x) + g(x) = 0 (d) {x ∈ R / $f(x)^2 + g(x)^2 = 0$}

23. If n(A)=3, n(B)=5 and n(A ∩ B)=2 then n [(A × B) ∩ (B × A)] = ...

(a) 5 (b) 3 (c) 4 (d) 6

24. If A and B are twe sets then B - (B - A) =....

(a) (A-B)-B (b) A-(A-B) (c) A (d) B

25. If A={(x,y) / $x^2 + y^2 = 25$} and B{(x,y) / $x^2 + 9y^2 = 144$} then A ∩ B contains

(a) two points (b) one points (c) three points (d) four points

26. If A = {1,3,5,7,9,11,13,15,17}, B = {2,4,...18} and N is the universal set then A' ∪ (A ∪ (B ∩ B')) is

(a) A (b) B (c) A ∪ B (d) N

27. Let U be the universal set and A ∪ B ∪ C = U. Then $\left[(A-B)\cup(B-C)\cup(C-A)\right]'$ equals

(a) A ∪ B ∪ C (b) A ∩ B ∩ C (c) A ∪ (B ∩ C) (d) A ∩ (B ∪ C)

28. The set (A ∪ B ∪ C) ∩ (A ∩ B' ∩ C')' ∩ C' equals

(a) B ∩ C' (b) B ∪ C' (c) A ∩ C (d) A ∪ C

29. Let A = {(x,y) : y = e^x, x ∈ R}, B = {(x,y) : y = e^{-x}, x ∈ R} then

(a) A ∩ B = φ (b) A ∩ B ≠ φ (c) A ∪ B = R (d) A ∪ B = A

30. Taking U = [1,5], A = {x / x ∈ N, $x^2 - 6x + 5 = 0$} A' = ...

(a) {1,5} (b) (1,5) (c) [1,5] (d) [-1, -5]

31. Let R be a reflexive relation of a finite set A having n elements and let there be m ordered pairs in R. Then

(a) $m \geq n$ (b) $m \leq n$ (c) $m = n$ (d) None of these

32. Let R be a relation in N defined by $R=\{(1 + x, 1 + x^2) / x \leq 5, x \in N\}$ which of the following is false ?

(a) R = {(2,2), (3,5), (4,10), (5,17), (6,25)}

(b) Domain of R = {2,3,4,5,6}

(c) Range of R = {2,5,10,17,26}

(d) (b) and (c) are true

33. The relation $\leq$ on numbers has the following properties.

(i) $a \leq a \ \forall \ a \in R$ (Reflexivity)

(ii) If $a \leq b$ and $b \leq a$ then $a = b \ \forall \ a, b \in R$ (Antisymmetry)

(iii) If $\mathbf{a} \leq b$ and $b \leq c$ then $a \leq c \ \forall \ a, b \in R$ (Transitivity)

Which of the above properties the relation $\subset$ on p(A) has ?

(a) (i) and (ii) (b) (i) and (iii)

(c) (ii) and (iii) (d) (i), (ii) and (iii)

34. A Relation R is defined in the set of integers I as follows $(x,y) \in R$ if $x^2+y^2 =9$ which of the following is false ?

(a) R = {(0,3), (0,-3), (3,0), (-3,0)}

(b) Domain of R = {-3,0,3}

(c) Range of R = {-3,0,3}

(d) None of the above

35. Let R be the real line consider the following subsets of the plane $R \times R$

$S = \{(x, y) / y = x + 1 \text{ and } 0 < x < 2\}$, $T = \{(x, y) / x - y \text{ is an integer}\}$

Which one of the following is true ?

(a) T is an equivalence relation on R but S is not.

(b) Neither S nor T is an equivalance relation on R

(c) Both S and T are equivalence relations on R

(d) S is an equivalence relation on R but T is not.

36. If A is the set of even natural numbers less then 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is

(a) 2^9 (b) 9^2 (c) 3^2 (d) $2^9 - 1$

37. Let W denotes the words in the English disctionary Define the relation R by R = {(x, y) $\in$ W $\times$ W the ward x and y have at least one letter in comman}

Then R is

(a) Not reflexive, symmetric and transitive

(b) Reflexive, symmetric and not transitive

(c) Reflexive, symmetric and transitive

(d) Reflexive, not symmetric and transitive

38. Given the relation on R = {(a, b), (b, c)} in the set A = {a, b, c} Then the minimum mumber of ordered pairs which added to R make it an equivalence relation is

(a) 5 (b) 6 (c) 7 (d) 8

39. Let R be the relation over the set N $\times$ N and is defined by

(a,b) R (c,d) $\Rightarrow a + d = b + c$ Then R is

(a) Reflexive only (b) Symmetric only

(c) Transitive only (d) An equivalence relation

40. Which one of the following relations on R is an equivalence relation ?

(a) $a \, R_1 \, b \Leftrightarrow |a| = |b|$ (b) $a \, R_2 \, b \Leftrightarrow a \geq b$

(c) $a \, R_3 \, b \Leftrightarrow a$ divides b (d) $a \, R_4 \, b \Leftrightarrow a < b$

41. R = {(x, y) / x, y $\in$ I, $x^2 + y^2 \leq 4$} is a relation in I then domain of R is

(a) {0,1,2} (b) {-2,-1,0} (c) {-2,-1,0,1,2} (d) {-2,-1}

42. An integer m is said to be related to another integer n if m is a multiple of n then the relation is

(a) Reflexive and symmetric (b) Reflexive and transitive

(c) Symmetric and transitive (d) Equivalence relation

43. Which of the following defined on Z is not an equivalence relation.

(a) $(x, y) \in S \Leftrightarrow x \geq y$ (b) $(x, y) \in S \Leftrightarrow x = y$

(c) $(x, y) \in S \Leftrightarrow x - y$ is a multiple of 3 (d) $(x, y) \in S$ if $|x - y|$ is even

44. If S is defined on R by $(x,y) \in S \Leftrightarrow xy \geq 0$. Then S is......

(a) an equivalence relation (b) reflexive only

(c) symmetric only (d) transitive only

45. If A = {1,2,3}, then the number of equivalence relation containing (1,2) is

(a) 1 (b) 2 (c) 3 (d) 8

46. A = [-1,1], B = [0,1], C = [-1,0]

$S_1 = \{(x,y) / x^2 + y^2 = 1, x \in A, y \in A\}$

$S_2 = \{(x,y) / x^2 + y^2 = 1, x \in A, y \in B\}$

$S_3 = \{(x,y) / x^2 + y^2 = 1, x \in A, y \in C\}$

$S_4 = \{(x,y) / x^2 + y^2 = 1, x \in B, y \in C\}$ then

(a) S_1 is not a graph of a function (b) S_2 is not a graph of a functicn

(c) S_3 is not a graph of a function (d) S_4 is not a graph of a function

47. For n, m $\in$ N n/m means that n is a factor of m, the relation / is

(a) reflexive and symmetric

(b) transitive and symmetric

(c) reflexive transitive and symmetric

(d) reflexive transitive and not symmetric

48. The relatien R defined on the let A = {1,2,3,4,5} by

$R = \{(x, y) / |x^2 - y^2| < 16\}$ is given by

(a) {(1,1), (2,1), (3,1), (4,1), (2,3)} (b) {(2,2), (3,2), (4,2), (2,4)}

(c) {(3,3), (4,3), (5,4), (3,4)} (d) None of these

49. Let R be a relation on N defind by $R = \{(x, y) / x + 2y = 8\}$ The domain of R is

(a) {2,4,8} (b) {2,4,6,8} (c) {2,4,6} (d) {1,2,3,4}

50. Let f : (-1, 1) $\to$ B be a functions defined by $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ then f is both one- one and onto then B is in the

(a) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (c) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

51. If f(x) is a polynomial Satisfying $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and f(3) = 28 then f(4) =.......

(a) 65 (b) 17 (c) 63 (d) 15

52. Range of $f(x) = \frac{x^2 + 34x - 71}{x^2 + 2x - 7}$ is

(a) [5,9] (b) (5,9]

(c) $(-\infty, 5] \cup [9, \infty)$ (d) $(-\infty, 5) \cup (9, \infty)$

53. A real valued function f(x) satisfies the functional equation

f(x - y) = f(x) f(y) - f(3 - x) f (3 + y) where f(0) = 1, f(6 - x) is equal to

(a) f(x) (b) f(3) (c) f(3) + f(3 - x) (d) -f(x)

54. If f(x) = $\frac{x}{x-1}$, $x \neq 1$ then (fofof ... f) $_{(17 \text{ times})}$ (x) is equal to ....

(a) $\frac{x}{x-1}$ (b) x (c) $\left(\frac{x}{x-1}\right)^{17}$ (d) $\frac{17x}{x-1}$

55. If [x] stands for the greatest integer function then the value of

$\left[\frac{1}{5} + \frac{1}{1000}\right] + \left[\frac{1}{5} + \frac{2}{1000}\right] + ... + \left[\frac{1}{5} + \frac{999}{1000}\right]$ is

(a) 199 (b) 201 (c) 202 (d) 200

56. If f(2x + 3y, 2x - 3y) = 24xy then f(x, y) is

(a) 2xy (b) 2(x^2 - y^2) (c) x^2 - y^2 (d) none of these

57. If f : R→R satisfies f(x+y) = f(x) + f(y), for all x, y ∈ R, f(1) = $\frac{3}{2}$ then $\sum_{r=1}^{n} f(r)$ is.....

(a) $\frac{3}{2}$ n(n + 1) (b) $\frac{3(n+1)}{2}$ (c) $\frac{3n(n+1)}{4}$ (d) $\frac{3n}{4}$

58. If $f(x) = \begin{vmatrix} 1 & x & (x+1) \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & x(x-1)(x+1) \end{vmatrix}$ then

f(1) + f(2) + f(3) + ...+ f(100) is equal to

(a) 5 (b) 8 (c) 0 (d) 10

59. If the function $f:[1,\infty) \rightarrow [1,\infty)$ is defined by f(x) = $2^{x(x-1)}$ then $f^{-1}(x)$ is

(a) $\left(\frac{1}{3}\right)^{x}$ (b) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(c) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ (d) $2^{\frac{1}{x(x-1)}}$

60. Let f(x) = $\frac{x-[x]}{1+x-[x]}$ where [x] donotes the greatest integer less than or equal to x then the range of f is

(a) [0,1] (b) [0, $\frac{1}{2}$) (c) [0,1) (d) [0, $\frac{1}{2}$]

61. Let f(x) = sec$x$ + tan$x$, g(x) = $\frac{\tan x}{1-\sec(x)}$

Statement - 1 g is an odd function

Statement - 2 f is neither an odd function nor an even function

(a) Statement 1is true (b) Statement 2 is true

(c) 1 and 2 both are true (d) 1 and 2 both are false

62. The domain of the function $f(x)=\frac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{-x^2+2x+8}}$

(a) (2,4) (b) (-2,4) (c) [2,4) (d) [-2,4)

63. Let $f(x)=\frac{x\sin\alpha}{x+1}$, $x\neq-1$ f ≠ I, f(x) ≠ 0. Then what values of sinα is f(f(x)) = x ?

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 1 (d) -1

64. Let f : R→R, g : R→R be two functions such that

f(x) = 2x - 3, g(x) = x^3 + 5

The function $(fog)^{-1}$ (x) is equal to

(a) $\left(\frac{x+7}{2}\right)^{\frac{1}{3}}$ (b) $\left(x-\frac{7}{2}\right)^{\frac{1}{3}}$ (c) $\left(\frac{x-2}{7}\right)^{\frac{1}{3}}$ (d) $\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

65. The domain of $f(x)=\frac{1}{\sqrt{|\cos x|+\cos x}}$ is

(a) $\left(\frac{(4n-1)\pi}{2}, \frac{(4n+1)\pi}{2}\right)$ (b) $\left(\frac{(4n+1)\pi}{2}, \frac{(4n-1)\pi}{2}\right)$

(c) (nπ, (n + 1)π) (d) (-2nπ , 2nπ)

66. If f(x) = $\sin^2 x + \sin^2(x + \frac{\pi}{3})$ + (cos x cos (x + $\frac{\pi}{3}$) and g($\frac{5}{4}$) = 1

then gof(x) = ...

(a) 1 (b) 2 (c) -2 (d) -1

67. If f(x) = $\frac{1-x}{1+x}$ then f (f(cos 2θ)) = ...

(a) tan2θ (b) sec2θ

(c) cos2θ (d) cot2θ

68. Given the function f(x) = $\frac{3^x + 3^{-x}}{2}$, then f(x + y) + f(x - y) = ...

(a) f(x) + f(y)   (b) f(x) f(y)

(c) $\frac{f(x)}{f(y)}$ (d) 2 f(x) • f(y)

69. The range of the function f(x) = ${}_{5-x}P_{x-1}$

(a) {1,2}   (b) {1,2,3}

(c) {1,2,3,4,5}   (d) {1}

70. If f : R→S defined by f(x) = sin$x$ - $\sqrt{3}$ cos$x$ + 1 is onto then the interval of S is

(a) [0,3]   (b) [-1,1]   (c) [0,1]   (d) [-1,3]

71. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$ is

(a) (-1,0) ∪ (1,2) ∪ (2,∞)   (b) (-1,0) ∪ (1,0)

(c) (-2, 2)   (d) (1,2) ∪ (2,∞)

72. $\left|\frac{x}{x+1}\right| < 10^{-5}$ hold if

(a) $-10^{-5} < x + 1 < 10^{-4}$ (b) $-(100001)^{-1} < x < (99999)^{-1}$

(c) $\frac{1}{10000} < x < 1$ (d) $(99999)^{-1} < x < (100001)^{-1}$

73. A function f : R→R satisfies the equation f(x)f(y)-f(xy) = x + y for all x,y ∈ R and f(1) > 0, then

(a) f(x) = $x + \frac{1}{2}$ (b) f(x) = $\frac{1}{2}x + 1$

(c) f(x) = x + 1   (d) f(x) = $\frac{1}{2}x - 1$

74. Part of the domain of the function f(x) = $\sqrt{\frac{\cos x - \frac{1}{2}}{6 + 35x - 6x^2}}$ lying in the interval [-1,6] is

(a) $\left[-\frac{1}{6}, \frac{\pi}{3}\right]$ (b) $\left[\frac{5\pi}{3}, 6\right]$

(c) $\left[-\frac{1}{5}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right]$ (d) $\left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$

75. Two finite sets have m and n elements respectively The total number of subsets of first set is 48 more than the total number of subsets of the second set and f = I_N then f(m+n) is

(a) 9 (b) 10 (c) 48 (d) 15

76. The domain of the function $f(x) = \cos^{-1}(\log_3 \frac{x}{4})$ is

(a) [4,12] (b) [0,3] (c) $[\frac{4}{3}, 4]$ (d) $[\frac{4}{3}, 12]$

77. The set of all x for which $f(x) = \log_{\frac{x-3}{x+4}} 5$ and $g(x) = \frac{1}{\sqrt{x^2 - 16}}$ are both not defined is

(a) [-4,3] (b) [-4,4] (c) [0,3] (d) [0,4]

78. If f: R$\rightarrow$R defined by $f(x) = x^4 + 2$ then the value of $f^{-1}(83)$ and $f^{-1}(-2)$ respectively are.

(a) ϕ, {3,-3} (b) {3,-3}, ϕ

(c) {4,-4}, ϕ (d) {4,-4}, {2,-2}

79. The domain of the function $f(x) = {}^{21-x}C_{3x-1} + {}_{25-3x}P_{5x-3}$ is

(a) {1,2,3} (b) {2,3} (c) {2,3,4} (d) {2,3,4,5}

80. Let X = {a,b,c,d} then one - one mapping f : X $\rightarrow$ X such that

$f(a) = a$, f(b)$\neq$b, f(d)$\neq$d are given by

(i) {(a,a), (b,c), (c,d), (d,b)}

(ii) {(a,a,), (b,d), (c,c), (d,b)}

(iii) {(a,a), (b,d), (c,b), (d,c)}

(a) only (i) is true (b) (i) & (ii) is true

(c) (i), (ii) and (iii) are true (d) only (iii) is true

81. If $f(x) = \cos(\log x)$ then $f(x) \bullet f(y) - \frac{1}{2}\left(f\left(\frac{x}{y}\right) + f(xy)\right)$ has the value

(a) 1 (b) -1 (c) 0 (d) 3

82. The domain of the definition of the function f(x) given by the equation $a^x + a^y = a$ is ...... $(a > 1)$

(a) $0 \leq x \leq 2$ (b) $0 \leq x \leq 1$

(c) $-\infty < x \leq 0$ (d) $-\infty < x < 1$

83. f(x) = max {2 - x, 2 + x, 4} $x \in$ R is

(a) f(x) = $\begin{cases} 2-x & x \geq 2 \\ 4 & -2 < x < 2 \\ 2+x & x \leq -2 \end{cases}$ (b) f(x) = $\begin{cases} 2-x & -2 < x < 2 \\ 4 & x \geq 2 \\ 2+x & x \leq -2 \end{cases}$

(c) f(x) = $\begin{cases} 2-x & x \leq -2 \\ 4 & x \geq 2 \\ 2+x & -2 < x < 2 \end{cases}$ (d) f(x) = $\begin{cases} 2-x & x \leq -2 \\ 4 & -2 < x < 2 \\ 2+x & x \geq 2 \end{cases}$

84. The inverse of $\dfrac{7^x - 7^{-x}}{7^x + 7^{-x}}$ is

(a) $\frac{1}{2}\log_7 \frac{1+x}{1-x}$ (b) $\log_7 \frac{1-x}{1+x}$

(c) $\log_{\frac{1}{2}} \frac{1-x}{1+x}$ (d) $\frac{1}{2}\log_e \frac{1+x}{1-x}$

85. Let f(x) = tan $\sqrt{m}\,x$ where m = [P] = greatest integer less then or equal to P and Principal period of f(x) is π. then

(a) $2 \leq p \leq 3$ (b) $1 \leq p \leq 2$

(c) $1 \leq p < 2$ (d) $3 \leq p < 4$

86. The largest interval in which the function f(x) = 3 sin $\left[\sqrt{\frac{\pi^2}{9} - x^2}\right]$ assumes values is

(a) $[0, 3\sqrt{3}]$ (b) $\left[\frac{-3\sqrt{3}}{2}, \frac{3\sqrt{3}}{2}\right]$

(c) $\left[0, \frac{3\sqrt{3}}{2}\right]$ (d) $\left[\frac{-3\sqrt{3}}{2}, 0\right]$

87. The domain of f(x) = $\log_5 [\log_6 [\log_8 x]]$ is

(a) $x > 4$ (b) $x > 8$ (c) $x < 8$ (d) $x < 4$

88. let g(x) = 1+ x - [x] and

$$f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

Then for all x, f(g(x)) is equal to

(a) x (b) 1 (c) f(x) (d) g(x)

89. The domain of the function f(x) = $\dfrac{1}{\sqrt{|x|-x}}$ is

(a) $(-\infty, \infty)$ (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) $(-\infty, \infty) - \{0\}$

90. If f(x) = 2x and g is identity function, then

(a) (fog) (x) = g(x) (b) (gog) (x) = g(x)

(c) (fog) (x) = (g+g) (x) (d) (fog) (x) = (f+f) (x)

91. If function f(x) = $\dfrac{1}{2} - \tan\left(\dfrac{\pi x}{2}\right)$; $(-1 < x < 1)$ and g(x) = $\sqrt{3+4x-4x^2}$ then the domain of gof is

(a) (-1, 1) (b) $\left[\dfrac{-1}{2}, \dfrac{1}{2}\right]$ (c) $\left[-1, \dfrac{1}{2}\right]$ (d) $\left[\dfrac{1}{2}, -1\right]$

92. The range of f(x) = $6^x + 3^x + 6^{-x} + 3^{-x} + 2$ is subset of.....

(a) $[-\infty, -2)$ (b) (-2,3) (c) $(6, \infty)$ (d) $[6, \infty)$

93. If f is satisfied the condition 2 f(x) + f(1 - x) =x^2, $x \in$ R then f(x) = ...

(a) $\dfrac{x^2+2x-1}{6}$ (b) $\dfrac{x^2+4x-1}{3}$ (c) $\dfrac{x^2+2x-1}{3}$ (d) $\dfrac{x^2-3x+1}{6}$

94. If function f satisfies the equation 3f(x) +2f $\left(\dfrac{x+59}{x-1}\right)$ = 10x + 30, $x \neq 1$ then f(7)=........

(a) 8 (b) 4 (c) -8 (d) 11

95. If [x] is an integer function and {x} = x - [x] then $f(x) = [x] + \sum_{r=1}^{100} \dfrac{\{x+r\}}{100}$ =............

(a) 4 x (b) 2x (c) 4[x] + 100{x} (d) x

96. g : R→R, g(x) = 3 + $\sqrt[3]{x}$ and f(g(x)) = 2 - $\sqrt[3]{x}$ + x then f(x) =....

(a) $x^3 - x^2 + x - 5$ (b) $x^3 - 9x^2 + 26x + 22$

(c) $x^3 + 9x^2 - 26x + 5$ (d) $x^3 + x^2 - x + 5$

97. Which of the following relation is one-one

(a) $R_1 = \{(x,y) \,/\, x^2 + y^2 = 1,\ x, y \in R\}$

(b) $R_2 = \{(x, y) \,/\, y = e^{x^2} \,/\, x, y \in R\}$

(c) $R_3 = \{(x,y) \,/\, y = x^2 - 3x + 3,\ x, y \in R\}$

(d) None of these

98. If f : R → R, f(x) = $x^2 + 1$ then $f^{-1}(-2) \cup f^{-1}(17)$......

(a) $\{\pm 4\}$ (b) $\{\pm 1, \pm 4\}$ (c) $\{4\}$ (d) $\{1,4\}$

99. $f(x) = \sin^{-1}\left\{4-(x-7)^3\right\}^{\frac{1}{5}}$ then $f^{-1}(x)$ = ...

(a) $\left(7-\sin^5 x\right)^{\frac{1}{3}}$ (b) $7 + \left(4-\sin^5 x\right)^{\frac{1}{3}}$ (c) $7 + \left(4+\sin^5 x\right)^{\frac{1}{3}}$ (d) $\left(7+\sin^5 x\right)^{\frac{1}{3}}$

100. There are three function given in colum -A and its inverse in column - B

Colum-A	Colum-B
(1) $f(x) = 1 - 2^{-x}$	(a) $f^{-1}(x) = \dfrac{x}{\sqrt{1-x^2}}$
(2) $f(x) = \sin(\tan^{-1} x)$	(b) $f^{-1}(x) = -\log_2(1-x)$
(3) $f(x) = 2x + 3$	(c) $f^{-1}(x) = \dfrac{x-3}{2}$

which one of the following matches is correct ?

(a) (1) A (2) B (3) C

(b) (1) B (2) (C) (3) A

(c) (1) B (2) (A) (3) (C)

(d) (1) C (2) B (3) A

Hint

1. $bN = \{bx/x \in N\}$ = the set of positive integer multiples of b

$cN = \{cx/x \in N\}$ = the set of positive interger multiples of c

$\therefore$ $bn \cap cN$ = the set of positive integer multiples of $bc = bcN$ [$\because$ b and c are prime]

Hence $d = bc$

2. Hence $2^m = 2^n + 112$ is given

$\Rightarrow 2^n(2^{m-n} - 1) = 2^4 \times 7$

$\Rightarrow n = 4$ and $2^{m-n} - 1 = 7$

$\Rightarrow n = 4$ and $2^{m-4} = 8$

$\Rightarrow n = 4$ and $m = 7$

3. $n(A) = 70$ $n(B) = 82$ $n(A \cap B) = x$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(A \cap B) = 152 - n(A \cup B)$ But $n(A \cup B) \le 100$ so $n(A \cap B) \ge 52$

$\therefore$ $n(A \cap B) \le 70$

$\therefore$ $52 \le n(A \cap B) \le 71$

4. $X \cup \{3, 4\} - \{3, 4\} = \{1, 2, 5, 6\}$ is the smallest set

5. $$\begin{aligned} n((A \times B) \cap (B \times A)) &= n((A \cap B) \times (B \cap A)) \\ &= n(A \cap B) \cdot n(B \cap A) \\ &= 43 \times 43 \\ &= 43^2 \end{aligned}$$

6. X = {(1, 2, 7), (1, 3, 6), (1, 4, 5), (2, 3, 5)} element of X which belong to Y are (1, 4, 5) and (2, 3, 5) both so they belong to $X \cap Y$

7. Suppose x families live in the town

A = {families have scooter}

B = {families have car}

$\therefore$ $n(A) = \dfrac{30x}{100}$ $n(B) = \dfrac{40x}{100}$ and $n(A \cup B)' = \dfrac{50x}{100}$

$\therefore$ $n(A \cup B) = \dfrac{50x}{100}$

$\therefore$ $n(A \cap B) = \dfrac{20x}{100}$

$\therefore$ $\dfrac{20x}{100} = 2000$

$\therefore$ $x = 10000$

$\therefore$ both scooter and car have 20 %

8. Given that $\tan\theta + \sec\theta = \sqrt{2}\sec\theta$

$\Rightarrow \sin\theta = \sqrt{2} - 1$

now also given that $\sec\theta - \tan\theta = \sqrt{2}\tan\theta$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}+1}$$

$\Rightarrow \sin\theta = \sqrt{2} - 1$

$\therefore$ A = B

9. $n(c) = n\left(\bigcup_{i=1}^{60} Ai\right) = \frac{1}{20}(12 \times 60) = 36$

$n(c) = n\left(\bigcup_{j=1}^{n} Bj\right) = \frac{4n}{18} = \frac{2n}{9}$

$\Rightarrow 36 = \frac{2n}{9} \Rightarrow n = 162$

10. A is empty set then $n(P(A)) = 1$

11. $n(A - B) + n(B - A) = n(A) + n(B) - 2n(A \cap B)$

$\Rightarrow 8 + 2x + 6x = 2n(A) - 2x$ ($\therefore n(A) = n(B)$)

$\Rightarrow n(A) = 4 + 5x$

$\Rightarrow n(A - B) = n(A) - n(A \cap B)$

$\Rightarrow 4 = 2x$

$\Rightarrow x = 2$

12. $5 = 2(m) - 5$

$\therefore m = 5$

$n = 2(6) - 5$

$\Rightarrow n = 7$

13. $\{x/x$ is a letter of the world LATA$\}$ = {L, A. T}

$\{x/x \in Z, x^2 < 5\} = \{-2, -1, 0, 1, 2\}$

$\sin 0 = 0, \sin\frac{3\pi}{2} = -1, \tan\frac{5\pi}{4} = 1$

$\therefore \{\sin 0, \sin\frac{3\pi}{2}, \tan\frac{5\pi}{4} = \{0, -1, 1\}$

14. $S_1 \times S_2$ has $20 \times 4 = 80$ elements

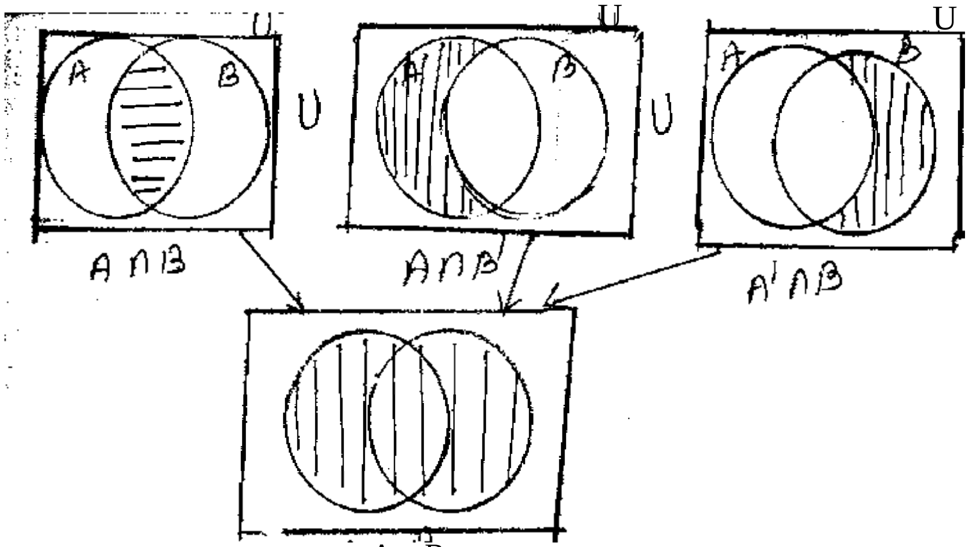
$S_1 \times S_3$ has $20 \times 4 = 80$ elements

No of comman elements = 20 × 2 = 40

∴ (b and d are two comman elements)

∴ No of elements of $(S_1 \times S_2) \cup (S_1 \times S_3)$ = 80 + 80 – 40 = 160 – 40 = 120

15. **Figure 1**



A ∪ B

16. If x be the number of news papers then $\frac{x}{5} \times 80 = 400$

$$\therefore x = \frac{400 \times 5}{80}$$

∴ $x = 25$

17. $x^2 = 1 \Rightarrow x = \pm 1$ ∴ A = {–1, 1}

Now $x^4 = 1 \Rightarrow x = \pm 1$ or $x = \pm i$

18. $4x^2 + 9y^2 - 32x - 54y + 109 \le 0$

$\Rightarrow 4(x^2 - 8x) + 9(y^2 - 6y) + 109 \le 0$

$\Rightarrow 4(x - 4)^2 + 9(y - 3)^2 \le 36$

$$\Rightarrow \left(\frac{x-4}{3}\right)^2 + \left(\frac{y-3}{2}\right)^2 \le 1$$

$$B = \{(x, y) / \left(\frac{x-4}{3}\right)^2 + \left(\frac{y-3}{2}\right)^2 \le 1, x, y \in R\}$$

Suppose $(x, y) \in A$

∴ $|x - 3| < 1$

$\Rightarrow -1 < x - 3 < 1$

$\Rightarrow -2 < x - 4 < 0$

$\Rightarrow 0 < (x - 4)^2 < 4$

$\Rightarrow 0 < \left(\frac{x-4}{3}\right)^2 < \frac{4}{9}$

Simillarly $0 \leq \left(\frac{y-3}{2}\right)^2 < \frac{1}{4}$

$\therefore \left(\frac{x-4}{3}\right)^2 + \left(\frac{y-3}{2}\right)^2 < \frac{4}{9} + \frac{1}{4} = \frac{25}{36}$

$\therefore \left(\frac{x-4}{3}\right)^2 + \left(\frac{y-3}{2}\right)^2 < 1$

$\therefore (x, y) \in B$

$\therefore A \subset B$

19. If $n = 1$ then $n^3 + (n + 1)^3 + (n + 2)^3 = 1^3 + 2^3 + 3^3$

$= 36$

$= 9 \times 4$

If $n = 2$ $n^3 + (n + 1)^3 + (n + 2)^3 = 9 \times 11$

If $n = 3$ $n^3 + (n + 1)^3 + (n + 2)^3 = 9 \times 24$

$\therefore n^3 + (n + 1)^3 + (n + 2)^3$ is a multiple of 9

$\therefore A \subset B$

20. U = {1, 2, 3}

$\therefore$ P(U) = {{1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3} ϕ}

and A = {1, 2}

Now [P(A)]' = {{3}, {2, 3}, {1, 3}, {1, 2, 3}}

21. Here $n(A) > n(B)$

$\Rightarrow n(A) - n(B) > 0$

$\Rightarrow n(A) - n(B) \leq n(A - B) \leq n(A)$

$\Rightarrow 2 \leq n(A - B) \leq 6$

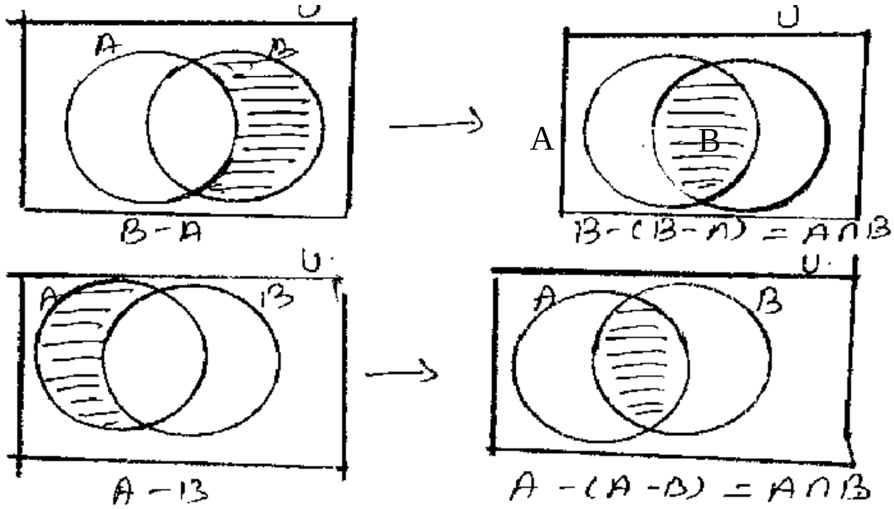
$\therefore$ minimum value of $n(A - B)$ is 2

22. Here $A \cap B = \{x \in R / f(x) = 0 \text{ and } g(x) = 0\}$

$= \{x \in R \mid f^2(x) + g^2(x) = 0\}$

23. We know that $n[(A \times B) \cap (B \cap A)] = [n(A \cap B)]^2 = [2]^2 = 4$

24.

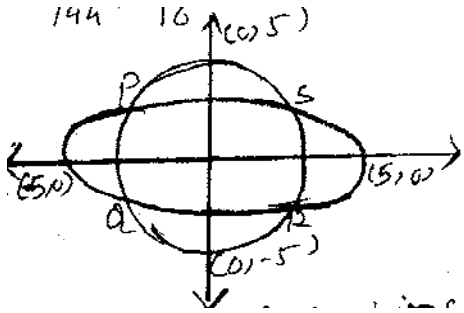


25. $x^2 + 9y^2 = 144$

$\Rightarrow \dfrac{x^2}{144} + \dfrac{y^2}{16} = 1$

$x^2 + y^2 = 25$

$\therefore\ x^2 + y^2 = 5^2$

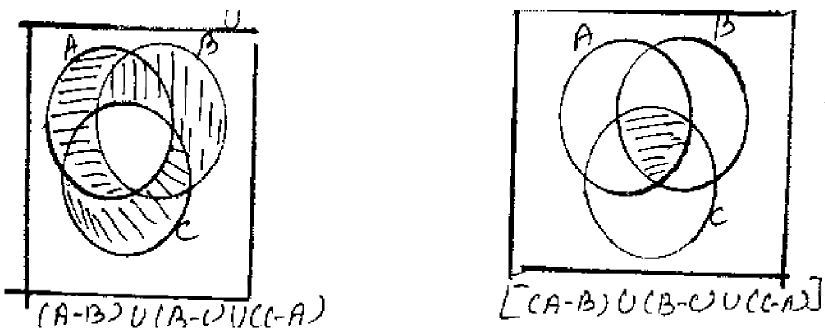


$\therefore$ clearly $A \cap B$ contains four points

26. We know that $A \cup (B \cap B') = A$

$A' \cup (A \cup (B \cap B')) = A' \cup A = N$

27.



U

28. $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$

$= (A \cup B \cup C) \cap (A' \cup B \cup C) \cap C'$

$= [(A \cap A') \cup (B \cup C)] \cap C'$

$= [\phi \cup (B \cup C)] \cap C' = (B \cup C) \cap C'$

$= (B \cap C') \cup (C \cap C')$

$= (B \cap C') \cup \phi$

$= B \cap C'$

29. y = e^x and y = e^{-x}

$\Rightarrow e^x = e^{-x} \Rightarrow e^{2x} = 1$

$\Rightarrow 2x = 0 \Rightarrow x = 0$

$\therefore\ y = e^0 = 1$

$\therefore$ A and B meet at (0, 1)

Hence $A \cap B \neq \phi$

30. $x^2 - 6x + 5 = 0$

$\therefore\ x = 5,\ x = 1$

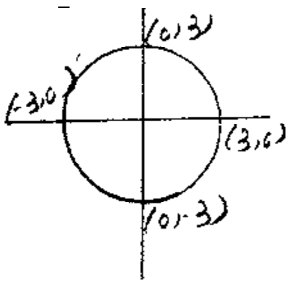
$\therefore$ A = {1, 5}

$\therefore$ A' = (1, 5)

31. Since R is reflexive relation on A

$\therefore\ (a, a) \in R$ for all $a \in A$

Fig



The minimum number of ordered pair in R is n $\therefore\ m \geq n$

32. R = {(2, 2), (3, 5), (4, 10), (5, 17), (6, 25)}

33. (i) $x \subset x$ ($\because$ every set is a subset of it self Reflexivity is trues)

(ii) If $x \subset y$ and $y \subset x$ then x = y $\therefore$ Antisymmetric hold

(iii) $x \subset y,\ y \subset z \Rightarrow x \subset z$ Transitivity hold

34. $x^2 + y^2 = 9,\ \ x = \sqrt{9 - y^2}$

R = {(0, 3), (0, –3), (3, 0), (–3, 0)}

$\therefore$ Domain ={–3, 0, 3}

$\therefore$ Range = {–3, 0, 3}

35. T is an equivalence relation

S = {1, 2} $\therefore$ (1, 1) $\notin$ S

$\therefore$ S is not reflexive

36. Here A = {2, 4, 6} and B = {2, 3, 5}

$\therefore$ A × B contains 3 × 3 = 9 elements

$\therefore$ No of relations = 2^9

37. $(x, x) \in$ R for $x \in$ W $\Rightarrow$ R is reflexive Let $(x, y) \in$ R $\Rightarrow$ $(y, x) \in$ R

[$\because$ x, y have at least one letter in common]

$\Rightarrow$ R is symmetric

But R is not transitive

($\because$ X = MITESH, Y = MUMBAI, Z = NAYAN Then $(x, y) \in$ R, $(y, z) \in$ R but $(x, z) \notin$ R)

38. R is reflexive if $(a, a), (b, b), (c, c) \in$ R

If (b,a) , (c, b) $\in$ R, then R is symmetric

Now if $(c, a), (a, c)$ R then R is transitive

39. R is Reflexive

Now (a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow a + d = b + c, c + f = d + e$

adding $a + d + c + f = b + c + d + e$

$\Rightarrow a + f = b + e$

$\Rightarrow$ (a, b) R (e, f)

$\Rightarrow$ R is transitive

40. $a\mathrm{R}_1\ a \Leftrightarrow |a| = |a|$ $\quad\therefore$ R is Reflexive

$a\mathrm{R}_1\ b \Leftrightarrow |a| = |b| \Leftrightarrow |b| = |a| \Leftrightarrow b\mathrm{R}_1\ a$

$\therefore$ R_1 is symmetric

$a\mathrm{R}_1\ b$ and $b\mathrm{R}_1\ c \Rightarrow |a| = |b|$ and $|b| = |c|$

$\Rightarrow |a| = |c| \Rightarrow a\mathrm{R}_1\ c$

$\therefore$ R is transitive

41. R = $\{(x, y) \mid x, y \in \mathrm{I}\ x^2 + y^2 \leq 4\}$

= {(0, 0), (0, –1), (0, 1), (0, –2), ... (–2, 0)}

$\therefore$ Domain of R = {–2, –1, 0, 1, 2}

42. R is Reflexive and transitive

Here 6 is a multiple of 2 but 2 is not a multiple of 6.

$\therefore$ R is not symmetric

43. $(x, y) \in$ S $\Rightarrow x \geq y \Rightarrow y \leq x \Rightarrow (y, x) \notin$ S

44. S is Relflexive and symmetric

Now $(x, y) \in$ S, $(y, z) \in$ S

$\Rightarrow xy \geq 0, yz \geq 0$

$\Rightarrow (x, y)(y\ z) \geq 0$

$\Rightarrow xy^2z \geq 0$

$\Rightarrow xz \geq 0$ ($\because$ $y^2 \geq 0$)

$\Rightarrow (x, z) \in$ S

45. If we take

S = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)} and

S’ = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3) (3, 2)}

then S and S' are equivalence relation

46. $0^2 + 1^2 = 1$ and $0^2 + (-1)^2 = 1 \Rightarrow (0, 1) \in S_1$ and $(0, -1) \in S_1$

$\therefore$ S_1 is not function.

47. *n* is a factor of *m* but not necessary *m* is a factor of *n* so R is not symmetric

48. $(1, 2) \in$ R but $(1, 2) \notin$ (a) or (b) or (c)

49. If $x = 8 - 2y$ then R = {(2, 3), (4, 2), (6, 1)}

50. Suppose $x = \tan\theta$, $x \in (-1, 1) \Rightarrow \theta \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$

$\therefore \tan^{-1} x = \theta$ now $\dfrac{2x}{1+x^2} = \dfrac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$

$\theta \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right) \Rightarrow 2\theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$... ... ... (1)

$\therefore f(x) = \sin^{-1}(\sin 2\theta)$

$= 2\theta$ (by(1))

$\therefore f(x) \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

51. Let $f(x) = a_0x^n + a_1x^{n-1} + ... + a_{n-1}\, x + a_n$

be a polynomial of degree *n*.

Since $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$

we have $[a_0x^n + a_1x^{n-1} + ... + a_n^{-1}\, x + a_n]\left[\frac{a_0}{x^n} + \frac{a_1}{x^{n-1}} + ... + \frac{a_{n-1}}{x} + an\right]$

$= a_0x^n + a_1x^{n-1} + ... + a_{n-1}\, x + a_n + \dfrac{a_0}{x^n} + \dfrac{a_1}{x^{n-1}} + ... + \dfrac{a_{x-1}}{x} + an$

comparing co-efficient to like power of *x*

we have $a_n = 1$ and $a_{n-1} = a_{n-2} = a_{n-3} = ... = a_1 = 0$

Also $a_0{}^2 = a_n{}^2$

$\therefore a_0{}^2 = 1$

$\therefore a_0 = \pm 1$

$f(x) = x^n + 1$ or $f(x) = -x^n + 1$

If $f(x) = x^n + 1$ then $f(3) = 3^n + 1 = 28 \Rightarrow n = 3$

If $f(x) = -x^n + 1$ then $f(3) = -3^n + 1 = 3^n = -27$

which is not possible

$\therefore f(x) = x^3 + 1$

$\therefore f(4) = 4^3 + 1 = 65$

52. Let $\dfrac{x^2 + 34x - 71}{x^2 + 2x - 7} = y$

$\Rightarrow x^2(1 - y) + 2(17 - y)x + (7y - 71) = 0$

$\therefore \Delta \geq 0$

$\Rightarrow (y - 9)(y - 5) \leq 0$

$\Rightarrow y \leq 9$ or $y \geq 5$

53. Now take $x = 0 = y$

then $f(0) = f(0)f(0) - f(3)(3)$

$1 = (1)(1) - (f(3))^2$

$\therefore f(3) = 0$ ... ... (1)

$f(6 - x) = f(3 - (x - 3))$

$= f(3)f(x - 3) - f(0)f(x)$

$= -f(x)$ by (1)

54. $f(x) = \dfrac{x}{x-1} \Rightarrow \mathrm{f(f(x))=f}\left(\dfrac{\mathrm{x}}{\mathrm{x}-1}\right) = \mathrm{x}$

$\therefore f(f(f(x))) = f(x) = \dfrac{x}{x-1}$

$\therefore \left(\text{fofof ... f}_{\text{(17 times)}}\right)(x) = \dfrac{x}{x-1}$

55. $\left[\dfrac{1}{5} + \dfrac{\mathrm{x}}{1000}\right] = \begin{cases} 0 \text{ if } 1 < \mathrm{x} < 800 \\ 1 \text{ if } 800 < \mathrm{x} < 999 \end{cases}$

$\therefore$ the value of the given expression = 200

56. $f(2x + 3y, 2x - 3y) = 24xy = (2x + 3y)^2 - (2x - 3y)^2$

$\therefore f(x, y) = x^2 - y^2$

57. $\sum_{r=1}^{n} f(r) = f(1) + f(2) + ... + f(n)$

$= f(1) + (f(1) + f(1)) + (f(2) + f(1)) + (f(3) + f(1)) + ... + (f(n - 1) + f(1))$

$= f(1) + 2f(1) + 3f(1) + 4f(2) + \ldots + nf(1)$

$= f(1)\ (1 + 2 + 3 \ldots n)$

$= \frac{3}{2}\ \frac{n\,(n+1)}{2}\ \left(\because f(1) = {}^{3}\!/_{2}\right)$

58. $\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & x(x-1)(x+1) \end{vmatrix}$ $C_{13}(-1); C_{23}(-1)$

$= 0$

59. Let $f(x) = y = 2^{x(x-1)}$

$\Rightarrow \log_2 y = x(x-1)\log_2 2$

$\Rightarrow x^2 - x = \log_2 y$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 = \frac{1 + 4\log_2 y}{4}$

$\Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{1 + 4\log_2 y}}{2}$

$\Rightarrow f^{-1}(x) = \frac{1 \pm \sqrt{1 + 4\log_2 x}}{2}$

$\Rightarrow f^{-1}(x) = \frac{1 + \sqrt{1 + 4\log_2 x}}{2}$ ($f^{-1}(x)$ 1)

60. If $[x]$ is greatest integer then $x - [x] = \frac{p}{q}$

Where p and q are positive integers and $p < q$ so $p + p < p + q$

$\Rightarrow \frac{p}{p+q} < \frac{1}{2}$

$\therefore f(x) = \frac{p}{p+q} < \frac{1}{2}$

61. $f(-x) = \sec(-x) + \tan(-x)$

$= \sec x - \tan x$

$\neq -f(x)$

$\therefore$ f is neither odd function nor an even function.

$g(-x) = -g(x)$ $\therefore$ g is an odd function.

62. $\log_{0.3}(x - 1) < 0$ for $x > 2$ also $-x^2 + 2x + 8 > 0$
if and only if $x \in (-2, 4)$, Hence the domain of the given function is (2, 4)

63. $f(x) = \dfrac{(\sin\alpha)(x)}{x+1}$

$\Rightarrow \text{fof}(x) = \dfrac{x\sin^2\alpha}{x(\sin\alpha+1)+1}$

but fof(x) = x is given

$\therefore \dfrac{x\sin^2\alpha}{x(\sin\alpha+1)+1} = x$

$\therefore (\sin\alpha + 1)\, x\, (\sin\alpha - 1 - x) = 0$

$\therefore \sin\alpha = -1$ or $x = 0$ or $\sin\alpha = 1 + x$

but $f(x) \neq 0$, $f(x) \neq I(x)$ $\quad \therefore \sin\alpha = -1$

64. $\text{fog}(x) = f(g(x)) = f(x^3 + 5) = 2x^3 + 7$

now $y = 2x^3 + 7$

$\Rightarrow x^3 = \dfrac{y-7}{2}$

$\Rightarrow x = \left(\dfrac{y-7}{2}\right)^{\frac{1}{3}}$

65. $|\cos x| + \cos x > 0 \Rightarrow \cos x > 0$

$\Rightarrow 2n\pi - \dfrac{\pi}{2} < x < 2n\pi + \dfrac{\pi}{2}$

$\Rightarrow \dfrac{(4n-1)\pi}{2} < x < \dfrac{(4n+1)\pi}{2}$

66. $f(x) = \sin^2 x + \left(\sin x\cos\dfrac{\pi}{3} + \cos x\sin\dfrac{\pi}{3}\right)^2 + \cos x\left(\cos x\cos\dfrac{\pi}{3} - \sin x\sin\dfrac{\pi}{3}\right)$

$= \sin^2 x + \left(\dfrac{\sin x}{2} + \dfrac{\sqrt{3}}{2}\cos x\right)^2 + \cos x\left(\dfrac{\cos x}{2} - \dfrac{\sqrt{3}}{2}\sin x\right)$

$= \dfrac{5}{4}\sin^2 x + \dfrac{5}{4}\cos^2 x = \dfrac{5}{4}$

67. We know that if $f(x) = \dfrac{1-x}{1+x}$ then fof(x) = x

68. $f(x + y) + f(x - y) = \frac{3^{x+y} + 3^{-x-y}}{2} + \frac{3^{x-y} + 3^{-x+y}}{2}$

$= \frac{1}{2}\left(\frac{3^x + 3^{-x}}{2}\right)\left(\frac{3^y + 3^{-y}}{2}\right)$ $= \frac{1}{2} f(x)f(y)$

69. ${}_{5-x}P_{x-1} = \frac{(5-x)!}{(6-2x)!}$

Now $5 - x > 0$, $x - 1 \geq 0$ and $5 - x \geq x - 1$

$\Rightarrow x < 5, x \geq 1, x \leq 3$

$\therefore\ x = 1, 2, 3$

70. We have $f(x) = \sin x - \sqrt{3} \cos x + 1$

$= 2\left(\sin x.\frac{1}{2} - \cos x\frac{\sqrt{3}}{2}\right) + 1$

$= 2\sin\left(x - \frac{\pi}{3}\right) + 1$

71. f is define when $(x \neq 2)$ and $x^3 - x > 0$

i.e. $x(x^2 - 1) > 0 \Rightarrow x > 0$ and $x^2 > 1$ or $x < 0$ and $x^2 - 1 < 0$

$\Rightarrow x > 0$ and $x > 1$ or $x < 0$ and $x > -1$

$\therefore$ Domain of $f = (-1, 0) \cup (1, 2) \cup (2, \infty)$

72. $\left|\frac{x}{x+1}\right| < 10^{-5}$

$\Rightarrow \left|\frac{x+1-1}{x+1}\right| < 10^{-5}$

$\Rightarrow 1 - 10^{-5} < \frac{1}{x+1} < 1 + 10^{-5}$

$\Rightarrow \frac{100000}{99999} > x + 1 > \frac{100000}{100001}$

$\Rightarrow \frac{-1}{100001} < x < \frac{1}{99999}$

$\Rightarrow -(100001)^{-1} < x < (99999)^{-1}$

73. Taking $x = y = 1$ we get

$f(1)f(1) - f(1) = 1 + 1 \Rightarrow f(1)^2 - f(1) - 2 = 0$

$\Rightarrow f(1) = 2 \quad (\because f(1) > 0)$

Taking $y = 1$ we get

$f(x)f(1) - f(x) = x + 1 \Rightarrow 2f(x) - f(x) = x + 1$

$\therefore f(x) = f(x + 1)$

74. $\cos x \geq \frac{1}{2}$, $(6 - x)(1 + 6x) > 0$ or $\cos x \leq \frac{1}{2}$, $(6 - x) \bullet (1 + 6x) < 0$

$$\therefore x \in \left(\frac{-1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$$

75. $2^m = 2^n + 48$

$\Rightarrow 2^n (2^{m-n} - 1) = 2^4 \times 3$

$\Rightarrow 2^{n-4} (2^{m-n} - 1) = 3$

since 3 is a prime number $\Rightarrow n = 4$

$\Rightarrow m = 6$

76. $-1 \leq \log_3 \frac{x}{4} \leq 1$

$\Rightarrow -\log 3 \leq \log \frac{x}{4} \leq \log 3$

$\Rightarrow \frac{1}{3} \leq \frac{x}{4} \leq 3$

$\Rightarrow \frac{4}{3} \leq x \leq 12$

77. f is not defined for $-4 \leq x \leq 3$ and g is not defined for $x^2 - 16 \leq 0$

i.e. f and g are not defined on $[-4, 3]$

78. $f(x) = x^4 + 2 = y \Rightarrow x = (y-2)^{\frac{1}{4}}$

$\Rightarrow f^{-1}(x) = (x-2)^{\frac{1}{4}}$

$\Rightarrow f^{-1}(83) = (81)^{\frac{1}{4}} = \pm 3$ and $f^{-1}(-2) = (4)^{\frac{1}{4}} = \phi$

79. $21 - x > 0$, $3x - 1 \geq 0$, $21 - x \geq 3x - 1$ and

$25 - 3x > 0$, $5x - 3 \geq 0$, $25 - 3x \geq 5x - 3$

$\Rightarrow \frac{3}{5} < x \leq \frac{7}{2}$

Hence domain of $f = \{1, 2, 3\}$

80. There ares three posibility

(i) (a, a), (b, c), (c, d), (d, b)

(ii) (a, a), (b, d), (c, c), (d, b)

(iii) (a, a), (b, d), (c, d), (d, c)

81. $\cos(\log x) \times \cos(\log y) - \frac{1}{2}\left[\cos\left(\log\left(\frac{x}{y}\right) + \cos(\log(xy))\right)\right]$

($\because$ $\cos(x - y) = \cos x \cos y + \sin x \sin y$), ($\cos(x + y) = \cos x \cos y - \sin x \sin y$)

82. $a^y = a - a^x >$ as expotenial function a^y is always positive

$\therefore\ a^x < a$

$\Rightarrow x < 1 = 1\ x \in (-\infty, 1)$

83. Case : 1 $x \le -2$ then $-x \ge 2 \Rightarrow 2 - x \ge 4,\ 2 - x \ge 2 + x$

$(\therefore 2 + x \le 0)$

$\therefore \max\{2 - x, 2 + x, 4\} = 2 - x$

Case : 2 $-2 < x < 2$ then $2 > -x > -2 \Rightarrow -2 < -x < 2$

$\Rightarrow 0 < 2 - x < 4$

Now $-2 < x < 2 \Rightarrow 0 < 2 + x < 4$

$\therefore \max\{2 - x, 2 + x, 4\}$ is 4

Case : 3

Now $x \ge 2 \Rightarrow x + 2 \ge 2 + 2$

$\Rightarrow x + 2 \ge 4$

Now $x \ge 2 \Rightarrow -x \le -2 \Rightarrow 2 - x \le 0$

$\therefore \max\{2 - x, 2 + x, 4\}$ is $2 + x$

84. take $y = \frac{7^x - 7^{-x}}{7^x + 7^{-x}} \Rightarrow 7^{2x} = \frac{y + 1}{1 - y} \Rightarrow x = \frac{1}{2}\log\frac{y + 1}{1 - y}$

85. Principal period of $f(x) = \frac{\pi}{\sqrt{m}}$ but period of $f(x)$ is π

$\therefore\ \frac{\pi}{\sqrt{m}} = \pi \Rightarrow 1 = \sqrt{m} \Rightarrow m = 1$

$\Rightarrow [p] = 1 \Rightarrow 1 \le p < 2$

86. Function $f(x) = 3\sin\left[\sqrt{\frac{\pi^2}{9} - x^2}\right]$ given f is defined only if $\frac{\pi^2}{9} - x^2 \ge 0$

$\Rightarrow x^2 \le \frac{\pi^2}{9}$

$\Rightarrow |x| \le \frac{\pi}{3}$

since $f\left(\frac{-\pi}{3}\right) = 0$ $f(0) = 3 \sin \frac{\pi}{3} = \frac{3}{2}\sqrt{3}$

$\left[0, \frac{3\sqrt{3}}{2}\right]$

87. The domain of $f(x) = \log_5 [\log_6[\log_8 x]]$ is defined if $\log_6 [\log_8 x] > 0$
$\Rightarrow \log_8 x > 6^0$
$\Rightarrow x > 8$

88. $g(x) = 1 + x - [x] = 1 + \{x\}$ where $\{x\}$ is fractional part of $x > 0$ Hence $f(g(x)) = 1$
for $x \in R$

89. For Df, $|x| - x > 0 \Rightarrow |x| > x$ i.e. $x < |x|$
Which is true if $x < 0$ $D_f = (-\infty, 0)$

90. $(fog)(x) = 2[g(x)]$ and $(g + g)(x) = 2[g(x)]$
$\therefore$ $fog(x) = (g + g)(x)$

91. since domain of f and domain of composite function gof are same (–1, 1)

92. $f(x) = 6^x + 3^x + 6^{-x} + 3^{-x} + 2 = \left(6^x + \frac{1}{6^x}\right) + \left(3^x + \frac{1}{3^x}\right) + 2$

$\ge 2\sqrt{6^x \frac{1}{6^x}} + 2\sqrt{3^x \frac{1}{3^x}} + 2$ ($\because$ AP > GP)

$= 2 + 2 + 2 = 6$

93. Here $2f(x) + f(1 - x) = x^2$ ... ... (1)
Now put $1 - x$ in place of x we get
$2f(1 - x) + f(x) = (1 - x)^2$ ... ... (2)

multiplying (1) and (2) we get $f(x) = \frac{x^2 + 2x - 1}{3}$

94. Here $3f(x) + 2f\left(\frac{x + 59}{x - 1}\right) = 10x + 30$ ... ... (1)

Now put $x = 7$ and $x = 11$ we get two equation and solved them we get $f(7) = 4$

95. Here $f(x) = [x] + \sum_{r=1}^{100} \frac{x+r}{100}$

$= [x] + \frac{1}{100} \sum_{r=1}^{100} ((x + r) - [x] - r)$ $(\because\ r\ )$

$= [x] + x - [x]$ $\because \left[\frac{1}{100}\sum_{i=1}^{100} 1 = 1\right]$ $= x$

96. $f(3 + \sqrt[3]{x}) = 2 - \sqrt[3]{x} + x$ now take $3 + \sqrt[3]{x} = y$

then $x = (y - 3)^3$ and $\sqrt[3]{x} = y - 3$

$\therefore\ f(y) = 2 - (y - 3) + (y - 3)^3$

97. for (0, 1), (0– 1) R_1 is not one-one

for (1, e), (–1, e) R_2 is not one-one

for (0, 3), (3, 3) R_3 is not one-one

98. Here $f^{-1}(-2) = \{x \in R \mid f(x) = -2\}$

$= \{x \in R \mid x^2 + 1 = -2\} = \phi$

$f^{-1}(17) = \{x \in R \mid f(x) = 17\}$

$= \{x \in R \mid x^2 = 16\}$

$= \{\pm 4\}$

99. suppose $y = \sin^{-1} \{4 - (x - 7)^3\}^{\frac{1}{5}}$

$\therefore\ 4 - \sin^5 y = (x - 7)^3$

$\therefore\ x = 7 + (4 - \sin^5 x)^{\frac{1}{3}}$

$\therefore\ f^{-1}(x) = 7 + \left(4 - \sin^5 x\right)^{1/3}$

100. $y = \sin\left(\tan^{-1} x\right) \Rightarrow y = \sin\left(\sin^{-1} \frac{x}{\sqrt{1+x^2}}\right) = \frac{x}{\sqrt{1+x^2}}$

$\Rightarrow x = \frac{y}{\sqrt{1-y^2}}$

$\Rightarrow f^{-1}(x) = \frac{x}{\sqrt{1-x^2}}$

$y = f(x) = 1 - 2^{-x}$ $\therefore\ x = \log_2 \frac{1}{1-y} = -\log_2(1-y)$

$y = 2\times + 3 \Rightarrow x = \frac{y-3}{2}$

Answers

1	a	26	d	51	a	76	d
2	c	27	b	52	c	77	a
3	b	28	a	53	d	78	b
4	a	29	b	54	a	79	a
5	a	30	b	55	d	80	c
6	d	31	a	56	c	81	c
7	d	32	a	57	c	82	d
8	a	33	d	58	c	83	d
9	a	34	d	59	b	84	a
10	d	35	a	60	b	85	c
11	d	36	a	61	c	86	c
12	a	37	b	62	a	87	b
13	c	38	c	63	d	88	b
14	b	39	d	64	d	89	c
15	b	40	a	65	a	90	c
16	a	41	c	66	a	91	a
17	c	42	b	67	c	92	d
18	a	43	a	68	d	93	c
19	a	44	a	69	b	94	b
20	b	45	b	70	d	95	d
21	a	46	a	71	a	96	b
22	d	47	d	72	b	97	d
23	c	48	d	73	c	98	a
24	b	49	c	74	d	99	b
25	d	50	b	75	b	100	c

●●●

Unit - 2
Complex Numbers
Important Points

Complex number : A number of form x + iy where x, y $\in$ R and $i=\sqrt{-1}$ us called a complex number.

A complex number may also be defined as an order pair $z=(x_1y)$ of real number

$z=x+iy=(x_1y)$

$\rightarrow$ $i=\sqrt{-1}$ is called an imaginary number

$\rightarrow$ $i^2=-1,\quad i^3=-i\ ,\ i^4=1$

$\rightarrow$ $i=-\frac{1}{i}$

$\rightarrow$ $\sqrt{-a}\times\sqrt{-b} \quad = i\sqrt{a}\times i\sqrt{b} \quad =-\sqrt{ab}$

$\rightarrow$ Set of complex number is denoted by C

$N\subset Z\subset Q\subset R\subset C$

$\rightarrow$ z = x + iy

x is called real part of z = Re(z)

y is called imaginary part of z = Im(z)

z = x + iy= Re(z) + i Im(z)

If x = 0, $y\neq 0$ then z = iy (Purely imaginary)

If $x\neq 0$, y = 0 then z = x (Purely real)

$\rightarrow$ $z_1 = z_2$ i.e. $x_1 + iy_1 = x_2+iy_2 \Leftrightarrow x_1=x_2,\ y_1=y_2$

$\rightarrow$ Algebra of complex numbers :

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ be two complex numbers, $x_1, y_1\ x_2, y_2\in R$ then

(1) $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

(2) $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

(3) $z_1z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$

(4) $\frac{z_1}{z_2}=\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}+i\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}\ ,\ x_2^2+y_2^2\neq 0$

$\rightarrow$ Conjugate of complex number.

conjugate of complex number z = a + ib is defined as $\bar{z}=a-ib$

properties : Let z = x + iy

(I) $\overline{(\bar{z})}=Z$

(II) $z=\bar{z}\Leftrightarrow y=0$

(III) $z = -\bar{z} \Leftrightarrow x = 0$

(IV) $z + \bar{z} = 2\,\text{Re}(z)$, $\quad z - \bar{z} = 2\,\text{Im}(z)$

(V) $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

(VI) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

(VII) $\overline{\left(\frac{z_1}{z_2}\right)} = \overline{z_1} / \overline{z_2}$, $\qquad z_2 \neq 0$

(VIII) If $z = f(z_1)$ $\qquad$ then $\bar{z} = f(\overline{z_1})$

(IX) $\left(\overline{z^n}\right) = \left(\bar{z}\right)^n$

(X) $z_1\overline{z_2} + z_2\overline{z_1} = 2\,\text{Re}(\overline{z_1}z_2) = 2\,\text{Re}(z_1\overline{z_2})$

→ modulus (absolute value) of complex number. If $z = x + iy$ be complex number then its modulus denoted by $|z|$ or r defined as $r = |z| = \sqrt{x^2 + y^2}$

<u>properties :</u>

let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ be two complex numbers, then

(1) $|z| \geq 0$

(2) $|z| = 0 \Leftrightarrow z = 0$

(3) $|z| = |\bar{z}| = |-z| = |-\bar{z}|$

(4) $z\bar{z} = |z|^2$

(5) $-|z| \leq \text{Re}(z) \leq |z|$ and $-|z| \leq \text{Im}(z) \leq |z|$

(6) $|z^n| = |z|^n$

(7) $|z_1 z_2| = |z_1||z_2|$

(8) $\left|\frac{z_1}{z_2}\right| = |z_1| / |z_2|$

(9) $|z_1 + z_2| \leq |z_1| + |z_2|$

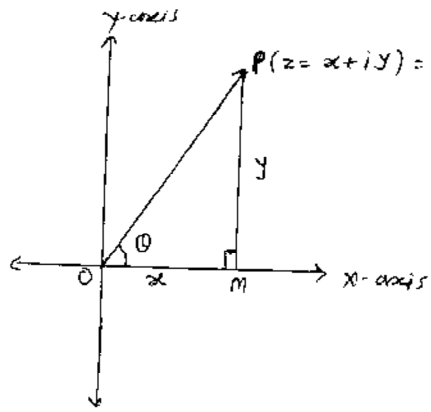
(10) $|z_1 - z_2| \geq |z_1| \sim |z_2|$

(11) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\,\text{Re}(z_1\overline{z_2})$

(12) $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\,\text{Re}(z_1\overline{z_2})$

(13) $|z_1+z_2|^2+|z_1-z_2|^2 = 2\left(|z_1|^2+|z_2|^2\right)$

→ If z = x + iy is a non zero complex number, which is represented by p(x,y) in the argand plane



→ $|z| = r = op = \sqrt{x^2+y^2}$

→ Argument (Amplitude) of z is the angle, which is $\overline{OP}$ makes with the +ve direction of x-axis. It is denoted by arg(z) (or Amp(z)) i.e. $\theta = \arg(z)$

→ (i) $P(z = x+iy) = P(x,y)$ lies in I'st guadrant then $\theta = \arg(z) = \tan^{-1} y/x$ $(x > 0, y > 0)$

(ii) $P(z = x + iy) = P(x,y)$ lies in II'nd guadrant then $\theta = \arg(z) = \pi - \tan^{-1} y/|x|$ $(x < 0, y > 0)$

(iii) $P(z = x + iy) = P(x,y)$ lies in III'rd guadrant then $\theta = \arg(z) = -\pi + \tan^{-1}\left(|y|/|x|\right)$ $(x < 0, y < 0)$

(iv) $P(z = x + iy) = P(x,y)$ lies in IVth guadrant then $\theta = \arg(z) = -\tan^{-1}\left(\frac{|y|}{x}\right)(x>0, y<0)$

If θ is principal argument of z then $\theta \in (-\pi, \pi]$

→ Properties of arguments :

(1) $\arg(z_1z_2) = \arg(z_1) + \arg(z_2)$

(2) $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

(3) $\arg\left(z/\bar{z}\right) = 2\arg(z) = \arg(z^2)$

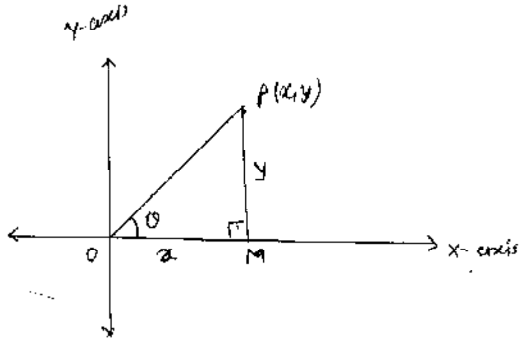
(4) $\arg(z^n) = n \arg z$

(5) $\arg \bar{z} = -\arg(z)$

(6) If $\arg\left(\frac{z_2}{z_1}\right) = \theta$ then $\arg\left(\frac{z_1}{z_2}\right) = 2k\pi - \theta$, $k \in z$

→ Polar form of a complex number :

Let z = x + iy be a complex number represented by the point P(x,y) in Argand plane From figure



$OP = r = |z| = \sqrt{x^2 + y^2}$

OM = x, PM = y and $m < XOP = \theta$ then z = x +iy

$z = r(\cos\theta + i\sin\theta)$. This form of z is called polar or trigonometric form.

$z = x + iy = r(\cos\theta + i\sin\theta)$

$x = r\cos\theta,\ \ y = r\sin\theta \qquad \theta \in (-\pi, \pi]$

→ z = x + iy

$= r\cos\theta + i\,r\sin\theta = r.e^{i\theta}$ is called exponential form of a complex number z

→ DE-MOIVER'S THEOREM :

$*$ If n is an integer, then $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$*$ If n is not an integer, then $\cos n\theta + i\sin n\theta$ is one of the value of $(\cos\theta + i\sin\theta)^n$

→ Euler's Theorem : $e^{i\theta} = \cos\theta + i\sin\theta$

→ Logarithm of complex number z = x + iy

$* \log(x + iy) = \frac{1}{2}\log\left(x^2 + y^2\right) + i\tan^{-1}\left(\frac{y}{x}\right), x \neq 0$

i.e.logz = log |z| + iarg (z)

$*$ $\log(iy) = \log y + i\,\pi/2$

$*$ $i = \cos\pi/2 + i\sin\pi/2 = e^{i\pi/2}$

$\log i = i\,\pi/2$

$\log(\log i) = \log i + \log(\pi/2) = i\frac{\pi}{2} + \log(\pi/2)$

→ Roots of Unity :

$*$ cube root of unity

$z = \sqrt[3]{1}$

$\therefore z^3 - 1 = 0$

$\therefore (z-1)\ (z^2+z+1)=0$

$\therefore z=1,\ \ z=\dfrac{-1+\sqrt{3}i}{2},\ \ z=\dfrac{-1-\sqrt{3}i}{2}$

take $w=\dfrac{-1+\sqrt{3}i}{2}$ then $w^2=\dfrac{-1-\sqrt{3}i}{2}$

$\therefore$ The cube roots of unity are 1, w, w^2

$*$ Properties :

(i) $1 + w + w^2 = 0$

(ii) $w^3 = 1$

(iii) $w^{3n} = 1,\quad w^{3n+1} = w,\quad w^{3n+2} = w^2$

(iv) $w^2=\overline{w}$, $\left(\overline{w}\right)^2=w$

(v) if $a + bw + cw^2 = 0$, then a=b=c, $a, b, c \in R$

→ Square root of complex number :

Let z = x +iy be complex number then

$$\sqrt{z}=\pm\left[\sqrt{\frac{|z|+x}{2}}+i\sqrt{\frac{|z|-x}{2}}\right]\quad \text{for } y>0$$

$$=\pm\left[\sqrt{\frac{|z|+x}{2}}-i\sqrt{\frac{|z|-x}{2}}\right]\quad \text{for } y<0$$

→ Geometry of complex numbers :

(1) Distance formula :

If $P(z_1)$ & $Q(z_2)$ be two distinct points in argand plane then

$PQ=|z_1-z_2|$

(2) Three points $P(z_1)$, $Q(z_2)$ & $R(z_3)$ are collinear if there exists a relation $az_1+bz_2+cz_3=0$ ($a, b, c \in R$), such that a + b + c = 0

(3) Equation of straight line :

Equation of line through $P(z_1)$ & $Q(z_2)$ is given by

$$\frac{z-z_1}{z_2-z_1}=\frac{\overline{z}-\overline{z_1}}{\overline{z_2}-\overline{z_1}}$$

OR
$$\begin{vmatrix} z & \overline{z} & 1 \\ z_1 & \overline{z_1} & 1 \\ z_2 & \overline{z_2} & 1 \end{vmatrix}=0$$

(4) The equation of perpendicular bisector of the line segmnet joining $P(z_1)$ & $Q(Z_2)$ is

$$z\left(\overline{z_1}-\overline{z_2}\right)+\overline{z}\left(z_1-z_2\right)=|z_1|^2-|z_2|^2$$

(5) Equation of a circle :

* equation of circle with center z_1 and radius r is $|z - z_1| = r$

* $\left|\frac{z - z_1}{z - z_2}\right| = k$ replesents a st. line if k = 1, and represents a circle if $k \neq 1$

* The equation $|z - z_1|^2 + |z - z_2|^2 = k$ represent circle if $k \geq \frac{1}{2}|z_1 - z_2|^2$

* If $A(z_1)$ & $B(z_2)$ are and points of diameter than equ. of circle is

$(z - z_1)(\overline{z} - \overline{z_2}) + (z - z_2)(\overline{z} - \overline{z_1}) = 0$

* Equation of the circle passing through three points $A(z_1)$, $B(z_2)$, $C(z_3)$ is

$$\left(\frac{z - z_3}{z - z_1}\right)\left(\frac{z_2 - z_1}{z_2 - z_3}\right) = \left(\frac{\overline{z} - \overline{z_3}}{\overline{z} - \overline{z_1}}\right)\left(\frac{\overline{z_2} - \overline{z_1}}{\overline{z_2} - \overline{z_3}}\right)$$

* If four points $A(z_1)$, $B(z_2)$, $C(z_3)$, $D(z_4)$ are concyclic then

$\left(\frac{z_1 - z_2}{z_1 - z_4}\right)\left(\frac{z_3 - z_4}{z_3 - z_2}\right)$ is purely real

(6) If $|z - z_1| + |z - z_2| = 2a$, where $2a > |z_1 - z_2|$ then z describes an ellipse with foci z_1 and z_2, $a \in R$

(7) If $|z - z_1| - |z - z_2| = 2a$, where $2a < |z_1 - z_2|$ then z describes a hyperbola with foci z_1 & z_2, $a \in R$

→ Multiplicative Inverse of a non-zero complex number

let z = a + ib = (a,b) $\neq 0$ then

$$z^{-1} = \frac{1}{z} = \frac{a}{a^2 + b^2} - i\frac{b}{a^2 + b^2}$$

Question Bank

1. If z = x+iy, x,y $\in$ R and 3x+(3x-y) i=4-6 i then z = _____

   (a) $\frac{4}{3}$ + i 10 (b) $\frac{4}{3}$ - i 10 (c) $\frac{-4}{3}$ + i 10 (d) $\frac{-4}{3}$ - i 10

2. Evahate $\left[i^{19}+\left(\frac{1}{i}\right)^{25}\right]^2$

   (a) 4 (b) -4 (c) 5 (d) -5

3. $i^1+i^2+i^3+i^4+......+i^{1000}=$ ______

   (a) -1 (b) 0 (c) 1
   (d) None

4. The expresion of comlex mmber $\frac{1}{1+-\cos\theta-i\sin\theta}$ in the form a+ib is ________

   (a) $\frac{\text{Sin}\theta}{2(1+\cos\theta)}+i\frac{1}{2}$ (b) $\frac{1}{2}-i\frac{\sin\theta}{2(1+\cos\theta)}$

   (c) $\frac{1}{2}+i\frac{1}{2}\tan\frac{\theta}{2}$ (d) $\frac{1}{2}\tan\frac{\theta}{2}$ - $i\frac{1}{2}$

5 If $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$ then (x,y) = ______

   (a) (3,1) (b) (3,-1) (c) (-3,1) (d) (-3,-1)

6. If Z = $\frac{4+3i}{5-3i}$ then Z^{-1} = _________

   (a) $\frac{11}{25}$ - $\frac{27}{25}$ i (b) $\frac{-11}{25}-\frac{27}{25}i$ (c) $\frac{-11}{25}+\frac{27}{25}i$ (d) $\frac{11}{25}+\frac{27}{25}i$

7. If z = x + y i and l3zl = lz-4l then x^2 + y^2 + x = _______

   (a) 1 (b) -1 (c) 2
   (d) -2

8. lt z be a complex number an ol l z+3 l $\leq$ 8 Then the value of l z-2 l lies in ______

   (a) [-2,13] (b) [0,13] (c) [2,13] (d) [-13,2]

9. If the cube roots of unity are 1,w,w^2 then 1+w+w^2 = _______

   (a) 1 (b) 0 (c) -1 (d) w

10. The complex numbers sinx + i cos 2 x and cos x - i sin 2 x are conjugate of each other for _____

(a) $x = k\pi,\ k \in z$ (b) $x = o$ (c) $x = (k + \frac{1}{2})\ \pi, k \in z$ (d) no value of x

11. If z= x- iy and $z^{\frac{1}{3}}$ = p+i q then $\frac{\frac{x}{p}+\frac{y}{q}}{p^2+q^2}$ = ______

(a) 2 (b) -1 (c) 1 (d) -2

12. Let z, w be complex mumbers such that $\bar{z}+i\bar{w}=o$ and Arg (zw) = π then Arg (z) =_____

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{5\pi}{4}$

13 If $W = \frac{z}{z-\left(\frac{1}{3}\right)^{i}}$ and $|w| = 1$ then z lies on ________

(a) circle (b) an ellipse (c) Parabola (d) a straight line

14 It $z^2 + z + 1 = 0$ where z is a complex mumber, then the value of

$\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+......+\left(z^6+\frac{1}{z^6}\right)^2$ *is* _______

(a) 18 (b) 54 (c) 6 (d) 12

15. If |z- 4/z | = 2 then the maximum value of $|z|$ is _______

(a) $\sqrt{5}+1$ (b) 2 (c) $2+\sqrt{2}$ (d) $\sqrt{3}+1$

16. If $\frac{1+2i}{2+i}$ = r (cosθ + i sinθ), then

(a) r =1, $\theta = \tan^{-1}\left(\frac{4}{3}\right)$ (b) $r=\sqrt{5}, \theta = \tan^{-1}\left(\frac{5}{4}\right)$

(c) r =1, $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ (d) $r=2, \theta = \tan^{-1}\left(\frac{3}{4}\right)$

17. The small positive integer 'n' for which $(1+i)^{2n} = (1-i)^{2n}$ is _______

(a) 4 (b) 8 (c) 2 (d) 12

18. If z_1, z_2 are complex mumbers and $| z_1+z_2 | = | z_1 | + | z_2 |$ then ______

(a) arg (z_1) + arg (z_2) = 0 (b) arg (z_1 z_2) =0

(c) arg (z_1) = arg (z_2) (d) None of these

19. The number of complex numbers z such that | z - | | = | z + 1 | = | z - i | equal ________

(a) 0 (b) 1 (c) 2 (d) ∞

20. Let α, β be real and z be a complex mumber if $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re(z) = 1 then it is necessary that

(a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$

(c) $\beta \in [1, \infty)$ (d) $\beta \in (0,1)$

21. If z= $\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$ then $z^2 - z + 1 =$ ____

(a) - 2i (b) 2 (c) 0 (d) -2

22. (1 + i) (2+ ai) + (2 + 3 i)(3 + i) = x + iy, x, x y ∈ R and x=y then a =_________

(a) 5 (b) -4 (c) -5 (d) 4

23. If z_1 = 2-i and z_2 = 1+i then $\left|\frac{z_1 - z_2 + 1}{z_1 + z_2 + i}\right| =$ ____________

(a) $\sqrt{\frac{5}{3}}$ (b) $\sqrt{\frac{3}{5}}$ (c) $\sqrt{\frac{4}{5}}$ (d) $\sqrt{\frac{5}{4}}$

24. The principal argument of $-2\sqrt{3} - 2i$ is ______

(a) $\frac{-5\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{-2\pi}{3}$ (d) $\frac{2\pi}{3}$

25. If x>1 then the squareroots of 1 - $\sqrt{x^2 - 1}$ i is ________

(a) $\pm\left(\sqrt{\frac{x+1}{2}} - i\sqrt{\frac{x-1}{2}}\right)$ (b) $\pm\left(\sqrt{\frac{x+1}{2}} + i\sqrt{\frac{x-1}{2}}\right)$

(c) $\pm\left(\sqrt{\frac{x-1}{2}} - i\sqrt{\frac{x+1}{2}}\right)$ (d) $\pm\left(\sqrt{\frac{x-1}{2}} + i\sqrt{\frac{x+1}{2}}\right)$

26. If 1, w and w^2 are cube root of 1 then $(1-w)(1-w^2)(1-w^4)(1-w^8)$ = _______

(a) 16 (b) 8 (c) 9 (d) 64

27. $w \neq 1$ is cube root of 1 and $(1+w^2)^n = (1+w^4)^n$ then the smallest positive value of n is _______

(a) 2 (b) 3 (c) 5 (d) 6

28. If $|z|$ = 1 and w = $\frac{z-1}{z+1}$ ($z \neq -1$) then Re (w) = _______

(a) 0 (b) $\frac{1}{|z+1|^2}$ (c) $\frac{1}{|z+1|^3}$ (d) $\frac{\sqrt{2}}{|z+1|^2}$

29. If z=-1 then arg $\left(z^{\frac{2}{3}}\right)$ = ________

(a) $\frac{\pi}{3}, 2\pi$ (b) $0, \frac{2\pi}{3}, \frac{-2\pi}{3}$ (c) $\frac{10\pi}{3}$ (d) π , 2π

30. If Z=cos $\frac{4\pi}{7}$ + i sin $\frac{4\pi}{7}$ then Re $(z+z^2+z^3)$ =_____

(a) cos $\frac{\pi}{3}$ (b) cos $\frac{2\pi}{3}$ (c) cos $\frac{\pi}{6}$ (d) cos $\frac{5\pi}{6}$

31. $\left(\dfrac{1+\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}}{1+\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}}\right)^{36}$ = _______

(a) -1 (b) 1 (c) 0 (d) $\frac{1}{2}$

32. $\sqrt[4]{-8+8\sqrt{3}i}$ = ______

(a) $\pm\left(1+\sqrt{3}i\right)$ (b) $\pm\left(2+2\sqrt{3}i\right)$

(c) $\pm\left(\sqrt{3}+i\right)$ (d) $\pm\left(2-2\sqrt{3}i\right)$

33. If Z is complex mumber Then the locus of the point Z satisfying arg $\left(\dfrac{z-i}{z+i}\right)=\frac{\pi}{4}$ is a _____

(a) Circle with center (-1,0) and radius $\sqrt{2}$

(b) Circle with center (0,0) and radius $\sqrt{2}$

(c) Circle with center (0,1) and radius $\sqrt{2}$

(d) Circle with center (1,1) and radius $\sqrt{2}$

34. The area of the triangle in the Argand diagram formed by the Complex number z, iz and z + iz is

(a) $|z|^2$ (b) $\frac{\sqrt{3}}{2}|z|^2$ (c) $\frac{1}{2}|z|^2$ (d) $\frac{3}{2}|z|^2$

35. Let z_1 and z_2 be two roots of equation $z^2 + az + b = 0$. Z is complex number. Assume that origin, z_1 and z_2 form an equilateral triangle then

(a) $a^2 = 2b$ (b) $a^2 = 3b$ (c) $a^2 = 4b$ (d) $a^2 = b$

36. $\sqrt{-1-\sqrt{-1-\sqrt{-1-..........\infty}}}$ = ______

(a) 1 (b) w. w^2 (c) w (d) w^2

37. If a=cos α + i sin α , b = cos β + i sin β c = cosγ + i sinγ and $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}=1$

then cos (α - β) + cos (β - γ) + cos (γ - α) = _______

(a) $\frac{3}{2}$ (b) $-\frac{3}{2}$ (c) 0 (d) 1

38. The value of $(-i)^{(-i)}$ = ______

(a) $-\frac{\pi}{2}$ (b) $\frac{\pi}{2}$ (c) $e^{-\frac{\pi}{2}}$ (d) $e^{\frac{\pi}{2}}$

39. If cube root of unity are 1,w,w^2 then the roots of the equation $(x-1)^3 + 8 = 0$ are ________

(a) -1,-1,-1 (b) -1,-1+2w, -1-2w^2

(c) -1,1+2w, 1+2w^2 (d) -1, 1-2w, -1-2w^2

40. If f(x) = $4x^5 + 5x^4 - 8x^3 + 5x^2\ 4x - 34i$ and f $\left(\frac{-1+\sqrt{3}i}{2}\right)$ = a+ib then a:b=______

(a) 2 (b) -2 (c) 17 (d) -17

41. If z= cosθ+i sin θ then arg (z^2+z) =_______

(a) $\frac{3\theta}{2}$ (b) θ (c) $\frac{\theta}{2}$ (d) 3θ

42. If x = a+b, y = a$\propto$+bβ and z = aβ + bα, Where α, β $\neq 1$ are cube roots of unity, then $x$yz=___

(a) 2 ($a^3 + b^3$) (b) 2 ($a^3 - b^3$)

(c) $a^3 + b^3$ (d) $a^3 - b^3$

43. $\frac{(\cos 2\theta - i\sin 2\theta)^7 (\cos 3\theta + i\sin 3\theta)^{-5}}{(\cos 4\theta - i\sin 4\theta)^{12} (\cos 5\theta + i\sin 5\theta)^{-6}}$ = _______

(a) cos 33θ + i sin 33θ (b) cos 33θ - i sin 33θ

(c) cos 47θ + i sin 47θ (d) cos 47θ - i sin 47θ

44. If $z = \frac{1+7i}{(2-i)^2}$ then the polar form of z is_____

(a) $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ (b) $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(c) $\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$ (d) $\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

45. For any integer n, arg $\left(\frac{(\sqrt{3}+i)^{4n+1}}{(1-i\sqrt{3})^{4n}}\right)$ = __________

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

46. If the imaginary part of $\frac{2z-3}{iz+1}$ is -2 then the locus of the point representing Z in the complex plane is

(a) a circle (b) a straight line (c) a parabola (d) an ellipse

47. The inequality $|z-4| < |z-2|$ represent the region given by

(a) Re(z) >o (b) Re (z) <o (c) Re (z) >2 (d) Re (z) >3

48. The equation $|z-i| + |z+i| = k$ represents an ellipse if K=______

(a) 1 (b) 2 (c) 4 (d) -1

49. Let Z be complex number with modulus 2 and argument $\frac{-2\pi}{3}$ then z = ________

(a) -1+i $\sqrt{3}$ (b) $\frac{-1+i\sqrt{3}}{2}$ (c) -1-i $\sqrt{3}$ (d) $\frac{-1-i\sqrt{3}}{2}$

50. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$ = x+iy then _______

(a) $x=3, y=1$ (b) x=1, y=3 (c) x=o, y=3 (d) x=o, y=o

51. If z is a complex number satisfying $|z - i\,\mathrm{Re}(z)| = |z - \mathrm{Im}(z)|$ then Z lies on

(a) y = x (b) y = -x (c) y = x+1 (d) y = -x+1

52. If w is one of the cube root of 1 other than 1 then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-w^2 & w^2 \\ 1 & w^2 & w^4 \end{vmatrix}$ =____

(a) 3w (b) 3w (w-1) (c) $3w^2$ (d) 3w (1-w)

53. If z = x + iy, $x, y \in R$ and $|x|+|y| \leq k|z|$ then k = ____

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\sqrt{4}$

54. If a = $\cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$ then the quadratic equation whose roots are $\alpha = a+a^2+a^4$ and $\beta = a^3+a^5+a^6$ is ____

(a) x^2-x+2 (b) x^2+x-2 (c) x^2-x-2 (d) x^2+x+2

55. If z, iz and z+iz are the vertices of a triangle whose area is 2 units then the value of $|z|$ is_____

(a) 4 (b) 2 (c) -2 (d) None of these

56. A(z_1), B (z_2) and C(z_3) are vertices of ΔABC where $m\angle C = \frac{\pi}{2}$ and AC=BC, z_1, z_2, z_3 are complex number if $(z_1 - z_2)^2 = k\,(z_1 - z_3)\,(z_3 - z_2)$ then K =______

(a) 1 (b) 2 (c) 4 (d) non of these

57. If $x_n = \cos\left(\frac{\pi}{2^n}\right) + i\sin\left(\frac{\pi}{2^n}\right)$ then $x_1\, x_2\, x_3 \ldots\ldots \infty$ =_____

(a) $-i$ (b) -1 (c) i (d) 1

58. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ then $|z_1 + z_2 + z_3|$ = ...

(a) =1 (b) < 1 (c) > 1 (d) = 3

59. Let Z = $\cos\theta + i\sin\theta$ then the value of $\sum_{n=1}^{15} \text{Im}\left(Z^{2n-1}\right)$ at $\theta = 2^0$ is _____

(a) $\frac{1}{\sin 2^0}$ (b) $\frac{1}{3\sin 2^0}$ (c) $\frac{1}{2\sin 2^0}$ (d) $\frac{1}{4\sin 2^0}$

60. Let j Z= $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ and Z = $\overline{Z}$ then θ =____________

(a) (2k+1) $\frac{\pi}{2}$, k$\in$z (b) 2kπ , k $\in$ z

(c) kπ , k $\in$ z (d) None

61. For complex numbers z_1,z_2 if $|z_1|$=12 and $|z_2-3-4i|$=5 then the minimum value of $|z_1-z_2|$ is

(a) 0 (b) 2 (c) 7 (d) 17

62. Let z=*x*+*i*y be a comlpex number, where *x*,y are integers Then the area of the rectangle whose vertices are the roots of the equatio $\overline{z}z^3 + z\overline{z}^3 = 350$ is

(a) 48 (b) 32 (c) 40 (d) 80

63. If a,b,c are integers, not all equal, and w is a cube root of unity (w$\neq$1) Then the minimum value of $|a+bw+cw^2|$ is _________

(a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

64. The complex numbers z_1,z_2 and z_3 satistying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$ are the vertices of a triangle which is ____________

(a) of area zero (b) right angled isosceles

(c) equilateral (d) obtuse-angle isosceles

Hints

1. Ans : (a)

Z = x + iy and 3x + (3x-y)i = 4 - 6i

3x = 4, 3x-y = 6

$$\therefore z = \frac{4}{3} + i10$$

2. Hint :- $i^2 = -1$, $\frac{1}{i} = -i$

3. Hint :- $i^2 = -1$, $\frac{1}{i} = -i$

4. Ans : (c)

$$z = \frac{1}{1 + \cos\theta - i\sin\theta}$$

$$= \frac{1 + \cos\theta + i\sin\theta}{(1+\cos\theta)^2 + \sin^2\theta}$$

$$= \frac{2\cos^2\frac{\theta}{2} + i\,2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2 \cdot 2\cos^2\frac{\theta}{2}}$$

$$= \frac{1}{2} + i\,\frac{\tan\frac{\theta}{2}}{2}$$

5. Ans : (b)

$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

(4x + 9y - 3) + i (2x -7y-3) = 0+10 i

$$\left.\begin{aligned} \therefore 4x + 9y - 3 &= 0 \\ 2x - 7y - 3 &= 10 \end{aligned}\right\} \text{Solving the Equation}$$

x = 3, y = -1

$$\therefore (x_1 y) = (3, -1)$$

6. Ans : (a)

$$z = \frac{11 + 27i}{34} = \frac{11}{34} + \frac{27}{34}i$$

$$z^{-1} = \frac{11/34}{\left(11/34\right)^2 + \left(27/34\right)^2} - i\frac{27/34}{\left(11/34\right)^2 + \left(27/34\right)^2}$$

$$= \frac{11}{25} - \frac{27}{25}i$$

7. Ans : (c)

z = x + iy, $|3z| = |z-4|$

$\therefore$ $|3x + i3y| = |x - 4 + iy|$

$\therefore$ $x^2+y^2+x = 2$

8. Ans : (b)

$|z+3| \le 8$

$\therefore |z-2| = |z+3-5|$

$\le |z+3| + |-5|$

$\le 8+5$

≤ 13 ________(i)

now $|z-2| = |5-(z-3)|$

$\ge |5| \sim |z-3|$

$\ge 5 \sim 8$

≥ 3

but $|z+3| \le 8$

$\therefore -11 \le z \le 5$ and z = 2 $\therefore$ $|z-2| = 0$

$\therefore$ least value of $|z-2|$ is 0

$\therefore 0 \le |z-2| \le 13$

9. Ans : (b)

let $z = \sqrt[3]{1}$

$\therefore z^3 = 1$ $\therefore z^3 - 1 = 0$

$\therefore$ $(z - 1)(z^2 + z + 1) = 0$

$\therefore$ z = 1, $z = \frac{-1+\sqrt{3}i}{2}$, $z = \frac{-1-\sqrt{3}i}{2}$

z = 1 = w = w^2

1, w, w^2 are given roots

$\therefore 1+w+w^2 \;=1+\frac{-1+\sqrt{3}i}{2}+\frac{-1-\sqrt{3}i}{2}$

$=0$

10. Ans : (d)

$\sin x+i\cos 2x$ and $\cos x-i\sin 2x$

are complex conjugate of each other

$\therefore \text{Im}(z)=0$

$\therefore \cos 2x=0,\ \sin 2x=0$

which is not possible

$\therefore$ the value of x does not exist.

11. Ans : (d)

z = x - iy and $z^{1/3}=p+iq$

$z=(p+iq)^3$

$=p^3+(iq)^3+3(p)(iq)(p+iq)$

$x-iy=\left(p^3-3pq^2\right)-i(q^3-3p^2q)$

$\therefore x=p^3-3pq^2,\quad y=q^3-3p^2q$

$\therefore \frac{x}{p}=p^2-3q^2,\qquad \frac{y}{p}=q^2-3p^2$

$\therefore \frac{\frac{x}{p}+\frac{y}{q}}{p^2+q^2}=-2$

12. Ans : (a)

$\bar{z}+i\overline{w}=0$

$\bar{z}=-i\overline{w}$

z = iw $\qquad \therefore w=-iz$

$\therefore$ $\text{Arg}(zw)=\pi$ given

$\therefore$ $\text{Arg}\left((z)(-iz)\right)=\pi$

$\therefore$ $\text{Arg}(-iz^2)=\pi$

$\therefore$ $\text{Arg}(-i)+2\text{Arg}(z)=\pi$

$\therefore \frac{-\pi}{2}+2\text{Arg}(z)=\pi$

$\therefore$ $\text{Arg}(z)=3\pi/4$

13. Ans : (d)

$$w=\frac{z}{z-\frac{1}{3}i}$$ †ÃÜ $|w|=1$

$$\therefore |w|=\left|\frac{z}{z-\frac{1}{3}i}\right|=1$$

let z = x + iy

$$\therefore \frac{|x+iy|}{\left|x+(y-\frac{1}{3}i)\right|}=1$$

$\therefore$ $x^2 + y^2 = x^2+(y-\frac{1}{3})^2$

$\therefore$ 6y = 1

$\therefore$ z lies on line.

14. Ans : (d)

$z^2 + z + 1 = 0$

$\therefore$ $(z - 1)(z^2 + z + 1) = 0$ $\therefore$ $z^3 - 1 = 0$

$\therefore$ $z^3 = 1$, $z^2 + z = -1$

$$\therefore\left(z+\frac{1}{z}\right)^2+(z^2+\frac{1}{z^2})^2+(z^3+\frac{1}{z^3})^2+(z^4+\frac{1}{z^4})^2+(z^5+\frac{1}{z^5})^2+(z^6+\frac{1}{z^6})^2$$

$$=(z+z^2)^2+(z^2+z)^2+(1+1)^2+(z+z^2)^2+(z^2+z)^2+(1+1)^2$$

$= (-1)^2 + (-1)^2 + 4 + (-1)^2 + (-1)^2 + 4$

= 12.

15. Ans : (a)

$$\left|z-\frac{4}{z}\right|=2$$

$$|z|=\left|z-\frac{4}{z}+\frac{4}{z}\right|$$

$$\leq\left|z-\frac{4}{z}\right|+\left|\frac{4}{z}\right|$$

$$\therefore |z|\leq 2+\frac{4}{|z|}$$

$$\therefore |z|^2\leq 2|z|+4$$

$$\therefore |z|^2-2|z|+1\leq 5$$

$$\therefore (|z|-1)^2\leq 5$$

$\therefore \left||z|-1\right| \le \sqrt{5}$

$-\sqrt{5} \le |z|-1 \le \sqrt{5}$

$\therefore -\sqrt{5}+1 \le |z| \le \sqrt{5}+1$

maximum value of $|z|$ is $\sqrt{5}+1$

16. Ans : (c)

$\frac{1+2i}{2+i}$ = $r\cos\theta + i\,r\sin\theta$

$\therefore \frac{(1+2i)(2-i)}{5}$ = $r\cos\theta + i\,r\sin\theta$

$\therefore 4/5 + i\,3/5$ = $r\cos\theta + i\,r\sin\theta$

$r^2 = 1$, $r = 1$

$\tan\theta = 3/4$

$\theta = \tan^{-1}(3/4)$

$r = 1$, $\theta = \tan^{-1}(3/4)$

17. Ans : (c)

$(1+i)^{2n} = (1+i)^{2n}$

$\therefore \left(\frac{1+i}{1-i}\right)^{2n} = 1$

$(i)^{2n} = 1$

$(-1)^n = 1$

which is possible if n = 2

$\therefore$ least positive integer n = 2

18. Ans : (c)

$|z_1+z_2| = |z_1|+|z_2|$

let $z_1 = r_1\cos\theta_1 + i\,r_1\sin\theta_1$ $\quad |z_1| = r_1$

$z_2 = r_2\cos\theta_2 + i\,r_2\sin\theta_2$ $\quad |z_2| = r_2$

$\therefore |z_1+z_2|^2 = (r_1\cos\theta_1 + r_2\cos\theta_2)^2 + (r_1\sin\theta_1 + r_2\sin\theta_2)^2$

$= r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1-\theta_2)$

$|z_1+z_2|^2 = (|z_1|+|z_2|)^2$

$\therefore r_1^2 + r_2^2 + 2r_1r_2 \cos(\theta_1 - \theta_2) = (r_1 + r_2)^2$

$$\therefore \cos(\theta_1 - \theta_2) = 1 = \cos 0$$

$$\therefore \theta_1 - \theta_2 = 0$$

$$\therefore \theta_1 = \theta_2$$

$\therefore$ arg (z_1) = arg (z_2)

19. Ans : (b)

$|z-1| = |z+1| = |z-i|$

let z = $(x_1 y)$ = x + iy

$\therefore \sqrt{(x-1)^2 + y^2} = \sqrt{(x+1)^2 + y^2} = \sqrt{x^2 + (y-1)^2}$

$\therefore$ d ((x, y), (1, 0)) = d ((x, y), (-1, 0)) = d ((x, y), (0, 1))

let A (1, 0), B (-1, 0), C (0, 1) and z = p(x, y)

then AP = BP = CP

$\therefore$ z is circum centre which is unique.

20. Ans : (c)

$z^2 + \alpha z + \beta = 0$, $\alpha, \beta \in R$, $z \in c$

let z = x + iy

Re(z) = 1 (given) $\therefore$ x = 1

$\therefore$ z = 1 + iy

let 1 + iy & 1 - iy are two distinct roots

product of roots = β

$\therefore \beta = (1+iy)(1-iy) = 1 + y^2$

now $1 + y^2 \geq 1$ $\therefore \beta \geq 1$

$$\therefore \beta \in (1, \infty)$$

21. Ans : (c)

$z = (\cos \pi/3 - i \sin \pi/3) = \frac{1}{2} - i\frac{\sqrt{3}}{2}$

$\therefore z^2 = (\cos \pi/3 - i \sin \pi/3)^2$

$= \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3} = \frac{-1}{2} - \frac{i\sqrt{3}}{2}$

$\therefore z^2 - z + 1 = \frac{-1}{2} - i\frac{\sqrt{3}}{2} - \frac{1}{2} + i\frac{\sqrt{3}}{2} + 1 = 0$

22. Ans : (b)

(1+i) (2+ai) + (2+3i) (3+i) = x + iy

$\therefore$ (5-a) + i (13+a) = x + iy

$\therefore$ x = 5-a, y = 13+a

x = y given

$\therefore$ 5-a = 13+a $\therefore$ a = -4

23. Ans : (c)

$z_1 = 2 - i,\ z_2 = 1 + i$

$$\therefore \left|\frac{z_1 - z_2 + 1}{z_1 + z_2 + i}\right| = \left|\frac{2 - i - 1 - i + 1}{2 - i + 1 + i + i}\right|$$

$$= \left|\frac{2 - 2i}{3 + i}\right| = \frac{\sqrt{4+4}}{\sqrt{9+1}} = \sqrt{\frac{4}{5}}$$

24. Ans : (a)

let $z = -2\sqrt{3} - 2i = x + iy$

r = |z| = 4

let arg(z) = θ, $\theta \in (-\pi, \pi]$

$\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{x}{r}$

$= \frac{-2\sqrt{3}}{4}$ $= \frac{-1}{4}$

$= \frac{-\sqrt{3}}{2} < 0$ $= \frac{-1}{2} < 0$

$$\therefore \theta = -\left(\pi - \pi/6\right) = -5\pi/6$$

25. Ans : (a)

$z = 1 - i\sqrt{x^2 - 1}$ $\therefore |z| = x = r$

let $\theta = \arg(z)$

$\therefore \cos\theta = 1/x > 0,$ $\sin\theta = \frac{-\sqrt{x^2-1}}{x} < 0$

$\therefore -\pi/2 < \theta < 0$ 4^{th}quadrant

$\cos\theta/2 = \sqrt{\frac{1+\cos\theta}{2}}$ $\sin\theta/2 = -\sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1 + 1/x}{2}}$ $= -\sqrt{\frac{1 - 1/x}{2}}$

$$=\sqrt{\frac{1+x}{2x}} \qquad =-\sqrt{\frac{x-1}{2x}}$$

$$\therefore \sqrt{z}=\pm\sqrt{r}\,(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})$$

$$=\pm\sqrt{x}\left(\frac{\sqrt{1+x}}{\sqrt{2}\sqrt{x}}-i\frac{\sqrt{x-1}}{\sqrt{2}\sqrt{x}}\right)$$

$$\sqrt{z}=\pm\left(\frac{\sqrt{x+1}}{\sqrt{2}}-i\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

26. Ans : (c)

1, w, w^2, are cube roots of 1

$\therefore 1+w+w^2=0$ & $w^3 = 1$

$(1 - w)(1 - w^2)(1 - w^4)(1 - w^8)$

$= (1 - w)(1 - w^2)(1 - ww^3)(1 - w^2.(w^3)^2)$

$= (1 - w)(1 - w^2)(1 - w)(1 - w^2)$

$= (1 - w^2 - w + w^3)^2$

$= 3^2 \quad = 9$

27. Ans : (b)

$w\neq 1$ is cube root of 1 $\quad \therefore w^3=1$

$(1 + w^2)^n = (1 + w^4)^n$

$\therefore (1 + w^2)^n = (1 + w)^n$

$$\therefore\left(\frac{1+w^2}{1+w}\right)^n=1 \qquad \therefore\left(\frac{-w}{-w^2}\right)^n=1$$

$$\therefore\left(\frac{w^2}{w^3}\right)^n=1 \qquad \therefore w^{2n}=1$$

If n = 2, $\quad w^{2n} = w^4 = w\neq 1$

n = 3, $w^{2n} = w^6 = 1^2 = 1$

n = 4, $w^{2n} = w^8 = w^6. w^2 = w^2\neq 1$

$\therefore n=3$

28. Ans : (a)

$$|z|=1 \quad \text{and} \quad w=\frac{z-1}{z+1}$$

let $z = x + iy \qquad |z|=1\Rightarrow x^2+y^2=1$

$$w = \frac{x+iy-1}{x+iy+1}$$

$$= \frac{(x-1)+iy}{(x+1)+iy}$$

$$= \frac{[(x-1)+iy][(x+1)-iy]}{(x+1)^2+y^2}$$

$$= \frac{x^2-1-iy(x-1-x-1)+y^2}{x^2+y^2+2x+1}$$

$$= \frac{x^2+y^2-1+i2y}{x^2+y^2+1+2x}$$

$$= \frac{i2y}{2(1+x)}$$

$$w = {}^{iy}\!/_{1+x}$$

$\therefore$ Re (w) = 0

29. ANS : (b)

If z = -1 then $Z^{2/3} = (-1)^{2/3} = \sqrt[3]{1}$

let $Z_1 = Z^{2/3} = \sqrt[3]{1}$

$\therefore Z_1 = 1$ or $Z_1 = \dfrac{-1+i\sqrt{3}}{2}$ or $Z_1 = \dfrac{-1-i\sqrt{3}}{2}$

If $Z_1 = 1 \quad \therefore \arg(z_1) = \arg(1) = 0$

If $Z_1 = \dfrac{-1}{2} + \dfrac{\sqrt{3}}{2}i$, $\arg(z_1) = 2\pi/3$

If $Z_1 = \dfrac{-1}{2} - \dfrac{\sqrt{3}}{2}i$, $\arg(z_1) = -2\pi/3$

$$\therefore \arg(Z^{2/3}) = 0,\ 2\pi/3,\ -2\pi/3$$

30. ANS : (b)

$$\text{Re}(z+z^2+z^3) = \frac{(z+z^2+z^3)+(z+z^2+z^3)}{2}$$

$$=\frac{1}{2}\left[z+z^2+z^3+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}\right]$$

$$=\frac{1}{2z^3}\left[z^6+z^5+z^4+z^2+z+1\right]$$

$$=\frac{1}{2z^3}\left[1+z+z^2+z^3+z^4+z^5+z^6-z^3\right]$$

$$=\frac{1}{2z^3}\left[\frac{1-z^7}{1-z}-z^3\right]-----(i)$$

$$z=\cos\frac{4\pi}{7}+i\ \sin\frac{4\pi}{7}$$

$$\therefore z^7=\cos\ 4\pi+i\ \sin\ 4\pi=1+0=1$$

$$\therefore \text{Re}(z+z^2+z^3)=\frac{1}{2z^3}\left[\frac{1-1}{1-z}-z^3\right]$$

$$=-1/2$$

$$=\cos\frac{2\pi}{3}$$

31. ANS : (a)

let $z=\cos\pi/12+i\ \text{Sin}\ \pi/12$

$$\therefore \bar{z}=\cos\pi/12-i\ \sin\ \pi/12$$

$$\therefore\left[\frac{1+\cos\frac{\pi}{12}+i\ \sin\frac{\pi}{12}}{1+\cos\frac{\pi}{12}-i\ \sin\frac{\pi}{12}}\right]^{36}=\left[\frac{1+z}{1+\bar{z}}\right]^{36}$$

$$=z^{36}\qquad (\because \bar{z}=1/z)$$

$$=\left(\cos\frac{\pi}{12}+i\ \sin\frac{\pi}{12}\right)^{36}$$

$$=\cos\ 3\pi+i\ \sin 3\pi$$

$= -1 + i(0)$

$= -1$

32. ANS : (c)

let $z=-8+8\sqrt{3}i=a+ib$

$\therefore a=-8,\ \ b=8\sqrt{3}$

$|z|=\sqrt{64\times 4}=16$

$$\therefore \sqrt{z}=\pm\left[\sqrt{\frac{|z|+a}{2}}+i\sqrt{\frac{|z|+a}{2}}\right]$$

$=\pm(\sqrt{4}+i\sqrt{12})$

$=\pm(2+i\,2\sqrt{3})$

let $w=\sqrt{z}=2+i\ 2\sqrt{3}=a+ib$

$a=2,\ b=2\sqrt{3}$

$\therefore |w|=\sqrt{4+12}=4$

$$\therefore \sqrt{w}\pm\left[\sqrt{\frac{|w|+a}{2}}+i\ \sqrt{\frac{|w|-a}{2}}\right]$$

$\therefore \sqrt[4]{z}=\pm(\sqrt{3}+i)$ ans.

33. ANS : (a)

let $z=x+iy$

also $agr\left(\frac{z-i}{z+i}\right)=\pi/4$

$\therefore agr(x+i(y-1))-arg(x+i(y+1))=\pi/4$

$$\therefore \tan^{-1}\frac{y-1}{x}-\tan^{-1}\frac{y+1}{x}=\pi/4$$

$$\therefore \tan^{-1}\left(\frac{\frac{y-1}{x}-\frac{y+1}{x}}{1+\frac{y^2-1}{x^2}}\right)=\pi/4$$

$$\therefore \frac{y-1-y-1}{x}\times\frac{x^2}{x^2+y^2-1}=\tan\pi/4=1$$

$\therefore -2x = x^2 + y^2 - 1$

$\therefore x^2 + y^2 + 2x = 1$

which is a circle with centre (-1, 0) and radius $\sqrt{2}$

34. ANS : (c)

let z = x+iy

and P = p(z) = (x, y)

Q = Q(iz) = (-y, x)

R = R(z+iz) = (x-y, x+y)

The area of $\Delta PQR = \frac{1}{2}|D|$

where $D = \begin{vmatrix} x & y & 1 \\ -y & x & 1 \\ x-y & x+y & 1 \end{vmatrix}$

$= -(x^2 + y^2)$

$= -|z|^2$

$\therefore$ area of $\Delta PQR = \frac{1}{2}|z|^2$

35. ANS : (b)

$z^2 + az + b = 0,$ z is complex number

$z_{1,}\ z_2$ are roots.

$\therefore z_1 + z_2 = -a \qquad , z_1z_2 = b$

Hint : z_1, z_2, z_3 complex number a triangle formed by z_1, z_2, z_3 is and equiletteral then $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

here $z_3 = 0$

$z_1^2 + z_2^2 + o^2 = z_1z_2 + o + o$

$\therefore (z_1 + z_2)^2 - 2z_1z_2 = z_1z_2$

$\therefore (-a)^2 - 2b = b$

$\therefore a^2 = 3b$

36. ANS : (b)

let $\sqrt{-1-\sqrt{-1-\sqrt{-1-----\infty}}}$

$\therefore Z=\sqrt{-1-z}$

$\therefore z^2+z+1=0$

$\therefore z=\frac{-1+i\sqrt{3}}{2}$ or $\frac{-1-i\sqrt{3}}{2}$

$\therefore$ Z = W or w^2

37. ANS : (d)

$a=\cos\alpha+i\sin\alpha=e^{i\alpha}$

$b=\cos\beta+i\sin\beta=e^{i\beta}$

$c=\cos\gamma+i\sin\gamma=e^{i\gamma}$

$\therefore \frac{a}{b}+\frac{b}{c}+\frac{c}{a}=1\Rightarrow e^{i(\alpha-\beta)}+e^{i(\beta-\gamma)}+e^{i(\gamma-\alpha)}=1$

$\Rightarrow[\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)]$

$+i[\sin(\alpha-\beta)+\sin(\beta-\gamma)+\sin(\gamma-\alpha)]=1$

$\Rightarrow \cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=1$

38. ANS : (c)

let $z=-(-i)^{(-i)}$

$\therefore \log z=(-i)\ \log(-i)$

$=(-i)\ \log(0+i(-1))$

$=(-i)\ \log(\cos \pi/2+i\sin\ \pi/2)$

$\therefore \log z=(-i)\ \log\ e^{i(-\pi/2)}$

$=(-i)i\left(-\pi/2\right)\ \log e$

$=i^2\cdot \pi/2$

$\log z=-\pi/2$

$\therefore z=e^{-\pi/2}$

39. ANS : (d)

$$(x-1)^3+8=0$$

$$\therefore \left(\frac{x-1}{-2}\right)^3=1$$

$$\therefore \frac{x-1}{-2}=\sqrt[3]{1}$$

$\therefore \frac{x-1}{-2}=1$ or $\therefore \frac{x-1}{-2}=w,\ \frac{x-1}{-2}=w^2$

$$\therefore x=-1,\quad x=1-2w,\quad x=1-2w^2$$

$\therefore$ roots are -1, 1-2w, 1-2w^2

40. ANS : (c)

$$f(x)=4x^5+5x^4-8x^3+5x^2+4x-34i$$

$$f\left(\frac{-1+\sqrt{3}i}{2}\right)=a+ib$$

let $\frac{-1+\sqrt{3}i}{2}=w$ $\quad\quad \therefore w^3=1,\ w+w^2=-1$

$$f\left(\frac{-1+\sqrt{3}i}{2}\right)=f(w)$$

$$=4w^5+5w^4-8w^3+5w^2+4w-34i$$

$$a+ib=-17-34i$$

$$a:b=(-17):(-34)$$

$$a:b=1:2$$

41. ANS : (c)

$$z=\cos\theta+i\sin\theta$$

$$\therefore \bar{z}=\cos\theta-i\sin\theta$$

$$\therefore z^2=\cos^2\theta-\sin^2\theta+i\,2\cos\theta\sin\theta$$

$$\therefore z^2=\cos 2\theta+i\sin 2\theta$$

$$\therefore z^2+\bar{z}=(\cos 2\theta+\cos\theta)+i(\sin 2\theta-\sin\theta)$$

$$=2\cos\frac{3\theta}{2}\cos\theta/2+i\,2\cos\frac{3\theta}{2}\sin\theta/2$$

$\therefore \arg(z^2+\bar{z}) = \tan^{-1}\left(\tan \theta/2\right) = \theta/2$

42. ANS : (d)

here $\alpha, \beta \neq 1$ and are cube roots of 1

$\therefore \alpha+\beta = w+w^2 = -1 \quad , \alpha\beta = 1, \ \alpha^2+\beta^2 = -1$

$xyz = (a+b)(a\alpha+b\beta)(a\beta+b\alpha)$

$= (a+b)[(a^2+b^2)\alpha\beta + ab(\alpha^2+\beta^2)]$

$= (a+b)[a^2+b^2-ab]$

$xyz = a^3 - b^3$

43. ANS : (d)

expression= $\dfrac{(\cos\theta+i\sin\theta)^{-14}(\cos\theta+i\sin\theta)^{-15}}{(\cos\theta+i\sin\theta)^{48}(\cos\theta+i\sin\theta)^{-30}}$

$= (\cos\theta + i\ \sin\theta)^{-47}$

$= \cos(-47)\theta + i\sin(-47)\theta$

$= \cos 47\theta - i\sin 47\theta$

44. ANS : (a)

$z = \dfrac{1+7i}{(2-i)^2} = \dfrac{1+7i}{3-4i}$

$\therefore z = \dfrac{(1+7i)(3-4i)}{9+16} = -1+i$

a = -1, b = 1, $\theta \in (\pi/2,\ \pi)$

$r = |z| = \sqrt{2}$ and $\theta = \tan^{-1}(-1) = 3\pi/4$

$z = r(\cos\theta + i\ \sin\theta)$

$= \sqrt{2}\left(\cos\dfrac{3\pi}{4} + i\ \sin\dfrac{3\pi}{4}\right)$

45. ANS : (b)

let $\quad z = \dfrac{\left(\sqrt{3}+i\right)^{4n+1}}{\left(1-i\sqrt{3}\right)^{4n}}$

$$= \frac{\left(\sqrt{3}+i\right)^{4n}}{\left(1-i\sqrt{3}\right)}(\sqrt{3}+i)$$

$$z = i^{4n}(\sqrt{3}+i) = \sqrt{3}+i$$

$$\therefore \arg z = \arg(\sqrt{3}+i)$$

$$= \tan^{-1}\left(1/\sqrt{3}\right) = \pi/6$$

46. ANS : (b)

$$\frac{2z-3}{iz+1} = \frac{2(x+iy)-3}{i(x+iy)+1}$$

$$= \frac{(2x-3)+2yi}{(1-y)+ix}$$

$$= \frac{[(2x-3)+i\,2y][(1-y)-ix]}{(1-y)^2+x^2}$$

$$\therefore \operatorname{Im}\left(\frac{2z-3}{iz+1}\right) = \frac{2y(1-y)-x(2x-3)}{x^2+(1-y)^2} = -2$$

$\therefore$ $3x - 2y + 2 = 0$

which is represent equation of line.

47. ANS : (d)

$$|z-4| < |z-2|$$

$$\therefore |x+iy-4| < |x+iy-2|$$

$$\therefore (x-4)^2+y^2 < (x-2)^2+y^2$$

$$\therefore 12 < 4x$$

$x > 3$ $\therefore \operatorname{Re}(z) > 3$

48. ANS : (c)

$$|z-i|+|z+i| = k$$

$$\therefore |x+iy-i|+|x+iy+i| = k$$

$$\therefore \sqrt{x^2+(y-1)^2}+\sqrt{x^2+(y-1)^2} = k$$

$\therefore -4y = k\left(\sqrt{x^2+(y-1)^2} - \sqrt{x^2+(y+1)^2}\right)$

$\therefore \frac{-4y}{k} = \sqrt{x^2+(y-1)^2} - \sqrt{x^2+(y+1)^2}$

and $k = \sqrt{x^2+(y-1)^2} + \sqrt{x^2+(y+1)^2}$

$\therefore k - \frac{4y}{k} = 2\sqrt{x^2+(y-1)^2}$

$\therefore 4\left[x^2+(y-1)^2\right] = (k - \frac{4y}{k})^2$

$\therefore 4x^2+4y^2-8y+4 = k^2-8y+{}^{16y^2}/_{k^2}$

$\therefore x^2+y^2+1 = \frac{k^2}{4}+\frac{4y^2}{k^2}$

$\therefore x^2+\left(1-{}^{4}/_{k^2}\right)y^2 = \frac{k^2}{4}-1$

$\therefore \frac{x^2}{\left(k^2-4\right)/4}+\frac{y^2}{k^2/4} = 1$

for ellips $k^2-4>0$

$\therefore k^2>4$

$\therefore k^2>2$

k = 3, 4 k=4

49. ANS : (c)

$r = |z| = 2$ and $\theta = \arg z = -{}^{2\pi}/_{3}$ given

$\therefore \theta$, lies in 3[rd] quadrand

$\cos\theta = {}^{x}/_{r}$, $\sin\theta = {}^{y}/_{r}$

$x = r\cos\theta$, $y = r\sin\theta$

$x = 2\cos\left(-2\,{}^{\pi}/_{3}\right)$, $y = 2\sin\left(-2\,{}^{\pi}/_{3}\right)$

$= -1$ $= -\sqrt{3}$

$z = x+iy = -1-i\sqrt{3}$

50. ANS : (d)

$$\begin{vmatrix} 6i & -3i & -1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$$

$0 = x + iy$

$\therefore x = 0,\ y = 0$

51. ANS : (c)

$z = x + iy$

$|z - i\ \mathrm{Re}(z)| = |z - \mathrm{Im}(z)|$

$\therefore |x + i(y - x)| = |x - y + iy|$

$\therefore x^2 + (y - x)^2 = (x - y)^2 + y^2$

$\therefore x^2 = y^2$

$\therefore x = \pm y$

52. ANS : (c)

let $w = -\frac{1}{2} + i\frac{\sqrt{3}}{2},\ w^2 = \frac{-1}{2} - \frac{i\sqrt{3}}{2},\ w^3 = 1$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-w^2 & w^2 \\ 1 & w & w^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w & w \end{vmatrix} \quad (\because 1 + w + w^2 = 0)$$

$= 1(w^2 - w^3) - 1(w - w^2) + 1(w - w)$

$= 3w^2$

53. ANS : (b)

Hint $\left\{\forall\ \ a \in R,\ |a| = \sqrt{a^2} \quad |a|^2 = a^2\right\}$

$\left(|x| - |y|\right)^2 \geq 0$

$\therefore |x|^2 - |y|^2 \geq 2|x|\ |y|$

$\therefore 2|x|\ |y| \leq |x|^2 + |y|^2$

$\therefore |x|^2 + 2|x|\ |y| + |y|^2 \leq 2|x|^2 + |y|^2$

$\therefore (|x|+|y|)^2 \le 2(|x|^2+|y|^2)$

$\therefore (|x|+|y|)^2 \le 2(x^2+y^2)$

$\therefore (|x|+|y|)^2 \le 2|z|^2$

$\therefore |x|+|y| \le \sqrt{2}|z|$

$\therefore k=\sqrt{2}$

54. ANS : (d)

$a=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7} \qquad \therefore a^7=1$

$\alpha+\beta=a+a^2+a^4+a^3+a^5+a^6$

$=a(a^6-1)/a-1$

$=\frac{a^7-a}{a-1}$

$=\frac{(1-a)}{(a-1)}$

$\alpha+\beta=-1$

$\alpha\beta=(a+a^2+a^4)(a^3+a^5+a^6)$

$=a^4+a^6+a^7+a^5+a^7+a^8+a^7+a^9+a^{10}$

$=2a^7+a^4+a^5+a^6+a^7+a^8+a^9+a^{10}$

$=2+a^4(1-1)$

$\alpha\beta=2$

$\therefore$ required quadratic equation

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$\therefore x^2+x+2=0$

55. ANS : (b)

let z = x+iy iz = ix-y

=(x, y) =(-y, x)

z+iz = x+iy + ix-y = (x-y, x+y)

$\Delta=2$ given. $\therefore \frac{1}{2}|D|=2 \qquad \therefore |D|=4$

where $D = \begin{vmatrix} x & y & 1 \\ -y & x & 1 \\ x-y & x+y & 1 \end{vmatrix}$

$D = -(x^2 + y^2)$

now $|D| = 4 \qquad \therefore x^2 + y^2 = 4$

$\therefore |z|^2 = 4 \qquad \therefore |z| = 2$

56. ANS : (b)

In ΔABC $m\angle C = \pi/2$ and AC =BC

$AC^2 + BC^2 = AB^2$

here $AB = |z_1 - z_3|,\ BC = |z_3 - z_2|,\ AC = |z_1 - z_3|$

AC = BC

$\therefore |z_1 - z_3| = |z_3 - z_2|$

$\therefore (z_1 - z_3)^2 = \pm i (z_3 - z_2)^2$

$\therefore z_1^2 + z_3^2 - 2z_1z_3 = -(z_3^2 + z_2^2 - 2z_2z_3)$

$\therefore z_1^2 + z_2^2 - 2z_1z_2 = 2z_2z_3 + 2z_1z_3 - 2z_1z_2 - 2z_3^2$

$\therefore (z_1 - z_2)^2 = 2(z_1z_3 - z_3^2 - z_1z_2 + z_2z_3)$

$= 2(z_2 - z_3)(z_3 - z_2)$

$\therefore k = 2$

57. ANS : (b)

$x_n = \cos\left(\pi/2^n\right) + i\ \sin\left(\pi/2^n\right)$

$\therefore x_1 = \cos \pi/2 + i\ \sin \pi/2 = e^{i\pi/2}$

$\therefore x_2 = \cos \pi/4 + i\ \sin \pi/4 = e^{i\pi/4}$

$\therefore x_3 = \cos \pi/8 + i\ \sin \pi/8 = e^{i\pi/8}$

$\therefore x_1 \cdot x_2 \cdot x_3 -----\infty = e^{i\pi/2} \cdot e^{i\pi/4} \cdot e^{i\pi/8} ---$

$$= e^{i\left(\frac{\pi}{2}+\frac{\pi}{4}+\frac{\pi}{8}+----\infty\right)}$$

$$= e^{\pi i\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+---\infty\right)}$$

$$= e^{\pi i}\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right) = e^{\pi i}$$

$$= \cos\pi + i\ \sin\pi$$

$$= -1$$

58. ANS : (a)

$$|z|^2 = z\cdot\bar{z}$$

$$|z| = 1 \qquad \therefore \bar{z} = \frac{1}{z}$$

$$\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = \left|\bar{z_1}+\bar{z_2}+\bar{z_3}\right|$$

$$1 = \left|\overline{z_1+z_2+z_3}\right|$$

$$\therefore |z_1+z_2+z_3| = 1$$

59. ANS : (d)

$$z = \cos\theta + i\sin\theta = e^{i\theta}$$

$$z^{2n-1} = (\cos\theta + i\sin\theta)^{2n-1}$$

$$= e^{i(2n-1)\theta}$$

$$\sum_{n=1}^{15}\mathrm{Im}\left(z^{2n-1}\right) = \sum_{n=1}^{15}\mathrm{Im}\left(e^{i(2n-1)\theta}\right)$$

$$= \sum_{n=1}^{15}\ \sin(2n-1)\theta$$

$$= \sin\theta + \sin 3\theta + -- + \sin 29\theta$$

$$\therefore \sum_{n=1}^{15}\mathrm{Im}\left(z^{2n-1}\right) = \frac{1}{2\sin\theta}\left[2\sin\theta\ \sin\theta + 2\sin 3\theta\sin\theta + -- + 2\sin\ 29\theta\sin\theta\right]$$

$$= \frac{1}{2\mathrm{Sin}\theta}\left[(\cos\theta-\cos 2\theta)+(\cos 2\theta-\cos 4\theta)+---+(\cos 28\theta-\cos 30\theta)\right]$$

$$= \frac{1}{2\,\text{Sin}\theta}[\cos 0 - \cos 30\,\theta]$$

put $\theta = 2^o$

$$\frac{1}{2\sin 2^\circ}\left[1-\cos\ 60\right] = \frac{1}{2\sin 2^\circ}(1-\tfrac{1}{2})$$

$$\frac{1}{4\sin 2^\circ}$$

60. ANS : (c)

$$z = \frac{3+2i\ \sin\theta}{1-2i\ \sin\theta}$$

$$\therefore z = \frac{(3+2i\ \sin\theta)(1-2i\ \sin\theta)}{1+4\ \sin^2\theta}$$

$$\therefore z = \frac{3-4\ \sin^2\theta}{1+4\ \sin^2\theta} + i\frac{8\ \sin\ \theta}{1+4\ \sin^2\theta}$$

$z = \bar{z}$ given

$$\therefore \text{Im}|z| = 0$$

$$\therefore \frac{8\ \sin\ \theta}{1+4\ \sin^2\theta} = 0$$

$$\therefore \text{Sin}\ \theta = 0 \qquad \therefore \theta = k\pi,\ k \in z$$

61. ANS : (b)

$$|z_1| = 12, \qquad |z_2 - 3 - 4i| = 5$$

$$|z_2 - (3+4i)| \geq |z_2| \sim |3-4i|$$

$$\therefore 5 \geq |z_2| \sim 5$$

$$\therefore 10 \geq |z_2| \qquad \therefore |z_2| \leq 10$$

now $|z_1 - z_2| \geq |z_1| \sim |z_2|$

$$\geq 12 \sim 10$$

$$\geq 2$$

$\therefore$ minimum value of $|z_1 - z_2|$ = 2

62. ANS : (a)

$\bar{z}\, z^3 + \bar{z}^3 = 350$ Where z = x + iy

$\therefore z\bar{z}\,(z^2 + \bar{z}^2) = 350$

$\therefore (x^2 + y^2)\left[(x+iy)^2 + (x-iy)^2\right] = 350$

$\therefore (x^2 + y^2)\cdot 2(x^2 - y^2) = 350$

$\therefore (x^2 + y^2)(x^2 - y^2) = 175 = 25\times 7$

$\therefore x^2 + y^2 = 25 \qquad x^2 - y^2 = 7$

by solving the Equation, $x^2 = 16\,, y^2 = 9$

$\therefore x = \pm 4,\ y = \pm 3$

$\therefore$ (4, 3), (-4, 3), (-4, -3), (4, -3)

are vertics of rectangle

$\therefore$ area of rectangle $= 8\times 6 = 48$

63. ANS : (b)

$w \neq 1,$ are cube roots of 1

$\therefore w = \dfrac{-1+\sqrt{3}i}{2}\ ,\ w^2 = \dfrac{-1-\sqrt{3}i}{2}$

$$\left|a + bw + cw^2\right| = \left|a + b\left(\frac{-1+\sqrt{3}i}{2}\right) + c\left(\frac{-1-\sqrt{3}i}{2}\right)\right|$$

$$= \frac{1}{2}\left|(2a-b-c) + (b\sqrt{3} - c\sqrt{3})i\right|$$

$$= \frac{1}{2}\left[(2a-b-c)^2 + (\sqrt{3}b - \sqrt{3}c)^2\right]^{1/2}$$

$$= \frac{1}{2}\left[4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac + 3b^2 + 3c^2 - 4ac\right]^{1/2}$$

$$= \frac{1}{2}\left[4a^2 + 4b^2 + 4c^2 - 4ab - 2bc - 4ac\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]^{1/2}$$

a, b, c are distinct $\therefore a \neq b \neq c$

and a, b, c are integers

$\therefore$ minimum difference between them is 1

$\therefore$ take a = b and $|b-c| = 1,\ |c-a| = 1$

then $|a + bw + cw^2| \geq \frac{1}{\sqrt{2}}(0+1+1)^{1/2}$

$$\geq \frac{1}{\sqrt{2}}(2)^{1/2}$$

$$\geq 1$$

$\therefore$ minimum value of $|a + bw + cw^2|$, is 1

64. ANS : (c)

$$\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$$

$$\therefore \left|\frac{z_1 - z_3}{z_2 - z_3}\right| = \left|\frac{1 - i\sqrt{3}}{2}\right| = 1$$

$$\therefore |z_1 - z_3| = |z_2 - z_3| ------(1)$$

now $\frac{z_1 - z_3}{z_2 - z_3} - 1 = \frac{-1 - i\sqrt{3}}{2} - 1$

$$\therefore \frac{z_1 - z_3 - z_2 - z_3}{z_2 - z_3} = \frac{-1 - i\sqrt{3}}{2}$$

$\therefore$ $|z_1 - z_2| = |z_2 - z_3|$ --------------(2)

from (1) & (2)

$$|z_1 - z_2| = |z_2 - z_3| = |z_1 - z_3|$$

$\therefore$ given triangle is an equilateral

Answers

1	a	21	c	41	c	61	b
2	b	22	b	42	d	62	a
3	b	23	c	43	d	63	b
4	c	24	a	44	a	64	c
5	b	25	a	45	b		
6	a	26	c	46	b		
7	c	27	b	47	d		
8	b	28	a	48	c		
9	b	29	b	49	c		
10	d	30	b	50	d		
11	d	31	a	51	b		
12	a	32	c	52	c		
13	d	33	a	53	b		
14	d	34	c	54	d		
15	a	35	b	55	b		
16	c	36	b	56	b		
17	c	37	d	57	b		
18	c	38	c	58	a		
19	b	39	d	59	d		
20	c	40	a	60	c		

●●●

Unit - 2
Quadratic Equation
Important Points

Standard form quadratic equation $ax^2 + bx + c = 0\ (a \neq 0)$, a, b, c $\in R$

α, β are roots of quadratic equation

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Discriminant Δ OR D $\qquad \Delta = b^2 - 4aC$

Natures of roots:

(i) If $\Delta > 0$ and perfect square, then roots are real, rational and distinct

(ii) If $\Delta > 0$ and not perfect square, then roots are real, irrational and distinct

(iii) If $\Delta = 0$ roots are real and equal

(iv) If $\Delta < 0$, then roots are complex conjugate numbers

Sum and product of roots : When α and β are roots

$$\alpha + \beta = \frac{-b}{a} \qquad \alpha\beta = \frac{c}{a}$$

Formation of quadratic equation with given roots :

α & β are roots of quadratic equation, then quadratic equation

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Common roots :

Let $a_1x^2 + b_1x + 9 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ are two distirict q.e. $(a_1, a_2 \neq 0)$

(i) If one root is common then $(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$

(ii) If both roots are common then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Limiting Values :

(i) If $(x - \alpha)(x - \beta) < 0$ then $x \in (\alpha, \beta)$

(ii) If $(x - \alpha)(x - \beta) \leq 0$ then $x \in [\alpha, \beta]$

(iii) If $(x - \alpha)(x - \beta) > 0$ then $x \notin (\alpha, \beta)$

(iv) If $(x-\alpha)(x-\beta) \geq 0$ then $x \notin [\alpha, \beta]$

Nature of roots :

$ax^2 + bx + c = 0 \; (a \neq 0)$ a, b, c $\in$ R

(i) $a + b + c = 0$ then 1 is root of the equation $ax^2 + bx + c = 0$

(ii) a & c have different sign, then the roots must be of opposite sign.

(iii) If $\alpha = -\beta$ then b = 0, ac > 0

(iv) $\alpha = \frac{1}{\beta}$ then c = a

(v) If $ax^2 + bx + c = 0$ has one root $p + iq$ then other will be $p - iq$

(vi) If $ax^2 + bx + c = 0$ where a, b, c are rational has one root $p + \sqrt{q}$, then other will be $p - \sqrt{q}$

Cubic equation :

If α, β, r are roots of the cubic equation,

$$ax^3 + bx^2 + cx + d = 0 \quad (a \neq 0)$$

then (i) $\alpha + \beta + r = -\frac{b}{a}$

(ii) $\alpha\beta + \beta r + r\alpha = \frac{c}{a}$

(iii) $\alpha\beta r = -\frac{d}{a}$

Formation of cubic equation α, β, r are given roots then

$$x^3 - (\alpha + \beta + r)x^2 + (\alpha\beta + \beta r + r\alpha)\,x - \alpha\beta r = 0$$

Maximum and Minimum values of Quadratic polynomial

$P(x) = ax^2 + bx + c \; (a \neq 0)$ be polynomial

(i) If $a > 0$, then the minimum value of $p(x)$ is $\frac{4ac - b^2}{4a}$ at $x = \frac{-b}{2a}$

(ii) If a < 0, then the maximum value of $p(x)$ is $\frac{4ac - b^2}{4a}$, at $x = \frac{-b}{2a}$

Question Bank

1. If 81 is the discriminant of $2x^2+5x-k=0$ then the value of k is________

   (a) 5 (b) 7 (c) -7 (d) 2

2. Discriminant of the quadratic equation $\sqrt{5}x^2 - 3\sqrt{3}x - 2\sqrt{5} = o$ is _______

   (a) 67 (b) 76 (c) -67 (d) -76

3. The value of k for which the quadratic equation $k\,x^2+1 = k\,x + 3\,x - 11\,x^2$ has real and equal roots are

   (a) {-11,-3} (b) {5,7} (c) {5,-7} (d) {-5,-7}

4. If the sum of the roots of $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then bc^2, ca^2, ab^2 are in

   (a) A.P (b) G,P (c) H.P (d) None of these

5. If the equation $x^2 - m(2x-8) - 15=0$ has equal roots then m=___

   (a) 3,-5 (b) -3,5 (c) 3,5 (d) -3,-5

6. The solution set of the equation (x+1) (x+2) (x+3) (x+4) = 120 is ______

   (a) $\left\{-6,1\frac{-5\pm\sqrt{39}i}{2}\right\}$ (b) $\left\{6,-1\frac{-5\pm\sqrt{39}i}{2}\right\}$

   (c) $\left\{-6,-1\frac{-5\pm\sqrt{39}i}{2}\right\}$ (d) $\left\{6,1\frac{-5\pm\sqrt{39}i}{2}\right\}$

7. The solution set of the Equation $x^4-5x^3-4x^2-5x+1= 0$ is_______

   (a) $\left\{3\pm2\sqrt{2},\frac{-1\pm\sqrt{3}i}{2}\right\}$ (b) $\left\{\frac{3\pm2\sqrt{2}}{2},-1\pm\sqrt{3}i\right\}$

   (c) $\left\{-3\pm2\sqrt{2},\frac{1\pm\sqrt{3}i}{2}\right\}$ (d) $\left\{\frac{-3\pm2\sqrt{2}}{2},1\pm\sqrt{3}i\right\}$

8. The solution set of equation $\frac{x-a}{x-b}+\frac{x-b}{x-a}=\frac{a}{b}+\frac{b}{a},(a\neq b)$ is _____

   (a) {a-b, o} (b) $\left\{\frac{a}{b},o\right\}$ (c) {a+b, o} (d) {ab, o}

9. If α & β are roots of quadratic equation $x^2+13x+8 = 0$ then the value of $\alpha^4+\beta^4$=_____

   (a) 23281 (b) 23218 (c) 23128 (d) 23182

10. The quadratic equation with rational, coeffcient the sum of the squares of whose roots is 40 and the sum of the cubes of whose root is 208 is ________

(a) $x^2+4x+12=0$ (b) $x^2-4x-12=0$ (c) $x^2-4x+12=0$ (d) $x^2+4x-12=0$

11. If the ratio of the roots of the quadratic equation $2x^2 + 16x + 3k = 0$ is 4:5 then k=____

(a) $\frac{2560}{243}$ (b) $\frac{243}{2560}$ (c) $\frac{-2560}{243}$ (d) $\frac{-243}{2560}$

12. If α & β are the roots of the equation $x^2-x+1 = 0$ then $\alpha^{2009} + \beta^{2009}$ = ______

(a) -1 (b) 1 (c) -2 (d) 2

13. If one root of the equation $ax^2-6x+c+9=0$ ($a, c \in R, a \neq 0$) is $3-5i$ then a=___ c=___

(a) 1,25 (b) -1,25 (c) 1,-25 (d) -1,-25

14. The roots of equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal, then a, b, c are in ____

(a) A. P. (b) G. P. (c) H. P. (d) None of these

15. If the roots of the eruation $bx^2+cx + a=0$ be imaginary then for all real values of x the expression $3b^2x^2 + 6bcx + 2c^2$ is ____

(a) < 4 ab (b) > - 4 ab (c) , - 4 ab (d) > 4 ab

16. If $\propto$ & β are roots of $4x^2 + 3x + 7=0$ then the value of $\frac{1}{\alpha^3}+\frac{1}{\beta^3}$ =_______

(a) $\frac{-27}{64}$ (b) $\frac{225}{343}$ (c) $\frac{63}{16}$ (d) $\frac{63}{64}$

17. If one root of the eruation $4x^2-6x+p=0$ is $q + 2i$, where p, q $\in$ R then p + q =____

(a) 10 (b) 19 (c) -24 (d) -32

18. If the roots of the qualrathe eruation $(2k+3)x^2 + 2(k+3)x + (k+5) = 0$ $\left(k \in R, k \neq \frac{-3}{2}\right)$ are equal, then K =_____

(a) 1,6 (b) -1,-6 (c) -1,6 (d) 1, -6

19. The quadratic equations havineg the roots $\frac{1}{10-\sqrt{72}}$ & $\frac{1}{10+6\sqrt{2}}$ is _______

(a) $28x^2 - 20x + 1 = 0$ (b) $20x^2 - 28x + 1 = 0$

(c) $x^2 - 20x + 28 = 0$ (d) $x^2 - 28x + 20 = 0$

20. If the roots of equation $a^2x^2 + b^2x + c^2=0$ are the squares of the roots of the eruation $ax^2+bx+c=0$ then a,b,c are in _______ ($a,b,c, \in R - \{0\}$)

(a) G. P. (b) H. P. (c) A. P. (d) None of these

21. The solution set of equation $3\left(x^2+\frac{1}{x^2}\right)$ + 16 $\left(x+\frac{1}{x}\right)$ + 26 = 0 is _____

(a) $\left\{-1,\frac{-1}{3},-3\right\}$ (b) $\left\{1,\frac{1}{3},3\right\}$ (c) $\left\{-1,\frac{1}{3},3\right\}$ (d) $\left\{1,\frac{-1}{3},3\right\}$

22. If the difference of the roots of the equation $x^2 - px + q = 0$ is 1 then______

(a) $p^2+4q^2 = (1+2q)^2$ (b) $q^2+4p^2 = (1+2q)^2$

(c) $p^2-4q^2 = (1+2q)^2$ (d) $q^2+4p^2 = (1-2q)^2$

23. If the sum of the two roots of the equation $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{k}$ is zero then their Product is ____________

(a) $\frac{1}{2}\left(a^2+b^2\right)$ (b) $\frac{-1}{2}\left(a^2+b^2\right)$

(c) $\left(\frac{a+b}{2}\right)^2$ (d) None

24. For the equation $\ell x^2+mx+n+0$, $\ell \neq o$ If α & β are roots of equation and m^3 + $\ell^2 n$ + $\ell n^2 = 3\ \ell mn$ then

(a) $\alpha = \beta^2$ (b) $\alpha^3 = \beta$ (c) $\alpha + \beta = \alpha\beta$ (d) $\alpha\beta = 1$

25. If $b^2 = ac$, equation $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have common roots then $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in _______

(a) A. P. (b) G. P. (c) H. P. (d) None of these

26. a, b, $\in$R, $a \neq b$ roots of equation $(a-b)x^2 + 5(a+b)x - 2(a-b) = 0$ are

(a) Real and distinct (b) Complex

(c) real and equal (d) None

27. Solution set of equation $x = \sqrt{12+\sqrt{12+\sqrt{12.......upto\ \infty}}}$ is _______

(a) 4 (b) -4 (c) 3 (d) -3

28. The solution set of equation $\left(5+2\sqrt{6}\right)^{x^2-3}+\left(5-2\sqrt{6}\right)^{x^2-3}$ = 10 is ________

(a) $\left\{\pm2,\pm\sqrt{2}\right\}$ (b) $\left\{\pm3,\pm\sqrt{3}\right\}$ (c) $\left\{\pm5,\pm\sqrt{5}\right\}$ (d) $\left\{\pm6,\pm\sqrt{6}\right\}$

29. Construct the guadratic equation whose roots are three times the roots of $5x^2-3x+3 = 0$

(a) $5x^2 - 9x + 27 = 0$ (b) $5x^2 + 9x + 27 = 0$ (c) $5x^2 - 9x - 27 = 0$ (d) $5x^2 + 9x - 27=0$

30. For equation $2x^2 + 16x + 3k = 0$ sum of the squares of roots is 10 then k = ______

(a) 12 (b) 15 (c) 18 (d) 21

31. For the equation $x^2+k^2 = (2k+2)x$, $k \in R$ roots are complex then __________

(a) $k = \frac{-1}{2}$ (b) $k > \frac{-1}{2}$ (c) $k < \frac{-1}{2}$ (d) $\frac{-1}{2} < k < o$

32. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 =0$ are greater then -2 but less than 4 lie in interval

(a) $m > 3$ (b) $-1 < m < 3$ (c) $1 < m < 4$ (d) $-2 < m < o$

33. If $o < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$ then tanx is ________

(a) $\frac{4-\sqrt{7}}{3}$ (b) $-\left(\frac{4+\sqrt{7}}{3}\right)$ (c) $\frac{1+\sqrt{7}}{3}$ (d) $\frac{1-\sqrt{7}}{3}$

34. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$ then the set of possible values of a is __________

(a) (-3, 3) (b) $(-3, \infty)$ (c) $(3, \infty)$ (d) $(-\infty, -3)$

35. The quadratic equation $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common The other roots of the first and second equations are integers in the ratio 4:3 Then, the common root is

(a) 1 (b) 4 (c) 3 (d) 2

36. Hardik and Shivang attempted to Solve a quadratic equation Hardik made a mistake in writting down the constant term and ended up in roots (4,3) Shivang made a mistake in writting down coefficient of x to get roots (3,2) The correct roots of equation are

(a) -4, 3 (b) 6, 1 (c) 4, 3 (d) -6, -1

37. In ΔABC, $m\angle C = \frac{\pi}{2}$ If $\tan\left(\frac{A}{2}\right)$ and $\tan\left(\frac{B}{2}\right)$ are the roots of $ax^2 + bx + c =0$, $a \neq o$ then

(a) b=a+c (b) b=c (c) c=a+b (d) a=b+c

38. Let two mumbers have arithmetic mean 9 and geometric mean 4 then these numbers are the roots of the quadratic equation _____

(a) $x^2 + 18x + 16 = 0$ (b) $x^2 - 18x + 16 = 0$

(c) $x^2 + 18x - 16 = 0$ (d) $x^2 - 18x - 16 = 0$

39. It one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is _________

(a) $\frac{49}{4}$ (b) 12 (c) 3 (d) 4

40. The sum of the roots of the equation x^2 - 3 l x l - 10 = 0 is ________

(a) 3 (b) -3 (c) -10 (d) 0

41. The quadtaric equation whose roots are A. M and positive G. M of the roots of $x^2 - 5x + 4 = 0$ is _________

(a) $x^2 + 9x + 5 = 0$ (b) $2x^2 + 9x + 10=0$

(c) $2x^2 - 9x + 10=0$ (d) $2x^2 - 9x - 10=0$

42. The minimum value of $(x+a)^2 + (x+b)^2 + (x+c)^2$ will be at x equal to

(a) $\frac{a+b+c}{3}$ (b) $-\frac{a+b+c}{3}$ (c) $\sqrt{abc}$ (d) $a^2 + b^2 + c^2$

43. The mumber of real values of x satistying the equation $3\left(x^2+\frac{1}{x^2}\right)-16\left(x+\frac{1}{x}\right)+26=0$ is

(a) 1 (b) 2 (c) 3 (d) 4

44. The only value of x satistying the equation $\sqrt{\frac{x}{x+2}}-\sqrt{\frac{x+2}{x}}=\frac{3}{2}$ is ________

(a) $\frac{8}{3}$ (b) $\frac{-8}{3}$ (c) -4 (d) 4

45. The sum of all the roots of $|x-5|^2-|x-5|$ - 6 = 0 is ________

(a) 10 (b) 6 (c) 0 (d) None

46. If $\alpha + \beta = 5$ $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{p}{\propto}$ is ______

(a) $3x^2 - 19x + 3 = 0$ (b) $x^2 + 5x - 3 = 0$

(c) $x^2 - 5x + 3=0$ (d) $3x^2 - 25x + 3 = 0$

47. If 2a + 3b + 6c = 0 then atleast one root of the equation $ax^2 + bx + c = 0$ lies in the interval _______

(a) (2 ,3) (b) (1, 2) (c) (0, 1) (d) (1, 3)

48. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers then $b^2 - 4c$ = ___________

(a) -2 (b) -3 (c) 3 (d) 1

49. The mumber of values of x in the interval [0, 3π] Satisfying the equation 2 Sin^2 x + 5 Sin x - 3 = 0 is

(a) 6 (b) 1 (c) 2 (d) 4

50. Find the value of a, for which the sum of square of roots of equation x^2 - (a - 1) x-a-1 = 0 assume the least value

(a) 0 (b) 1 (c) 2 (d) 3

51. If α, β are roots of equation $ax^2 + bx + c = 0$ then value of $(\alpha a + b)^{-2} + (\beta a + b)^{-2}$ is ___________

(a) $\frac{b^2 - 4ac}{a^2c^2}$ (b) $\frac{b^2 - ac}{a^2c^2}$ (c) $\frac{b^2 - 2ac}{a^2c^2}$ (d) $\frac{b^2 + 2ac}{a^2c^2}$

52. If tanA & tanB are roots of $x^2 - px + q = 0$ then the value of $\cos^2(A+B)$ = ________

(a) $\frac{(1-q)^2}{p^2+(1-q)^2}$ (b) $\frac{p^2}{p^2+(1-q)^2}$

(c) $\frac{(1-q)^2}{p^2+q^2}$ (d) $\frac{p^2}{p^2+q^2}$

53. The mumber of real solution of the equation $27^{\frac{1}{x}} + 12^{\frac{1}{x}} = 2.8^{\frac{1}{x}}$

(a) 1 (b) 2 (c) 3 (d) 0

54. If α, β are roots of $8x^2 - 3x + 27 = 0$ then $\left(\frac{\alpha^2}{\beta}\right)^{\frac{1}{3}} + \left(\frac{\beta^2}{\alpha}\right)^{\frac{1}{3}}$ = ___________

(a) $\frac{1}{3}$ (b) $\frac{7}{2}$ (c) 4 (d) $\frac{1}{4}$

55. For all $X \in R$, the value of expression $\frac{x^2+2x+1}{x^2+2x+7}$ lies in _______

(a) [2, 3] (b) [0, 1] (c) [1, 2] (d) [0, 2]

56. If $o \le x \le \pi$ and $16^{Sin^2 x} + 16^{Cos^2 x} = 10$ then x = ________

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

57. If $f(x) = 2x^3 + mx^2 - 13x + n$ and $(x-2)$, $(x-3)$ are factor of $f(x)$ then (m, n) = _______

(a) (-5, -30) (b) (-5, 30) (c) (5, 30) (d) (5, -30)

58. $a^{\log_a (x^2-4x+5)} = 3x - 5$ then the solution set is ______

(a) {5, -2} (b) {-5, 2} (c) {-5, -2} (d) {5, 2}

59. If α, β are roots of $x^2 + px + q = 0$ and $x^{2n} + p^n x^n + q^n = 0$ and if $\frac{\alpha}{\beta}$ is one root of $x^n + 1 + (x+1)^n = 0$ then n must be __________

(a) even integer (b) odd integer

(c) rational but not integer (d) None of these

60. If α, β are roots of the equation $x^2 + px + q = 0$ and γ, δ are roots of $x^2 + rx + s = 0$, then the vralue of $(\alpha - \gamma)^2 + (\beta - \gamma)^2 + (\alpha - \delta)^2 + (\beta - \delta)^2$ is __________

(a) $2(p^2 + r^2 - pr + 2q - 2s)$ (b) $2(p^2 + r^2 - pr + 2q + 2s)$

(c) $2(p^2 + r^2 - pr - 2q - 2s)$ (d) $2(p^2 + r^2 + pr - 2q + 2s)$

61. The roots of the equation $(5-x)^4 + (4-x)^4 = (q-2x)^4$ are

(a) all imaginary (b) all real

(c) Two real and two imaginary (d) None of these

62. If $\alpha, \beta,$ are roots of $24x^2 - 8x - 3 = 0$ and $S_n = \alpha^n + \beta^n$ then $\lim_{n\to\infty} \sum_{r=1}^{n} S_r =$ ....

(a) $\frac{14}{13}$ (b) $\frac{-14}{13}$ (c) $\frac{7}{13}$ (d) $\frac{-7}{13}$

63. If $ax + by = 1,\ px^2 + qy^2 - 1 = 0$ have only one root then

(a) $\frac{a^2}{p} + \frac{b^2}{q} = 1$ (b) $x = -\frac{a}{p}$ (c) $x = \frac{b}{q}$ (d) None of these

64. For all $x \in R$ the number of triplet (ℓ, m, n) satisfying equation $\ell \cos 2x + m \sin^2 x + n = 0$ ....

(a) 2 (b) 4 (c) 6 (d) infinite

65. If $f(x) = x - [x],\ x \in R - \{0\}$ where $[\ x\ ] =$ the greatest integer not greater than x, than number of solution $f(x) + f\left(\frac{1}{x}\right) = 1$............

(a) 0 (b) 1 (c) 2 (d) infinite

66. If the product of roots of equation $x^2 - 5kx + 4e^{4\log k} - 3 = 0$ is 61 then ....

(a) 1 (b) 2 (c) 3 (d) 4

67. If one root of equation $ax^2 + bx + c = 0$ is n power of other root then $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} =$........

(a) $-a$ (b) $-b$ (c) $-c$ (d) None of these

68. $\log_{10} a + \log_{10} \sqrt{a} + \log_{10} \sqrt[4]{a} + ... = b$ and $\frac{\sum_{n=1}^{b}(2n-1)}{\sum_{n=1}^{b}(3n+1)} = \frac{20}{7\log_{10}^{a}}$ than a = ...

(a) 10^5 (b) 10^4 (c) 10^3 (d) 10^2

69. If α, β are roots of equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, \alpha\beta$ are roots of equation $x^2 - qx + r = 0$ then $r =$ ....

(a) $\frac{2}{9}(p-q)(2q-p)$ (b) $\frac{2}{9}(q-p)(2p-q)$

(c) $\frac{2}{9}(q-2p)(2q-p)$ (d) $\frac{2}{9}(2p-q)(2q-p)$

70. For $x \in R$, $3^{72}\left(\frac{1}{3}\right)^{x}\left(\frac{1}{3}\right)^{\sqrt{x}} > 1$

(a) $x \in [0, 64]$ (b) $x \in (0, 64)$

(c) $x \in [0, 64]$ (d) None of these

71. If roots of equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and lessthan 3 then ....

(a) $a < 2$ (b) $2 \le a \le 3$

(c) $3 < a \le 4$ (d) $a > 4$

72. For which value of b, equation $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root.

(a) $-\sqrt{2}$ (b) $-i\sqrt{3}$ (c) $i\sqrt{5}$ (d) $\sqrt{2}$

73. If a, b, c are rational numbers then the roots of equation $abc^2x^2 + 3a^2cx + b^2cx - 6a^2 - ab + 2b^2 = 0$ are ....

(a) imeginary (b) equals (c) rational (d) irrational

74. If $a, b, c \in R$ and the roots of equations $ax^2 + bx + c = 0$ and $x^3 + 3x^2 + 3x + 2 = 0$ are common then ....

(a) $a = b \ne c$ (b) $a = b = -c$

(c) $a = b = c$ (d) None of these

75. If both the roots of equation $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are equal then the value of $2r - P$ is ....

(a) 0 (b)1 (c) -1 (d) None of these

76. If $\alpha \,\&\, \beta$ are roots of equation $x^2 + x + 1 = 0$ then the equation whose roots are $\alpha^{19} \,\&\, \alpha^7$ is............

(a) $x^2 - x + 1 = 0$ (b) $x^2 + x + 1 = 0$

(c) $x^2 + x + 3 = 0$ (d) $x^2 - x + 3 = 0$

77. If α & β are roots of $x^2 + nx - c = 0$ then equation whose roots are b and c is ....

(a) $x^2 + (\alpha + \beta + \alpha\beta)x - \alpha\beta(\alpha + \beta) = 0$

(b) $x^2 + (\alpha + \beta + \alpha\beta)x + \alpha\beta(\alpha + \beta) = 0$

(c) $x^2 - (\alpha + \beta + \alpha\beta)x - \alpha\beta(\alpha + \beta) = 0$

(d) $x^2 + 2x - \beta = 0$

78. If the roots of equation $ax^2 + bx + c = 0$ are in m : n then $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} + \frac{b}{\sqrt{ac}} =$ ....

(a) 0 (b) 1 (c) -1 (d) None of these

Hints

1. Ans. (b)

$2x^2 + 5x - k = 0$

$\Delta = b^2 - 4ac = 81$

$\therefore\ 25 - 8k = 81 \qquad \therefore\ k = 7$

2. Ans. (a)

$\sqrt{5}\,x^2 - 3\sqrt{3}x - 2\sqrt{5} = 0$

$a = \sqrt{b},\ b = -3\sqrt{3},\ c = -2\sqrt{5}$

$\Delta = b^2 - 4ac = 27 - 4\left(\sqrt{5}\right)\left(-2\sqrt{5}\right) = 67$

3. Ans. (c)

$kx^2 + 1 = kx + 3x - 11x^2$

$\therefore\ (k + 11)x^2 - (k + 3)\,x + 1 = 0$

$\Delta = b^2 - 4ac$

$= k^2 + 2k - 35$

roots are real and equal $\Delta \geq 0$

$k^2 + 2k - 35 \geq 6$

$\therefore\ \left|k + 1\right| \geq 6$

$\therefore\ -6 \geq k + 1 \geq 6$

$\therefore\ k \leq -7,\ k \geq 5$

$\therefore\ k = -7,\ k = 5 \qquad k \in \{-7, 5\}$

4. Ans. (a)

$ax^2 + bx + c = 0$

suppose α, β are roots of equation

$\alpha + \beta = \frac{-b}{a},\quad \alpha\beta = \frac{c}{a}$

also, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$\alpha + \beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$

$$\frac{-b}{a} = \frac{\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$\therefore\ a^2c = \frac{ab^2 + bc^2}{2}$$

$\therefore\ ab^2,\ a^2c,\ bc^2$ are in arithmatic progresion

5. Ans. (c)

$x^2 - m\,(2x - 8) - 15 = 0$

$\therefore\ x^2 - 2mx + 8m - 15 = 0$

roots are equal $\therefore\ \Delta = 0$

$\therefore\ 4m^2 - 32m + 60 = 0$

$\therefore\ m = 5,\ m = 3$

6. Ans. (a)

$(x + 1)(x + 2)\,(x + 3)\,(x + 4) = 120$

$(x^2 + 5x + 4)\,(x^2 + 5x + 6) = 120$

suppose $x^2 + 5x = m$

$(m + 4)\,(m + 6) = 120 = 10 \times 12$

$m + 4 = 10 \Rightarrow m = 6$

$m + 4 = -12 \Rightarrow m = -16$

$x^2 + 5x = 6,$ $\qquad$ $x^2 + 5x = -16$

$\therefore x^2 + 5x - 6 = 0$ $\qquad$ $x^2 + 5x + 16 = 0$

$x = -6,\ x = 1$ $\qquad$ $\Delta = -39 < 0$

$\sqrt{\Delta} = \sqrt{39}\ i$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x = \frac{-5 \pm \sqrt{39}\ i}{2}$$

solution set $\left\{-6, 1, \frac{-5 \pm \sqrt{39}\,i}{2}\right\}$

7. Ans. (a)

$$x^4 - 5x^3 - 4x^2 - 5x + 1 = 0$$

$$x^2 - 5x - 4 - \frac{5}{x} + \frac{1}{x^2} = 0$$

$$\therefore\ x^2 + \frac{1}{x^2} - 5\left(x + \frac{1}{x}\right) - 4 = 0$$

Suppose $x + \frac{1}{x} = m$ $\quad\therefore\ x^2 + \frac{1}{x^2} = m^2 - 2$

$\therefore\ m^2 - 5m - 6 = 0$ $\quad\therefore\ m = 6,\ m = -1$

If $m = 6 \Rightarrow x + \frac{1}{x} = 6$

$$\Rightarrow x^2 - 6x + 1 = 0$$

$$\Rightarrow\ x = 3 \pm 2\sqrt{2}$$

If $m = -1 \Rightarrow x + \frac{1}{x} = -1$

$\Rightarrow x^2 + x + 1 = 0$ $\quad$ Solution set is: $\left\{3 \pm 2\sqrt{2},\ \frac{-1 \pm \sqrt{3}\,i}{2}\right\}$

$$\Rightarrow x = \frac{-1 \pm \sqrt{3}\,i}{2}$$

8. Ans. (c)

$$\frac{x - a}{x - b} + \frac{x - b}{x - a} = \frac{a}{b} + \frac{b}{a}\ (a \neq b)$$

Suppose $\frac{x - a}{x - b} = m$

$$\therefore\ m + \frac{1}{m} = \frac{a^2 + b^2}{ab}$$

$$\therefore\ abm^2 - \left(a^2 + b^2\right)m + ab = 0$$

$$\therefore\ (bm - a)(am - b) = 0$$

$m = \frac{a}{b}, \quad m = \frac{b}{a}$

If $\quad m = \frac{a}{b} \Rightarrow \frac{x-a}{x-b} = \frac{a}{b}$

$\Rightarrow x = 0$

If $\quad m = \frac{b}{a} \Rightarrow \frac{x-a}{x-b} = \frac{b}{a}$

$\Rightarrow x = a + b$

Solution Set : $\{0, \; a + b\}$

9. Ans. (a)

$x^2 + 13x + 8 = 0$

$\alpha + \beta = -13, \; \alpha\beta = 8$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 153$

$\therefore \alpha^2 + \beta^2 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = 23281$

10. Ans. (b)

let α, β are roots

$\therefore \; \alpha^2 + \beta^2 = 40, \; \alpha^3 + \beta^3 = 208$ (given)

Suppose $\alpha + \beta = m, \; \alpha\beta = n$

$\alpha^2 + \beta^2 = 40 \Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 40$

$\Rightarrow m^2 - 2n = 40$

$\Rightarrow n = \frac{m^2 - 40}{2}$ ...(1)

$\alpha^3 + \beta^3 = 208 \Rightarrow (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = 208$

$\Rightarrow m(40 - n) = 208$ ...(2)

from (i) & (ii)

$m\left(40 - \frac{m^2 - 40}{2}\right) = 208$

$\therefore m^3 - 120m + 416 = 0$

$(m - 4)(m^2 + 4m - 104) = 0$

$\therefore m - 4 = 0$

$m = 4$

$n = \frac{m^2 - 40}{2} = -12$

$\therefore \alpha + \beta = 4, \quad \alpha\beta = n = -12$

$\therefore x^2 - 4x - 12 = 0$

11. Ans. (a)

$2x^2 + 16x + 3k = 0$

Suppose α, β the roots of the given equation

$\therefore \alpha + \beta = -8, \qquad \alpha\beta = \frac{3K}{2}$

also $\alpha : \beta = 4 : 5$

$\therefore \alpha = 4m, \beta = 5m$

$\therefore \alpha + \beta = -8 \Rightarrow 9m = -8$

$\Rightarrow m = -\frac{8}{9}$

$\therefore \alpha = -\frac{32}{9} \qquad \beta = \frac{-40}{9}$

$\alpha\beta = \frac{3K}{2} \Rightarrow K = \frac{2560}{243}$

12. Ans. (b)

$x^2 - x + 1 = 0$

$x^3 + 1 = (x+1)(x^2 - x + 1)$

$= 0$

$\therefore x^3 = -1$

If α, β are roots then $\alpha + \beta = 1, \alpha\beta = 1$

now, $x^3 + 1 = 0$

$\therefore \alpha^3 = -1, \qquad \beta^3 = -1$

$\alpha^{2009} + \beta^{2009} = \frac{\alpha^{2010}}{\alpha} + \frac{\beta^{2010}}{\beta}$

$$= \frac{(\alpha^3)^{670}}{\alpha} + \frac{\beta^{2010}}{\beta}$$

$$= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{1}{1} = 1$$

13. Ans. (a)

$ax^2 - 6x + c + 9 = 0$

Suppose $\alpha = 3 - 5i \qquad \beta = 3 + 5i$

$\therefore \ \alpha + \beta = 6 \quad \alpha\beta = 9 + 25 = 34$

$\therefore \ \frac{6}{a} = 6, \qquad \frac{c + 9}{a} = 34$

$\therefore a = 1, \qquad c = 25$

14. Ans. (c)

$a(b - c)x^2 + b(c - a)x + c(a - b) = 0$

roots are equal $\quad \therefore \ \Delta = 0$

$b^2(c - a)^2 - 4ac(b - c)(a - b) = 0$

$\therefore [b(a + c) - 2ac]^2 = 0$

$\therefore ab + bc - 2ac = 0$

$\therefore ab - ac = ac - bc$

$\therefore \frac{1}{c} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a}$

$\therefore \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$, are in athematic progression a, b, c, are in Harmonic Progression

15. Ans. (b)

$bx^2 + cx + a = 0$

roots are complex $\Delta < 0$

$\therefore c^2 - 4ab < 0 \qquad \therefore c^2 < 4ab$

$\therefore -c^2 > -4ab \qquad \ldots(1)$

$3b^2x^2 + 6bcx + 2c^2 = 3(bx + c)^2 - c^2$

$> -c^2$

$> -4ab$

16. Ans. (b)

$$4x^2 + 3x + 7 = 0$$

$$\alpha + \beta = \frac{-3}{4}, \quad \alpha\beta = \frac{7}{4}$$

$$\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{\alpha^3\beta^3}$$

$$= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{(\alpha\beta)^3}$$

$$= \frac{225}{343}$$

17. Ans. (b)

$$4x^2 - 6x + p = 0$$

$$\alpha + \beta = \frac{3}{2}, \alpha\beta = \frac{p}{4}$$

Suppose , $\alpha = q + 2i, \quad \beta = q - 2i$

$$\alpha + \beta = 2q \Rightarrow 2q = \frac{3}{2}$$

$$\Rightarrow q = \frac{3}{4}$$

$$\alpha\beta = q^2 + 4 \Rightarrow \frac{p}{4} = q^2 + 4$$

$$\Rightarrow p = \frac{73}{4}$$

$\therefore p + q = 19$

18. Ans. (b)

$$(2k + 3)x^2 + 2(k + 3)x + (k + 5) = 0$$

roots are equal $\quad \therefore \Delta = 0$

$$4(k + 3)^2 - 4(2k + 3)(k + 5) = 0$$

$$\therefore (k + 1)(k + 6) = 0$$

$$\therefore k = -1, k = -6$$

19. Ans. (a)

$$\alpha = \frac{1}{10-\sqrt{72}} = \frac{10+\sqrt{72}}{28}$$

$$\beta = \frac{1}{10+6\sqrt{2}} = \frac{10-\sqrt{72}}{28}$$

$$\therefore \quad \alpha+\beta = \frac{5}{7}, \quad \alpha\beta = \frac{1}{28}$$

$$x^2 - (\alpha+\beta)x + \alpha\beta = 0$$

$$\therefore \quad x^2 - \frac{5}{7}x + \frac{1}{28} = 0$$

$$\therefore \quad 28x^2 - 20x + 1 = 0$$

20. Ans. (a)

$\therefore$ α, β are roots of equation $ax^2 + bx + c = 0$

$$\therefore \quad \alpha+\beta = \frac{-b}{a}, \qquad \alpha\beta = \frac{c}{a}$$

Suppose $\alpha_1 \beta_1$ are roots of equation $a^2x^2 + b^2x + c^2 = 0$

$$\therefore \quad \alpha_1+\beta_1 = \frac{-b^2}{a^2} \qquad \alpha_1\beta_1 = \frac{c^2}{a^2}$$

Also, $\alpha_1 = \alpha^2, \ \beta_1 = \beta^2$

$$\alpha^2+\beta^2 = \frac{-b^2}{a^2}$$

$$\therefore \quad (\alpha+\beta)^2 - 2\alpha\beta = \frac{-b^2}{a^2}$$

$$\therefore \quad \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right) = -\frac{b^2}{a^2}$$

$$\therefore \quad b^2 = ac$$

a, b, c are in Geometric progression

21. Ans. (a)

$$3\left(x^2 + \frac{1}{x^2}\right) + 16\left(x + \frac{1}{x}\right) + 26 = 0$$

Suppose $x + \frac{1}{x} = m$ $\therefore\ x^2 + \frac{1}{x^2} = m^2 - 2$

$\therefore\ 3\left(m^2 - 2\right) + 16m + 26 = 0$

$\therefore\ m = -2,\ m = \frac{-10}{3}$

$m = -2 \Rightarrow x + \frac{1}{x} = -2$

$\Rightarrow (x+1)^2 = 0$

$\Rightarrow x = -1$

If $m = -\frac{10}{3} \Rightarrow x + \frac{1}{x} = \frac{-10}{3}$

$\Rightarrow 3x^2 + 10x + 3 = 0$

$\Rightarrow (x+3)(3x+1) = 0$

$\Rightarrow x = -3,\ x = \frac{-1}{3}$

Solution set : $\left\{-1,\ -3,\ \frac{-1}{3}\right\}$

22. Ans. (a)

$x^2 - px + q = 0$

α, β are roots of Equation

$\therefore\ \alpha + \beta = p,\ \alpha\beta = q$

$|\alpha - \beta| = 1$ (given)

$p^2 - 4q = 1$

$p^2 = 4q + 1$

$\therefore\ p^2 + 4q^2 = 4q^2 + 4q + 1 = (2q+1)^2$

23. Ans. (b)

$\frac{1}{x+a} + \frac{1}{a+b} = \frac{1}{k}$

$\therefore\ x^2 + (a + b - 2k)x + ab - (a+b)k = 0$

$\alpha + \beta = 0 \Rightarrow a + b - 2k = 0$

$$\Rightarrow k = \frac{a+b}{2}$$

$$\alpha\beta = ab - (a+b)k$$

$$= -\frac{a^2+b^2}{2}$$

24. Ans. (a)

$\ell x^2 + mx + n = 0$

$\therefore\ \alpha + \beta = \frac{-m}{\ell}, \qquad \alpha\beta = \frac{m}{\ell}$

$\therefore\ m = -\ell(\alpha+\beta), \quad n = \ell\alpha\beta$

also, $m^3 + \ell^2 n + \ell n^2 = 3\ell mn$

$\therefore\ -\ell^3(\alpha+\beta)^2 + \ell^2\alpha\beta + \ell^3\alpha^2\beta^2 = -3\ell^2(\alpha+\beta)(\alpha\beta)$

$\therefore\ (\beta - \alpha^2)(\alpha - \beta^2) = 0$

$\therefore\ \alpha^2 = \beta$ or $\beta^2 = \alpha$

$\alpha = \beta^2$

25. Ans. (a)

$ax^2 + 2bx + c = 0$ and $b^2 = ac$

$$x = \frac{-2b \pm \sqrt{4b^2 - 4ac}}{2a}$$

$$\therefore\ x = \frac{-b}{a}$$

now $x = \frac{-b}{a}$ is also roots of $dx^2 + 2ex + f = 0$

$\therefore\ d\left(\frac{-b}{a}\right)^2 + 2e\left(\frac{-b}{a}\right) + f = 0$

$\therefore\ \frac{2eb}{a} = \frac{db^2}{a^2} + f$

$\therefore\ \frac{2eb}{a} = \frac{dac}{a^2} + f$

$2eb = dc + af$

$\therefore \quad \frac{2e}{b} = \frac{d}{a} + \frac{f}{c}$

$\therefore \quad \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in Arithmetic progression

26. Ans. (a)

$(a-b)x^2 + 5(a+b)x - 2(a-b) = 0$

$\Delta = 25(a+b)^2 + 8(a-b)^2 > 0$

roots are real and distict

27. Ans. (a)

$x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots\ldots\ldots\ldots\infty}}}$

$\therefore \quad x = \sqrt{12 + x}$

$\therefore \quad x^2 - x - 12 = 0$

$\therefore \quad x = 4, x = -3$ but $x > 0$

$\therefore \quad x = 4$

28. Ans. (a)

$\left(5 + 2\sqrt{6}\right)^{x^2-3} + \left(5 - 2\sqrt{6}\right)^{x^2-3} = 10$

Suppose $\left(5 + 2\sqrt{6}\right)^{x^2-3} = m$

$\therefore \quad \left(5 - 2\sqrt{6}\right)^{x^2-3} = \frac{1}{m}$

$\therefore m + \frac{1}{m} = 10$

$\therefore \quad m = 5 \pm 2\sqrt{6}$

If $m = 5 + 2\sqrt{6} \Rightarrow \left(5 + 2\sqrt{6}\right)^{x^2-3} = 5 + 2\sqrt{6}$

$\Rightarrow \quad x^2 - 3 = 1$

$\Rightarrow \quad x^2 = 4 \qquad \therefore \quad x = \pm 2$

If $m = 5 - 2\sqrt{6} \Rightarrow x^2 - 3 = -1$

$\Rightarrow x = \pm\sqrt{2}$

Solution set : $\left\{ \pm 2, \pm\sqrt{2} \right\}$

29. Ans. (a)

Suppose $\alpha, \beta,$ are roots of equation $5x^2 - 3x + 3 = 0$

$$\therefore \ \alpha + \beta = \frac{3}{5}, \alpha\beta = \frac{3}{5}$$

Suppose α_1 & β_1 are roots of required equation

$\alpha_1 = 3\alpha, \ \beta_1 = 3\beta$ (given)

$$\therefore \ \alpha_1 + \beta_1 = \frac{9}{5} \qquad \alpha_1\beta_1 = \frac{27}{5}$$

$\therefore$ required equation

$$x^2 - \frac{9}{5}x + \frac{27}{5} = 0$$

$\therefore \ 5x^2 - 9x + 27 = 0$

30. Ans. (c)

$2x^2 + 16x + 3k = 0$

$$\alpha + \beta = -8, \alpha\beta = \frac{3k}{2}$$

also, $\alpha^2 + \beta^2 = 10 \Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 10$

$\Rightarrow 64 - 3k = 10$

$\Rightarrow k = 18$

31. Ans. (c)

$x^2 + k^2 = (2k + 2)x$

$\therefore \ x^2 - (2k + 2)x + k^2 = 0$

$\therefore \ \Delta = (2k + 2)^2 - 4k^2 = 4(2k + 1)$

now roots are complex $\therefore \ \Delta < 0$

$\therefore \ 4(2k + 1) < 0 \qquad \therefore \ k < \frac{-1}{2}$

32. Ans. (b)

$x^2 - 2mx + m^2 - 1 = 0$

$\Delta = 4$

$\therefore \ \alpha = m + 1, \beta = m - 1$

$\alpha < 4$ & $\beta > -2$

$m + 1 < 4,\ m - 1 > -2 \qquad \therefore\ -1 < m < 3$

33. Ans. (b)

$0 < x < \pi$ yLk $\cos x + \sin x = \frac{1}{2}$

$\therefore\ \frac{1 - t^2}{1 + t^2} + \frac{2t}{1 + t^2} = \frac{1}{2}$ Where $t = \tan\frac{x}{2}$

$\therefore\ 3t^2 - 4t - 1 = 0$

$\Delta = 28$

$t = \frac{2 \pm \sqrt{7}}{3}$

$0 < x < \pi \qquad \therefore\ 0 < \frac{x}{2} < \frac{\pi}{2} \qquad \therefore\ \tan\frac{x}{2} > 0$

$\therefore\ \tan\frac{x}{2} \neq \frac{2 - \sqrt{7}}{3}$

$\therefore\ \tan\frac{x}{2} = \frac{2 + \sqrt{7}}{3}$

34. Ans. (a)

$x^2 + ax + 1 = 0$

$\alpha + \beta = -a,\ \alpha\beta = 1$

$|\alpha - \beta| < \sqrt{5}$ (given)

$\therefore\ (\alpha - \beta)^2 < 9$

$a^2 < 9$

$\therefore |a| < 3 \quad \therefore\ a \in (-3, 3)$

35. Ans. (d)

Suppose $\alpha, \beta,$ are roots of $x^2 - 6x + a = 0$

$\alpha + \beta = 6,\ \alpha\beta = a$

Suppose $\alpha_1\ \beta_1$ are roots of $x^2 - cx + 6 = 0$

$\alpha_1 + \beta_1 = c,\ \alpha_1\ \beta_1 = 6$

$\alpha = \alpha_1, \beta : \beta_1 = 4:3$ (given) and $\alpha_1 \; \beta_1 \in z$

$\therefore \; \beta = 4k, \; \beta_1 = 3k, \; k \neq 0$

$\alpha, \beta, = 6 \Rightarrow \alpha(3k) = 6$ $\qquad \therefore \alpha = \frac{2}{k}$

$\alpha\beta = a \Rightarrow \alpha \, 14k = a$ $\qquad \therefore \alpha = \frac{a}{4k}$

$\therefore \; \frac{2}{k} = \frac{a}{4k}$ $\qquad \therefore \; a = 8$

now, $\alpha + \beta = 6, \alpha\beta = 8$

$\therefore \quad \alpha = 4, \beta = 2$ or $\alpha = 2, \beta = 4$ in

If $\alpha = 4$ than $4 = \frac{2}{k}$ $\quad \therefore \; k = \frac{1}{2}$

$\therefore \; \beta = 4k = 2$ and $\therefore \; \beta_1 = 3k = \frac{3}{2} \notin z$

$\therefore \; \alpha \neq 4$

If $\alpha = 2$ than $2 = \frac{2}{k}$ $\quad \therefore k = 1$

$\beta = 4k = 4 \in z$ and $\beta_1 = 3k = 3 \in z$

$\therefore \; \alpha = 2$

36. Ans. (b)

for Hardik roots are (4,3)

$\alpha + \beta = 7$

$\alpha \; \beta = 12$

Quadratic Equation $x^2 - 7x + 12 = 0$

hear constant is wrong (given)

$\{3, 2\}$

$\alpha + \beta = 5$

$\alpha \times \beta = 6$

Quadratic Equation $x^2 - 5x + 6 = 0$

Co-efficient x is wrong (given)

Correct solution $x^2 - 7x + 12 = 0$

$\therefore\ (x-6)(x-1)=0$

$\therefore\ x=6,\ x=1$

6, 1

37. Ans. (c)

In ΔABC, $m\angle C=\frac{\pi}{2}\quad \therefore\ A+B=\frac{\pi}{2}$

$\tan\frac{A}{2},\ \tan\frac{B}{2}$ are roots of equation $ax^2+bx+c=0$

$\text{sum of roots} = -\frac{b}{a}$

$\therefore\ \tan\frac{A}{2}+\tan\frac{B}{2}=\frac{-b}{a}$

$\therefore\ \sin\left(\frac{A}{2}+\frac{B}{2}\right)=\frac{-b}{a}\cos\frac{A}{2}\cos\frac{B}{2}$

$\therefore\ \cos\frac{A}{2}.\cos\frac{B}{2}=\frac{-a}{\sqrt{2}b}$ ...(i)

$\text{produt of roots} = \frac{c}{a}$

$\therefore\ \tan\frac{A}{2}.\ \tan\frac{B}{2}=\frac{c}{a}$

$\therefore\ \sin\frac{A}{2}.\ \sin\frac{B}{2}=\frac{-c}{\sqrt{2}\,b}$ ...(ii)

$\cos\left(\frac{A}{2}+\frac{B}{2}\right)=\cos\frac{A}{2}\cos\frac{B}{2}-\sin\frac{A}{2}\sin\frac{B}{2}$

$\cos\frac{a}{4}=\frac{-a}{b\sqrt{2}}+\frac{c}{b\sqrt{2}}$

$c-a=b \qquad \therefore\ c=a+b$

38. Ans. (b)

Suppose $\alpha,\ \beta$ two numbers

given $\frac{\alpha+\beta}{2}=9 \qquad \therefore\ \alpha+\beta=18$

and $\sqrt{\alpha\beta} = 4$ $\therefore$ $\alpha\beta = 16$

$\therefore$ required equation whose roots are

α, β is $x^2 - 18x + 16 = 0$

39. Ans. (a)

$x = 4$ is a one root of equation $x^2 + px + 12 = 0$

$\therefore$ $16 + 4p + 12 = 0$ $\therefore$ $P = -7$

for equation $x^2 + px + q = 0$ roots are equal

$\therefore$ $\Delta = 0$

$p^2 - 4q = 0$ $\therefore$ $q = \frac{49}{4}$

40. Ans. (d)

$x^2 - 3|x| - 10 = 0$

$|x^2| - 3|x| - 10 = 0$

Suppose $|x| = y > 0$

$\therefore$ $y^2 - 3y - 10 = 0$

$\therefore$ $(y - 5)(y + 2) = 0$

$y = 5$, $y = -2$ but $y \neq -2$

$|x| = 5$ $\therefore x = \pm 5$

$\therefore$ sum of roots $= 5 + (-5) = 0$

41. Ans. (c)

$x^2 - 5x + 4 = 0$

Suppose α, β are roots

$\therefore$ $\alpha + \beta = 5$, $\alpha\beta = 4$

Suppose α_1, β_1 are roots of required equation

$\therefore$ $\alpha_1 = \frac{\alpha + \beta}{2}$ & $\beta_1 = \sqrt{\alpha\beta}$

$= \frac{5}{2}$ $= 2$

$\alpha_1 + \beta_1 = \frac{9}{2}$ $\alpha_1\beta_1 = 5$

$\therefore$ required equation is

$2x^2 - 9x + 10 = 0$

42. Ans. (b)

Suppose $f(x) = (x + a)^2 + (x + b)^2 + (x + c)^2$

$\therefore \quad f(x) = 3x^2 + 2(a + b + c)x + a^2 + b^2 + c^2$

the co-efficient of $x^2 = 3 > 0$

the minimum value of f(x) will be at $x = \frac{-b}{2a}$

$\therefore \quad x = \frac{-b}{2a}$

$= \frac{-(a + b + c)}{3}$

43. Ans. (b)

$3\left(x^2 + \frac{1}{x^2}\right) - 16\left(x + \frac{1}{x}\right) + 26 = 0$

Suppose $\quad x + \frac{1}{x} = m \qquad \therefore \quad x^2 + \frac{1}{x^2} = m^2 - 2$

$\therefore \quad 3m^2 - 16m + 20 = 0$

$\therefore \quad m = 2, \ m = \frac{-10}{3}$

$m = 2 \Rightarrow x + \frac{1}{x} = 2$

$\Rightarrow \ x = 1$

$m = -\frac{10}{3} \Rightarrow x + \frac{1}{x} = \frac{-10}{3}$

$\Rightarrow (x + 3)(3x + 1) = 0$

$\Rightarrow x = -3, \ x = \frac{-1}{3}$

number of real roots $= 3$

44. Ans. (b)

$\sqrt{\frac{x}{x+2}} - \sqrt{\frac{x+2}{2}} = \frac{3}{2} = 2 - \frac{1}{2}$

$\therefore \quad \sqrt{\dfrac{x}{x+2}} = 2 \qquad \therefore \quad x = \dfrac{-8}{3}$

45. Ans. (a)

$|x-5|^2 - |x-5| - 6 = 0$

suppose $|x-5| = y, \quad y > 0$

$\therefore \quad y^2 - y - 6 = 0$

$\therefore \quad y = 3,\ y = -2$ but $y \neq -2$

$\therefore \quad y = 3 \qquad \therefore \quad |x-5| = 3$

$x = 8,\ x = 2$

$\therefore$ sum of roots $= 8 + 2 = 10$

46. Ans. (a)

$\alpha + \beta = 5,\ \alpha^2 = 5\alpha - 3,\ \beta^2 = 5\beta - 3$

$\alpha^2 + \beta^2 = 5(\alpha + \beta) - 6$

$\alpha\beta = 3$

Suppose $\alpha_1 = \dfrac{\alpha}{\beta},\ \beta_1 = \dfrac{\beta}{\alpha}$

$\therefore \quad \alpha_1 + \beta_1 = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{19}{3}$ & $\alpha_1\beta_1 = 1$

$\therefore$ required quadratic equation is $3x^2 - 19x + 3 = 0$

47. Ans. (c)

$ax^2 + bx + c = 0, \quad 2a + 3b + 6c = 0$

Suppose $f(x) = 2ax^3 + 3bx^2 + 6cx$

$f'(x) = 6(ax^2 + bx + c) = 0$

Also $f(0) = 0,\ f(1) = 2a + 3b + 6c = 0$

$\therefore$ by roll therom for some $x \in (0, 1)$, $f'(x) = 0$

$\therefore$ atleast one root of equation $ax^2 + bx + c = 0$ lies in $(0,1)$

48. Ans. (d)

$x^2 - bx + c = 0$

Suppose $\alpha = n,\ \beta = n + 1$

$\therefore\ \alpha + \beta = 2n + 1,\ \alpha\beta = n(n + 1)$

$b = 2n + 1 \quad c = n(n+1)$

$n = \frac{b-1}{2},\ n^2 + n = c$

$\therefore\ \left(\frac{b-1}{2}\right)^2 + \left(\frac{b-1}{2}\right) = c$

$\therefore\ b^2 - 4c = 1$

49. Ans. (d)

$2\sin^2 x + 5\sin x - 3 = 0$

$\therefore\ \sin x = -3,\ \sin x = \frac{1}{2}$

$\sin x \neq -3$

$\therefore\ \sin x = \frac{1}{2} = \sin\frac{\pi}{6},\ x \in [0, 3\pi]$

$\therefore\ x = \frac{\pi}{6},\ x = \pi - \frac{\pi}{6},\ x = 2\pi + \frac{\pi}{6},\ x = 3\pi - \frac{\pi}{6}$

$\therefore$ The number of value of x in $[0, 3\pi]$ is 4

50. Ans. (b)

$x^2 - (a-1)x - (a+1) = 0$

If α and β are roots than

$\alpha + \beta = a - 1,\ \alpha\beta = -(a + 1)$

now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a - 1)^2 + 5$

$\therefore$ minimum value $\alpha^2 + \beta^2$ will be at $a = 1$

51. Ans. (c)

$ax^2 + bx + c = 0$

$\therefore\ \alpha + \beta = \frac{-b}{a},\ \alpha\beta = \frac{c}{a}$

$(\alpha a + b)^{-2} + (a\beta + b)^{-2} = \frac{b^2 - 2ac}{c^2 a^2}$

52. Ans. (a)

$\tan B$ and $\tan B$ are roots of $x^2 - px + q = 0$

$\therefore\ \tan A + \tan B = P$ and $\tan A . \tan B = q$

$$\therefore\ \tan(A + B) = \frac{p}{1-q}$$

now $\cos^2 (A + B) = \dfrac{1 + \cos 2(A + B)}{2}$

$$= \frac{\left[1 + \dfrac{1 - \tan^2 (A + B)}{1 + \tan^2 (A + B)}\right]}{2}$$

$$= \frac{\left[\dfrac{2}{1 + \tan^2 (A + B)}\right]}{2}$$

$$= \frac{(1-q)^2}{p^2 + (1 - q)^2}$$

53. Ans. (d)

$$27^{\frac{1}{x}} + 12^{\frac{1}{x}} = 2.8^{\frac{1}{x}}$$

$$\therefore\ \left(\frac{27}{8}\right)^{\frac{1}{x}} + \left(\frac{12}{8}\right)^{\frac{1}{x}} = 2$$

$$\therefore\ \left(\frac{3}{2}\right)^{\frac{3}{x}} + \left(\frac{3}{2}\right)^{\frac{1}{x}} = 2$$

Suppose $\left(\dfrac{3}{2}\right)^{\frac{1}{x}} = m$

$\therefore\ m^3 + m - 2 = 0$

$\therefore\ m^3 - m + 2m - 2 = 0$

$\therefore\ m(m^2 - 1) + 2(m - 1) = 0$

$\therefore\ (m - 1)[m(m + 1) + 2] = 0$

$\therefore\ m^2 + m + 2 = 0$ OR $m - 1 = 0$

for $m^2 + m + 2 = 0$, $\Delta < 0$

$\therefore$ real roots does not exits

$\therefore \quad m - 1 = 0 \qquad\qquad \therefore \quad m = 1$

$\therefore \quad \left(\frac{3}{2}\right)^{\frac{1}{x}} = \left(\frac{3}{2}\right)^{0} \qquad\qquad \therefore \quad \frac{1}{x} = 0$ is not possible

$\therefore$ number of real roots = 0

54. Ans. (d)

$8x^2 - 3x + 27 = 0$

$\alpha + \beta = \frac{3}{8}, \ \alpha\beta = \frac{27}{8}$

$$\left(\frac{\alpha^2}{\beta}\right)^{\frac{1}{3}} + \left(\frac{\beta^2}{\alpha}\right)^{\frac{1}{3}} = \frac{\alpha + \beta}{(\alpha\,\beta)^{\frac{1}{3}}}$$

$$= \frac{318}{\left(\frac{27}{8}\right)^{\frac{1}{3}}}$$

$$= \frac{1}{4}$$

55. Ans. (b)

suppose $\frac{x^2 + 2x + 1}{x^2 + 2x + 7} = m$

$\therefore \quad (1 - m)\,x^2 + 2\,(1-m)\,x + 1 - 7\,m = 0$

$\Delta = 4\,(1 - m)^2 - 4\,(1 - m)\,(1 - 7m)$

x is real number. $\therefore \ \Delta \geq 0$

$\therefore \quad (1 - m)\,(6m) \geq 0$

$\therefore \ 0 \leq m \leq 1$

$\therefore m\,(m-1) \leq 0$

$\therefore \ 0 \leq m \leq 1, \ m \in [0, 1]$

56. Ans. (a)

$0 \leq x \leq \pi \qquad\qquad \therefore \ \sin x > 0$

$\therefore\ 16^{\sin^2 x} + 16^{\cos^2 x} = 10$

Suppose $16^{\sin^2 x} = m$

$\therefore\ m + \frac{16}{m} = 10$

$\therefore\ m = 8$ or $m = 2$

$\therefore 16^{\sin^{2x}} = 8$ or $16^{\sin^2 x} = 2$

$\therefore 4\sin^2 x = 3$ or $4\sin^2 x = 1$

$\therefore\ \sin^2 x = \frac{3}{4}$ or $\sin^2 x = \frac{1}{4}$

$\therefore\ \sin x = \frac{\sqrt{3}}{2}$ or $\sin x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$ or $x = \frac{\pi}{6}, \frac{5\pi}{6}$

57. Ans. (b)

$f(x) = 2x^3 + mx^2 - 13x + n$

$x - 2,\ x - 3$ are factors of $f(x)$

$\therefore\ f(2) = f(3) = 0$

$\therefore\ 16 + 4m - 26 + n = 0$

and $54 + 9m - 39 + n = 0$

$$\therefore\ \begin{array}{r} 4m + n - 10 = 0 \\ 9m + n + 15 = 0 \\ \hline -5m - 25 = 0 \end{array}$$

$m = -5,\ n = 30$

$\therefore\ (m, n) = (-5, 30)$

58. Ans. (d)

$a^{\log_a (x^2 - 4x - 5)} = 3x - 5 \quad a \in R^+ - \{1\}$

$\therefore\ x^2 - 4x - 5 = 3x - 5$

$\therefore\ x = 5, x = 2$

Soluation set : $\{5, 2\}$

59. Ans. (a)

$x^2 + px + q = 0$ and $x^{2n} + p^n x^n + q^n = 0$

$\alpha + \beta = -p,\ \alpha\beta = q$

$\alpha^n + \beta^n = -p^n,\ \alpha^n \beta^n = q^n$

Now, $x^n + 1 + (x+1)^n = 0$

$\therefore \left(\frac{\alpha}{\beta}\right)^n + 1 + \left(\frac{\alpha}{\beta - 1}\right)^n = 0$

$\therefore \frac{\alpha^n + \beta^n}{\beta^n} + \frac{(\alpha+\beta)^n}{\beta^n} = 0$

$\therefore -p^n + (-p)^n = 0$

$\therefore p^n \left[(-1)^n - 1\right] = 0$ which is possible for even value of n

60. Ans. (c)

$\therefore x^2 + px + q = 0$

$\alpha + \beta = -p,\ \alpha\beta = q$

$\rightarrow x^2 + rx + 5 = 0$

$\gamma + \delta = -\gamma,\ \gamma\delta = 5$

now $(\alpha - \gamma)^2 + (\beta - \gamma)^2 + (\alpha - \delta)^2 + (\beta - \delta)^2$

$= 2\left[p^2 + r^2 - pr - 2q - 2s\right]$

61. Ans. (c)

$(5-x)^4 + (4-x)^4 = (9-2x)^4$

$\therefore m^4 + n^4 = (m+n)^4$ where $m = 5 - x,\ n = 4 - x$

$\therefore 2mn\left(2m^2 + 3mn + 2n^2\right) = 0$

$\therefore m = 0,\ n = 0,\ 2m^2 + 3mn + 2n^2 = 0$

$m = 0 \Rightarrow x = 5$

$n = 0 \Rightarrow x = 4$

and $2m^2 + 3mn + 2n^2 = 0 \Rightarrow 7x^2 - 63x + 142 = 0$

$\Delta < 0$

$\therefore$ two roots are and real and two roots are complex

62. Ans. (a)

$$24x^2 - 8x - 3 = 0$$

$$\alpha + \beta = \frac{1}{3} \quad \alpha\beta = -\frac{1}{8}$$

$$\therefore \ |\alpha|, |\beta| < 1$$

$$\lim_{n \to \infty} \sum_{r=1}^{n} Sr = \lim_{n \to \infty} S_1 + S_2 + \ldots\ldots\ldots + S_n$$

$$= \lim_{n \to \infty} (\alpha + \beta) + (\alpha^2 + \beta^2) + \ldots\ldots\ldots + (\alpha^n + \beta^n)$$

$$= \lim_{n \to \infty} (\alpha + \alpha^2 + \ldots\ldots\ldots + \alpha^n) + (\beta + \beta^2 + \ldots\ldots\ldots + \beta^n)$$

$$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta}$$

$$= \frac{14}{13}$$

63. Ans. (a)

$$ax + by = 1 \Rightarrow y = \frac{1 - ax}{b}$$

$$px^2 + qy^2 - 1 = 0$$

$$\therefore \ px^2 + q\left(\frac{1 - ax}{b}\right)^2 - 1 = 0$$

$$\therefore \ (pb^2 + qa^2)x^2 - 2aqx + q - b^2 = 0$$

roots are equal $\therefore \ \Delta = 0$

$$(-2aq)^2 - 4(pb^2 + qa^2)(q - b^2) = 0$$

$$b^2(a^2q + b^2p - pq) = 0$$

$$a^2q + b^2p = pq$$

$$\therefore \ \frac{a^2}{p} + \frac{b^2}{q} = 1$$

64. Ans. (d)

$\ell \cos 2x + m \sin^2 x + n = 0$

$\therefore \ (m - 2\ell) \sin^2 x + (\ell + n) = 0$

$\therefore \ m - 2\ell = 0, \ \ell + n = 0$

$\therefore \ \ell = \frac{m}{2} = -n = k$, $k \in R$(suppose)

$\ell = k, m = 2k, n = -k$

$\therefore$ number of tripletes (ℓ, m, n) are infinite.

65. Ans. (d)

$f(x) = x - [x]$

$\therefore f(x) + f\left(\frac{1}{x}\right) = 1$

$\therefore x - [x] + \frac{1}{x} - \left[\frac{1}{x}\right] = 1$

$\therefore \ x + \frac{1}{x} - 1 = [x] + \left[\frac{1}{x}\right]$

$\therefore \ \frac{x^2 - x + 1}{x} = k$ where $k = [x] + \left[\frac{1}{x}\right]$ is an integer

$\therefore \ x^2 - (1 + k) x + 1 = 0$

as $x \in R - \{0\}$, $\Delta \geq 0$

$[-(1+k)]^2 - 4(1)(1) \geq 0$

$\therefore \ (1 + k)^2 \geq 4$

$\therefore \ |1 + k| \geq 2$

$\therefore \ -2 \geq 1 + k \geq 2$

$\therefore k \leq -3, \ k \geq 1$

$\therefore$ number of solution is an infinite.

66. Ans. (b)

$x^2 - 5kx + 4e^{4\log k} - 3 = 0$

As given, $\alpha\beta = 61 \Rightarrow 4e^{4\log k} - 3 = 61$

$\Rightarrow 4e^{\log k^4} = 64$

$\Rightarrow k = 2$

67. Ans. (b)

Suppose α, β are the roots of $ax^2 + bx + c = 0$

$$\alpha+\beta=-\frac{b}{a}, \quad \alpha\beta=\frac{c}{a}$$

also, $\alpha=\beta^n$

$$\alpha\beta = \frac{c}{a} \Rightarrow \beta^{n+1} = \frac{c}{a}$$

$$\therefore\ \beta = \left(\frac{c}{a}\right)^{\frac{1}{n+1}} \quad \alpha = \left(\frac{c}{a}\right)^{\frac{n}{n+1}}$$

$$\alpha + \beta = \frac{-b}{a}$$

$$\therefore\ \left(\frac{c}{a}\right)^{\frac{n}{n+1}} + \left(\frac{c}{a}\right)^{\frac{1}{n+1}} = \frac{-b}{a}$$

$$\therefore\ \left(ac^n\right)^{\frac{1}{n+1}} + \left(a^n\ c\right)^{\frac{1}{n+1}} = -b$$

68. Ans. (a)

$$\log_{10} a + \log_{10}\sqrt{a} + \log_{10}\sqrt[4]{a} + \ldots\ldots\ldots\ldots\ldots\ldots = b$$

$$\therefore\ b = \log_{10}\ a^{1+\frac{1}{2}+\frac{1}{4}} + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \log_{10} a^{\frac{1}{1-\frac{1}{2}}}$$

$$\therefore\ b = 2\log_{10} a$$

now $$\left[\frac{\sum_{n=1}^{b}(2n-1)}{\sum_{n=1}^{b}(3n+1)}\right] = \frac{20}{7\log_{10} a}$$

$$\therefore\ \frac{2.\frac{b}{2}(b+1) - b}{3.\frac{b}{2}(b+1) + b} = \frac{20\times 2}{7b}$$

$\therefore\ 7b^2 - 60b - 100 = 0$

$b = 10,\ b = -\dfrac{10}{7}$

$\log_{10} a = 5, \quad \left(b \neq -\dfrac{10}{7}\right)$

$\therefore\ a = 10^5$

69. Ans. (d)

for $x^2 - px + r = 0 \therefore \alpha + \beta = p,\ \alpha\beta = r$

for $x^2 - qx + r = 0 \therefore \dfrac{\alpha}{2} + 2\beta = q \quad \alpha\beta = r$

Solving,

$$\left.\begin{aligned} \alpha + 4\beta &= 2q \\ \alpha + \beta &= p \end{aligned}\right\}$$

$\alpha = \dfrac{2(2p-q)}{3},\ \beta = \dfrac{2q-p}{3}$

now $r = \alpha\beta = \dfrac{2(2p-q)}{3}\left(\dfrac{2q-p}{3}\right) = \dfrac{2}{q}(2p-q)(2q-p)$

70. Ans. (c)

for $x \in R \ \therefore\ 3^{72}\left(\dfrac{1}{3}\right)^{x}\left(\dfrac{1}{3}\right)^{\sqrt{x}} > 1$

$\therefore 3^{72-x-\sqrt{x}} > 3^{o}$

$\therefore 72 - x - \sqrt{x} > 0$

$\therefore x + \sqrt{x} - 72 < 0$

$\therefore\ (\sqrt{x}+9)(\sqrt{x}-8) < 0$

for $x \in R$ and $x \geq 0$, ineqalities is possible

$\therefore\ \sqrt{x} - 8 < 0 \quad \therefore \sqrt{x} < 8 \qquad x < 64$

$\therefore 0 \leq x < 64 \qquad x \in (0, 64)$

71. Ans. (a)

let $f(x) = x^2 - 2ax + a^2 + a - 3 = 0$

for $f(x) = 0$ roots are real and less than 3

$\therefore$ (i) $f(3) > 0$ (ii) $\Delta \geq 0$ (iii) $\alpha + \beta < 6$

(i) $f(3) > 0 \Rightarrow a^2 - 5a + 6 > 0$

$\Rightarrow (a-3)(a-2) > 0$

$\therefore a \in R - [2, 3]$ ....(1)

(ii) $\Delta \geq 0 \Rightarrow a \leq 3$ ....(2)

(iii) $\alpha + \beta < 6 \Rightarrow a < 3$ ...(3)

from (i), (ii) & (iii) $a < 2$

72. Ans. (b)

let α be the common root of $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$

$\therefore \alpha^2 + b\alpha - 1 = 0$ and $\alpha^2 + \alpha + b = 0$

$$\therefore \frac{\alpha^2}{b^2 + 1} = \frac{\alpha}{-1 - b} = \frac{1}{1 - b}$$

$$\therefore \alpha^2 = \frac{b^2 + 1}{1 - b},\ \alpha = \frac{1 + b}{b - 1}$$

$$\therefore \left(\frac{1 + b}{b - 1}\right)^2 = \frac{b^2 + 1}{1 - b} \Rightarrow b^3 + 3b = 0$$

$\Rightarrow b = -i\sqrt{3}$

73. Ans. (c)

$abc^2 x^2 + (3a^2c + b^2c) + (2b^2 - 6a^2 - ab) = 0$ a, b, c $\in Q$

$\therefore \Delta = c^2 \left[3a^2 - b^2 - 4ab\right]^2$ which is perfect square so roots are rational.

74. Ans. (c)

$x^3 + 3x^2 + 3x + 2 = 0$

$\therefore (x + 1)^3 + 1 = 0$

$\therefore (x + 2)(x^2 + x + 1) = 0$

$$x = -2,\ x = \frac{-1 + \sqrt{3}i}{2},\ x = \frac{-1 - \sqrt{3}i}{2}$$

$\therefore\ x = -2,\ x = w,\ x = w^2$

roots of the equations $ax^2 + bx + c = 0$ & $(x+1)^3 + 1 = 0$ are equal.

so, both the roots cannot be real and complex.

Both the roots are compelx and

sum of the roots $= -\frac{b}{a}$ and product of the roots $= \frac{c}{a}$

$\therefore\ w + w^2 = \frac{b}{a}, \qquad w - w^2 = \frac{c}{a}$

$\therefore\ -1 = \frac{-b}{a} \qquad w^3 = \frac{c}{a} = 1$

$\therefore\ a = b \qquad c = a$

$\therefore\ a = b = c$

75. Ans. (a)

$(6k + 2)\,x^2 + rx + 3k - 1 = 0 \qquad ....(1)$

$(12k + 4)x^2 + px + 6k - 2 = 0 \qquad ....(2)$

both the roots of equation (1) & (2) are equal.

$\therefore\ \frac{6k + 2}{12k + 4} = \frac{r}{p} = \frac{3k - 1}{6k - 2}$

$\therefore\ 2r - p = 0$

76. Ans. (b)

α, β are roots of $x^2 + x + 1 = 0$

$\therefore\ \alpha = w = \frac{-1 + \sqrt{3}\,i}{2},\ \beta = w^2 = \frac{-1 - \sqrt{3}\,i}{2}$

$\alpha + \beta = -1,\ \alpha\beta = 1$

let $\alpha_1 = \alpha^{19},\ \beta_1 = \beta^7$

$= w^{19} \quad = (w^2)^7 = w^{14}$

$\alpha_1 = w,\ \beta_1 = w^2 \qquad (\because\ w^3 = 1)$

$\alpha_1\beta_1 = w + w^2 = -1$

$\alpha_1\beta_1 = w.\,w^2 = w^3 = 1$

Required equation $x^2 + x + 1 = 0$

77. Ans. (b)

let α, β be roots of $x^2 + bx - c = 0$

$\therefore\ \alpha+\beta = -b,\ \alpha\beta = -c$

$\therefore\ b = -(\alpha+\beta)\ c = -(\alpha\beta)$

let $\alpha_1 = b,\ \beta_1 = c$

$\therefore\ \alpha_1\beta_1 = b + c = -(\alpha+\beta+\alpha\beta)$

$\alpha_1\beta_1 = bc = \alpha\beta(\alpha+\beta)$

Required equation

$x^2 + (\alpha + \beta + \alpha\beta)x + \alpha\beta(\alpha+\beta) = 0$

78. Ans. (a)

Roots of the equation $ax^2 + bx + c = 0$ are in ratio m : n

$\therefore\ \frac{\alpha}{\beta} = \frac{m}{n} \qquad \therefore\ \alpha = mk,\ \beta = nk$

$\therefore\ \alpha + \beta = \frac{-b}{a} \therefore (m+n)k = -\frac{b}{a},\ k = \frac{-b}{a(m+n)}$

$\alpha = \frac{-mb}{a(m+n)},\ \beta = \frac{-nb}{a(m+n)}$

now, $\alpha\beta = \frac{c}{a} \Rightarrow \frac{mnb^2}{a^2(m+n)^2} = \frac{c}{a}$

$\therefore \frac{mn}{(m+n)^2} = \frac{ca}{b^2}$

$\therefore \left(\frac{m+n}{\sqrt{m}\sqrt{n}}\right)^2 = \left(\frac{b}{\sqrt{ca}}\right)^2$

$\therefore \sqrt{\frac{m}{n}} + \sqrt{\frac{m}{n}} \pm \frac{b}{\sqrt{ca}} = 0$

Hint

1. Ans. (b)

$\Delta = b^2 - 4ac$

2. Ans. (a)

$\Delta = b^2 - 4ac$

3. Ans. (c)

Root are real and equal $\therefore \Delta \geq 0$

4. Ans. (a)

for $ax^2 + bx + c = 0$ α, β are root

$\therefore \alpha + \beta = -\frac{b}{a}, \ \alpha\beta = \frac{c}{a}$

a_1 b, c are in A.P. $\therefore b - a = c - b$

5. Ans. (c)

6. Ans. (a)

Suppose $x^2 + 5x = m$

7. Ans. (a)

Divinding both side by x^2

Suppose $x + \frac{1}{x} = m$

8. Ans. (c)

Suppose $\frac{x - a}{x - b} = m$

9. Ans. (a)

10. Ans. (b)

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

11. Ans. (a)

12. Ans. (b)

$\therefore x^2 - x + 1 = 0$

Multiplying both side by $x + 1$

$\therefore x^3 + 1 = 0$

α, β are roots $\quad \therefore \alpha^3 = -1, \ \beta^3 = -1$

13. Ans. (a)

14. Ans. (c)

a, b, c are in harmonic sequence

$\therefore \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

15. Ans. (b)

Roots are imaginary $\therefore \Delta < 0$

16. Ans. (b)
17. Ans. (b)
18. Ans. (b)
19. Ans. (a)

$$\alpha = \frac{1}{10 - \sqrt{72}} = \frac{10 + \sqrt{72}}{28}$$

$$\beta = \frac{1}{10 + 6\sqrt{2}} = \frac{10 - \sqrt{72}}{28}$$

Quadratic Equation $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

20. Ans. (a)

a, b, c Geometric sequence $\therefore b^2 = ac$

21. Ans. (a)
22. Ans. (a)
23. Ans. (b)
24. Ans. (a)
25. Ans. (a)
26. Ans. (a)

Roots are real and distinct. $\therefore \Delta > 0$

27. Ans. (a)
28. Ans. (a)

Suppose $\left(5 + 2\sqrt{6}\right)^{x^2-3} = m \quad \therefore \left(5 - 2\sqrt{6}\right)^{x^2-3} = \frac{1}{m}$

29. Ans. (a)
30. Ans. (c)
31. Ans. (c)
32. Ans. (b)
33. Ans. (b)

$\cos x = \frac{1-t^2}{1+t^2}, \sin x = \frac{2t}{1+t^2}$ Where $t = \tan\frac{x}{2}$

34. Ans. (a)
35. Ans. (d)
36. Ans. (b)
37. Ans. (c)
38. Ans. (b)
39. Ans. (a)
40. Ans. (d)
41. Ans. (c)
42. Ans. (b)

$a > 0$, minimum value of $f(x) = ax^2 + bx + c$ is at $x = \frac{-b}{2a}$

43. Ans. (c)
44. Ans. (b)
45. Ans. (a)

Suppose $|x-5| = y,\ y > 0$

46. Ans. (c)
47. Ans. (c)

$ax^2 + bx + c = 0$ & $2a + 3b + 6c = 0$

Suppose $f(x) = 2ax^3 + 3bx^2 + 6cx$

$f(x) = 6ax^2 + 6bx + 6c$

$= 6(ax^2 + bx + c)$

$f(0) = 0,\ f(1) = 2a + 3b + 6c = 0$

$\therefore\ f(0) = f(1)$

By Rolle's Thm, $\exists\ x \in (0, 1) \supset f'(x) = 0$

$\therefore ax^2 + bx + c = 0$

48. Ans. (d)
49. Ans. (d)

$\sin x = \sin\alpha$, General solution is $x = k\pi + (-1)^k\ \alpha,\ k \in z$

50. Ans. (b)

51. Ans. (c)

52. Ans. (a)

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

53. Ans. (d)

Suppose $\left(\frac{3}{2}\right)^{\frac{1}{x}} = m$

54. Ans. (d)

55. Ans. (b)

56. Ans. (a)

Suppose $16^{\sin^2 x} = m$

57. Ans. (b)

58. Ans. (d)

$a^{\log_a x} = x$

59. Ans. (a)

$x^2 + px + q = 0 \;\therefore\; \alpha + \beta = -p, \alpha\beta = q$

and $x^{2n} + p^n x^n + q^n = 0 \;\therefore\; \alpha^n + \beta^n = P^n, \alpha^n\beta^n = q^n$

60. Ans. (c)

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

61. Ans. (c)

$5 - x = m$ and $4 - x = n$ Taking

62. Ans. (a)

$$\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$$

$$a + ar + ar^2 + \ldots\ldots\ldots\ldots = \frac{a}{1-r}, |r|<1$$

63. Ans. (a)

Equation having one root $\Delta = 0$

64. Ans. (d)

$(m - 2\ell)\sin^2 x + (\ell + n) = 0$

$\therefore\ m - 2\ell = 0,\ \ell + n = 0$

$\frac{m}{2} = \ell = -n = (k)$ (Äkhíkk)

$(\ell, m, n) = (k, 2k, -k), k \in R$

65. Ans. (d)

$[x] + \left[\frac{1}{x}\right] =$ Some integer Suppose - k, Roots are real $\Delta \geq 0$

66. Ans. (b)

$\alpha\beta = \frac{c}{a},\ a^{\log_a x} = x$

67. Ans. (b)

$\Rightarrow\ \alpha + \beta = \frac{-b}{a},\ \alpha\beta^2 = \frac{c}{a}$

use of indices rules

68. Ans. (a)

$\log_{10} A + \log_{10} B = \log_{10} AB$

$\Rightarrow a + ar + ar^2 + \ldots\ldots\ldots\ldots = \frac{a}{1-r}, |r| < 1$

$\Rightarrow \sum_{r=1}^{n} r = \frac{n}{2}(n+1), \sum_{r=1}^{n} 1 = n$

69. Ans. (d)

Solve two linear equations for α, β

70. Ans. (c)

$x \geq 0$ Since that $72 - x - \sqrt{x} > 0, \sqrt{x} \in R$

71. Ans. (a)

Roots are ral and less then three :

(i) $f(3) > 0$ (ii) $\Delta \geq 0$ (iii) $\alpha + \beta < 3$

72. Ans. (b)

one root is common.

$\alpha^2 + b\alpha - 1 = 0$

$\alpha^2 + \alpha + b = 0$

$$\frac{\alpha^2}{\begin{vmatrix} b & -1 \\ 1 & b \end{vmatrix}} = \frac{-\alpha}{\begin{vmatrix} 1 & -1 \\ 1 & b \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & b \\ 1 & 1 \end{vmatrix}}$$

73. Ans. (c)

$\Delta = b^2 - 4ac > 0$ and perfect square there for roots are rational and , $a, b, c \in Q$

74. Ans. (c)

$$\frac{-1+\sqrt{3}\,i}{2} = w, \frac{-1-\sqrt{3}\,i}{2} = w^2$$

$w + w^2 = -1, \; w^3 = 1$

75. Ans. (a)

$$\left.\begin{aligned} a_1x^2 + b_1x + c_1 = 0 \\ a_2x^2 + b_2x + c_2 = 0 \end{aligned}\right\} \text{Both roots are common}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

76. Ans. (b)

$$w = \frac{-1+\sqrt{3}\,i}{2}, \; w^2 = \frac{-1-\sqrt{3}\,i}{2}$$

Sum of roots $= w + w^2 = -1$

Products of roots $= ww^2 = 1$

Required quadratic equation

$x^2 -$ (Sum of roots) x + Product of roots = 0

77. Ans. (b)

78. Ans. (a)

$$\alpha : \beta = m : n, \; \alpha + \beta = \frac{-b}{a}, \; \alpha\beta = \frac{c}{a}$$

Solving first two equations for α, β and substituting in $\alpha\beta = \frac{c}{a}$

Answers

1	b	25	a	49	d	73	c
2	a	26	a	50	b	74	c
3	c	27	a	51	c	75	a
4	a	28	a	52	a	76	b
5	c	29	a	53	d	77	b
6	a	30	c	54	d	78	a
7	a	31	c	55	b		
8	c	32	b	56	a		
9	a	33	b	57	b		
10	b	34	a	58	d		
11	a	35	d	59	a		
12	b	36	b	60	c		
13	a	37	c	61	c		
14	c	38	b	62	a		
15	b	39	a	63	a		
16	b	40	d	64	d		
17	b	41	c	65	d		
18	b	42	b	66	b		
19	a	43	c	67	b		
20	a	44	b	68	a		
21	a	45	a	69	d		
22	a	46	a	70	c		
23	b	47	c	71	a		
24	a	48	d	72	b		

●●●

Unit - 3

Matrices and Determinants

Important Points

- **Matrix :**

  Any rectangular array of numbers is called matrix. A matrix of order m × n having m rows and n columns. Its element in the i^{th} row and j^{th} column is a_{ij}. We denote matrix by A, B, C etc.

  $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ is a matrix of order 2 × 2.

  $\begin{bmatrix} -1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ is matrix of order 2 × 3.

  A matrix of order m × n is $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$

- **Algebra of Matrices**

  (1) Equality : If $A = \left[a_{ij}\right]_{m\times n}$, $B = \left[b_{ij}\right]_{p \times q}$ are said to be equal i.e. A = B if

  (i) $a_{ij} = b_{ij} \ \forall$ i & j

  (ii) order of A = order of B, i.e. m = p and n = q

- **Types of Matrices :** Let $A = \left[a_{ij}\right]_{m\times n}$

  (1) **Row matrix :** A 1 × n matrix $[a_{11} \ a_{12} \ a_{13} \ ..... \ a_{1n}]$ is called a row matrix (row vector)

  (2) **Column matrix :** A m × 1 matrix $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{n1} \end{bmatrix}$ is called column matrix

  (Column vector)

(3) **Square Matrix :** An n × n matrix is called a square matrix.

(4) **Diagonal matrix :** If in a square matrix $A = [a_{ij}]_{n \times n}$ we have $a_{ij} = 0$ whenever $i \neq j$ then A is called a diagonal matrix.

(5) **Zero (null) matrix :** A matrix with all elements are zero is called zero (null) matrix. It is denoted by $[0]_{m \times n}$ or $O_{m \times n}$ or O.

- **Algebra of Matrices**

(2) Sum and Difference : If A and B are of same oder

i.e. $A = \left[a_{ij}\right]_{m \times n}$, $B = \left[b_{ij}\right]_{m \times n}$, $C = \left[c_{ij}\right]_{m \times n}$

then A + B = C $\Rightarrow \left[a_{ij} + b_{ij}\right]_{m \times n} = \left[c_{ij}\right]_{m \times n}$

A - B = C $\Rightarrow \left[a_{ij} - b_{ij}\right]_{m \times n} = \left[c_{ij}\right]_{m \times n}$

Properties of addition

If matrices A, B, C and O are of same order, then

(i) A + B = B + A	(Commutative law)
(ii) A + (B + C) = (A + B) + C	(Associative law)
(iii) A + O = O + A	(Existence of Identity)
(iv) (-A) + A = A + (-A) = O	(Existence of Inverse)

(3) Product of Matrix with a Scalar

If $A = \left[a_{ij}\right]_{m \times n}$ and $k \in R$ then we define product of matrix with a scalar is

$kA = k\left[a_{ij}\right]_{m \times n} = \left[ka_{ij}\right]_{m \times n}$

Properties of Addition of Matrices and Multiplication of a Matrix by a scalar

Let $A = \left[a_{ij}\right]_{m \times n}$, $B = \left[b_{ij}\right]_{m \times n}$, $k, l \in R$

(i) k(A + B) = kA + kB

(ii) $(k + l)$ A = kA + lA

(iii) (kl)A = k(lA)

(iv) 1A = A

(v) (-1) A = -A

(4) Matrix Multiplication :

Let $A = \left[a_{ij}\right]_{m \times n}$, $B = \left[b_{ij}\right]_{n \times p}$. Then AB = C where $C_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + ... + a_{in}b_{nj}$

= Scalar product of i^{th} row of A and j^{th} Column of B.

(i) Product AB defined if and only if number of column of A = number of rows of B

(ii) If A is $m \times n$ matrix and B is $n \times p$ matrix then AB is $m \times p$ matrix.

- **Properties of Matrix Multiplication**

Let the matrices A, B, C and O have order compitable for the operations involved.

(i) A(B + C) = AB + AC

(ii) (A + B)C = AC + BC

(iii) A(BC) = (AB) C

(iv) AO = O = OA

(v) AB $\neq$ BA, generally

(vi) AB = O need not imply A = O or B = O

(vii) AB = AC need not imply B = C

- **Types of Matrices**

(6) Identity (unit) Matrix : In a diagonal matrix all elements of principal diagonal are 1 is called Identity (unit) Matrix and is denoted by I or I_n or $I_{n \times n.}$

(7) Scalar Matrix : If $k \in R$, then kI called a scalar matrix.

(8) Traspose of a Matrix : If all the rows of matrix $A = \left[a_{ij}\right]_{m \times n}$ are converted into corresponding column, the matrix so obtained is called the transpose of A. It is denoted by A^T or A'. $A^T = \left[a_{ji}\right]_{n \times m}$

Properties of Transpose

(i) $(A^T)^T = A$

(ii) $(A + B)^T = A^T + B^T$

(iii) $(kA)^T = kA^T$, $k \in R$

(iv) $(AB)^T = B^T A^T$

(9) Symmetric Matrix : For a square matrix A, if $A^T = A$, then A is called a symmetric matrix. Here $a_{ij} = a_{ji}$ for all *i* and *j*.

(10) Skew - Symmetric Matrix : For a square matrix A, if $A^T = -A$, then A is called a

Skew - symemtric matrix.

Here $a_{ij} = -a_{ji}$ for all *i* and *j* and $a_{ii} = 0 \ \forall \ i$

For square matrix A, $A + A^T$ is symmetric and $A - A^T$ is skew - symmetric matrix.

(11) Triangular Matrices :

(i) Upper Triangular Matrix : A sqare matrix whose element $a_{ij} = 0$ for $i > j$ is called an uppear triangular matrix.

e.g. $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}, \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$

(ii) Lower Triangular Matrix : A square matrix whose element $a_{ij} = 0$ for $i < j$ is called a lower triangular matrix.

e.g. $\begin{bmatrix} a & 0 \\ b & c \end{bmatrix}, \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$

Let A be a square matrix of order $n \times n$.

(12) Orthogonal matrix : A is called an orthogonal matrix if and only if $A^TA = I_n = A A^T$

(13) Idempotent Matrix : A is called an idempotgent matrix if $A^2 = A$

(14) Nilpotent Matrix : A is called a nilpotent matrix if $A^m = 0$, $m \in Z^+$

(15) Involutary Matrix : A is called an involutary matrix if $A^2 = I$, i.e. $(I + A)(I - A) = O$

(16) Conjugate of a Matrix : If $A = [a_{ij}]$ is a given matrix, then the matrix obtained on replacing all its elements by their corresponding complex cojugates is called the conjugate of the matrix A and is denoted by $\overline{A} = \left[\overline{a}_{ij}\right]$

Properties :

(i) $\overline{(\overline{A})} = A$

(ii) $\overline{(A + B)} = \overline{A} + \overline{B}$

(iii) $\overline{(kA)} = \overline{k} \cdot \overline{A}$, k being a complex number

(iv) $\overline{AB} = \overline{A} \cdot \overline{B}$

(17) Conjugate Transpose of a matrix : The conjugate of the transpose of a given matrix A is called the conjugate transpoe (Tranjugate) of A and is denoted by A^{θ}.

Properties :

(i) $A^{\theta} = \overline{\left(A^{T}\right)} = \left(\overline{A}\right)^{T}$

(ii) $\left(A^{\theta}\right)^{\theta} = A$

(iii) $\left(A + B\right)^{\theta} = A^{\theta} + B^{\theta}$

(iv) $\left(kA\right)^{\theta} = \overline{k} \cdot A^{\theta}$, k being a complex number

(v) $\left(AB\right)^{\theta} = B^{\theta} \cdot A^{\theta}$

Let A be a square matrix of oder $n \times n$

(18) Unitary Matrix : A is an unitary matrix if $AA^{\theta} = I_n = A^{\theta}A$.

(19) Hermitian Matrix : A is a hermitian matrix if $A^{\theta} = A$

(20) Skew - Hermitian Matrix : A is a skw-Hermitian matrix if $A^{\theta} = -A$

- **The determinant of a square matrix :**

If all entries of a square matrix are kept in their respective places and the determinant of this array is taken, then the detrminant so obtained is called the determinant of the given square matrix. If A is a square matrix, then determinant of A is denoted by | A | or det A.

Evaluation of Determinants (Expansion)

Second order determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Third order determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

$= a_1 (b_2c_3 - b_3c_2) - b_1 (a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$

$= a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1$

Some Symbols :

(1) $R_i \rightarrow C_i$: To convert every row (column) into corresponding column (row)

(2) R_{ij} $[c_{ij}]$ $(i \neq j)$: Interchange of i^{th} row (column) and j^{th} row (column)

(3) $R_i(k)$ $[c_i(k)]$: multiply i^{th} row (Column) by $k \in R - \{0\}$

(4) $R_{ij}(k)[c_{ij}(k)]$: Multiply i^{th} row (column) by $k \in R$ $(k \neq 0)$ and adding to the corresponding elements of j^{th} row(column)

Properties of Determinants of Matrices

(i) $|A^T| = |A|$

(ii) $|AB| = |A||B|$

$|A\,B\,C| = |A|\;|B|\;|C|$

(iii) $|kA| = k^n |A|$ (where A is $n \times n$ matrix)

(iv) $|I| = 1$

Value of some Determinants :

(i) Symmetric Determinant $\begin{vmatrix} x & p & q \\ p & y & r \\ q & r & z \end{vmatrix} = xyz + 2pqr - xp^2 - yq^2 - zr^2$

(ii) Skew - symmetric determinant of odd order : $\begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ y & -z & 0 \end{vmatrix} = 0$

(iii) Circular Determinant : $\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = -\left(x^3 + y^3 + z^3 - 3xyz\right)$

Area of a Triangle :

If the vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3) then,

Area of a triangle = $\Delta = \frac{1}{2}|D|$, where $D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Shifting of origin does not effect the area of a triangle.

If D = 0 $\Leftrightarrow$ all three points are collinear

Let the sides of the triangle be $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$

$\therefore$ Area of a triangle $= \dfrac{\Delta^2}{2|C_1C_2C_3|}$,

where C_1, C_2, C_3 are respetively the cofactors of c_1, c_2 and c_3 and $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

If A(x_1, y_1) and B(x_2, y_2) are two distinct points of $\overleftrightarrow{AB}$ then the cartesian equation of $\overleftrightarrow{AB}$ is

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Propeties of Determiants : (D = value or determinante)

(i) If a row (column) is a zero vector (i.e. all elements of a row or a column are zero), then D = 0

(ii) If two rows (Columns) are identical, then D = 0

(iii) If any two rows (columns) are interchanged, then D becoms - D (additive inverse of D)

(iv) If any two rows (columns) are interchanged, D is unchaged $\Rightarrow |A^T| = |A|$

(v) $$\begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(vi) $$\begin{vmatrix} a_1 + d_1 & b_1 + e_1 & c_1 + f_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & e_1 & f_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(vii) If any rows (columns) is multiplied by $k \in R$ ($k \neq 0$) and added to another rows (columns), then D is unchanged.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + ka_2 & b_1 + kb_2 & c_1 + kc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(vii) All rows of a determinant are converted into corresponding column, D is unchanged.

(viii) Determinants are multiplied in the same way as we multiply matrices.

$$\therefore\ |AB| = |A|\ |B| = |BA| = |AB^T| = |A^TB| = |A^TB^T|$$

(ix) $\Delta = \begin{vmatrix} f_1 & g_1 & h_1 \\ f_2 & g_2 & h_2 \\ f_3 & g_3 & h_3 \end{vmatrix}$, where f_r, g_r, h_r are functions of x for $r = 1, 2, 3$.

$$\therefore \frac{d\Delta}{dx} = \begin{vmatrix} f_1' & g_1' & h_1' \\ f_2 & g_2 & h_2 \\ f_3 & g_3 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & g_1 & h_1 \\ f_2' & g_2' & h_2' \\ f_3 & g_3 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & g_1 & h_1 \\ f_2 & g_2 & h_2 \\ f_3' & g_3' & h_3' \end{vmatrix}$$

(x) Let D(x) be a 3×3 determinant whose elements are polynomials.

If D(m) has two identical rows (columns), then x - m is a factor of D(x)

If D(m) has three identical rows (columns), then $(x - m)^2$ is a factor of D(x).

- **Minor and cofactor**

Let $A = \left[a_{ij}\right]_{3 \times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

The minor of the element a_{ij} (i, j = 1, 2, 3) in A

= M_{ij} = The determiant obtained from A on deleting the row and the column in which a_{ij} occurs.

The cofactor of the element *aij* (*i, j* = 1, 2, 3) in A = $A_{ij} = (-1)^{i+j} M_{ij}$

The value of any third order determinant can be obtained by adding the products of the elements of any of its rows (columns) by their correspdong co-factor.

If we multiply all the elements of any rows (columns) of any third order determinant by the cofactos of the corresponding elements of another row (column) and add the products, then the sum is zero.

or in Mathematical notation

$$\sum_{j=1}^{3} a_{ij} A_{kj} = \begin{cases} A & \text{if } i = k = 1, 2, 3 \\ 0 & \text{if } i \neq k = 1, 2, 3 \end{cases} \qquad \sum_{i=1}^{3} a_{ij} A_{ik} = \begin{cases} A & \text{if } j = k = 1, 2, 3 \\ 0 & \text{if } j \neq k = 1, 2, 3 \end{cases}$$

- **Adjoint of Matrix**

Adjoint Matrix of A = *adj* A $= \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$

= Tranpose of the matrix of cofactor = $[A_{ji}]_{3 \times 3}$

If $A = [a_{ij}]_{n \times n}$ then *adj* $A = [A_{ji}]_{n \times n}$

To obtain the adjoint of 2×2 matrix, interchange the elements on the principal diagonal and change the sign of the elements on the secondary diagonal.

Properties of Adjoint Matrix : If A is square matrix of order n,

(i) A(*adj* A) = (*adj* A) A = | A | I_n

(ii) *adj* $I_n = I_n$

(iii) *adj* $(kI_n) = k^{n-1} I_n$, k is a scalar.

(iv) $adj\ A^T = (adj\ A)^T$

(v) $adj\ (kA) = k^{n-1}\ adj A$, k is a scalar.

(vi) $adj(AB) = (adj\ B)(adj\ A)$

(vii) $adj\ (ABC) = (adj\ C)(adj\ B)(adj\ A)$

If A is a non singular matrix of order n, then

(i) $|\ adj\ A\ | = |\ A\ |^{n-1}$

(ii) $adj\ (adj\ A) = |\ A\ |^{n-2} A$

(iii) $|\ adj\ (adj\ A)\ | = |\ A\ |^{(n-1)^2}$

Adjoint of

(i) a diagonal matrix is diagonal

(ii) a triangular matrix is triangular

(iii) a symmetric matrix is symmetric

(iv) a hermitian matrix is thermitian

- **Inverse of a Matrix**

A square matrix A is said to be singular if | A | = 0 and non singular if | A | = 0

 If A is a square matrix of order n, if there exists another square matrix of order n such that $AB = I_n = BA$

Then B(A) is called inverse of A(B).It is denoted A^{-1}.

$$A^{-1} = \frac{1}{|A|}(adj\ A)$$

If inverse of matrix A exists, then it is unique.

A square matrix A is non-singular $\Leftrightarrow\ |A| \neq 0$

$$\Leftrightarrow\ A^{-1} \text{ exists.}$$

Results :

(i) $|A^{-1}| = |A|^{-1}$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

(iii) $(A^T)^{-1} = (A^{-1})^T$

(iv) $(A^k)^{-1} = (A^{-1})^k,\quad k \in Z$

(v) A = diag $[a_{11}\ \ a_{22}\ \ a_{33}\ \ .....\ \ a_{nn}]$ and $a_{11} \cdot a_{22} \cdot a_{33} \cdot ... \cdot a_{nn} \neq 0$ then
A^{-1} = diag $[a_{11}^{-1}\ \ a_{22}^{-1}\ \ a_{33}^{-1}\ ....\ a_{nn}^{-1}]$

(vi) Inverse of a symmetric matrix is symmetric.

- **Elementary Transformations (operations) of a matrix**

(i) Interchange of rows (columns)

(ii) The multiplication of the elements of a row (column) by a non - zero scalar.

(iii) The addition (subtraction) to the elements of any row (column) of the scalar multiples of the corresponding elements of any other row (column).

- **Test of Consistency**

If the system of equation possesses atleast one solution set (solution set is not empty) then the equations are said to be consistent.

If the system of equation has no solution they are said to inconsistent.

Solution of simultaneous linear equations in two (three) variables :

Trival solution :

Value of all the variables is zero i.e. $x = 0$, $y = 0$, $z = 0$

Non Triavial Solution :

Value of atleast one variable is non-zero

Homogeneous linear equation :

If constant term is zero, i.e. $ax + by = 0$ or $ax + by + cz = 0$

such equations is called homogenous linear eqaution.

Solutiuon of homogenoeous linear equation

Consider the eqautions

For three variables

$$a_{11}x + a_{12}y + a_{13}z = 0$$
$$a_{21}x + a_{22}y + a_{23}z = 0$$
$$a_{31}x + a_{32}y + a_{33}z = 0$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = O

For two variables

$$a_{11}x + a_{12}y = 0$$
$$a_{21}x + a_{22}y = 0$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

A X = O

(i) If $|A| \neq 0$ the system is consistent and has only trivial (unique) solution.

(ii) If $|A| = 0$ the system is consistent and has non trivial (infinite number of) Solution.

- **Solution of non-homogeneous linear equation :**

Let three equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Matrix inversion Method

Equations can be expressed as a matrix form AX = B.

$$\text{Where } A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

If $|A| \neq 0$ (A is non singular), A^{-1} exists.

The solution is $X = A^{-1}B$

Cramer's Rule

$$x = \frac{D_1}{D},\ y = \frac{D_2}{D},\ z = \frac{D_3}{D}$$

Where

$$D_1 = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix},\ D_2 = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix},\ D_3 = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix},\ D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

(i) If $D \neq 0$ then the system has a unique solution and said to be consistent.

(ii) If $D = 0$ and $D_1 = D_2 = D_3 = 0$ then the system has infinite number of solution and said to be consistent.

(iii) If $D = 0$ and atleast one determinants D_1, D_2, D_3 is non-zero, then the system has no solution (solution set is empty) and said to be inconsistent.

Above both method can be used to solve non-homogeneous linear equation in two variables.

- **Characteristic Equation :**

Let $A = [a_{ij}]_{n \times n}$ then $A - \lambda I$ is called characteristic matrix of A.

Equation $|A - \lambda I| = 0$ is called characteristic equation of A.

Homogeneous system of linear equation having non-trival solution if $|A - \lambda I| = 0$

Every square matrix A satisfies its characteristic equation $|A - \lambda I| = 0$

Question Bank

1. If the system of equations $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$ has non trivial solution, then $\frac{xy}{z^2} = ...$

(a) $\frac{5}{6}$ (b) $-\frac{5}{6}$ (c) $\frac{6}{5}$ (d) $-\frac{6}{5}$

2. $\begin{vmatrix} n & {}_{n}P_{n} & {}_{n}C_{n} \\ n+1 & {}_{n+1}P_{n+1} & {}_{n+1}C_{n+1} \\ n+2 & {}_{n+2}P_{n+2} & {}_{n+2}C_{n+2} \end{vmatrix} = ...$

(a) $(n^2 + n + 1)\, n!$ (b) $n(n + 1)!$ (c) $(n + 1)\, n!$ (d) $(n + 2)\, n!$

3. Let a, b, c be such that $b(a + c) \neq 0$.

If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$, then n is...

(a) Zero (b) any even integer (c) any odd interger (d) any integer

4. $\begin{vmatrix} \sin(x+p) & \sin(x+q) & \sin(x+r) \\ \sin(y+p) & \sin(y+q) & \sin(y+r) \\ \sin(z+p) & \sin(z+q) & \sin(z+r) \end{vmatrix} = ...$

(a) $\sin(x + y + z)$ (b) $\sin(p + q + r)$ (c) 1 (d) 0

5. $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$, then $x = ...$

(a) $\frac{3}{2}, \frac{3}{11}$ (b) $\frac{3}{2}, \frac{11}{3}$ (c) $\frac{2}{3}, \frac{11}{3}$ (d) $\frac{2}{3}, \frac{3}{11}$

6. $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = k(abc)(a + b + c)^3$, then $k = ...$

(a) 1 (b) -1 (c) -2 (d) 2

7. $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = kabc$, then $k = ...$

(a) 4 (b) 3 (c) 2 (d) 1

8. $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and the vectors $(1, a, a^2)$, $(1, b, b^2)$, $(1, c, c^2)$ are non-coplanar

then $abc = ...$

(a) 0 (b) 2 (c) -1 (d) 1

9. $\begin{vmatrix} \sqrt{11}+\sqrt{3} & \sqrt{20} & \sqrt{5} \\ \sqrt{15}+\sqrt{22} & \sqrt{25} & \sqrt{10} \\ 3+\sqrt{55} & \sqrt{15} & \sqrt{25} \end{vmatrix} = ...$

(a) $5(5\sqrt{3}-3\sqrt{2})$ (b) $5(3\sqrt{2}+5\sqrt{3})$

(c) $-5(5\sqrt{3}+3\sqrt{2})$ (d) $5(3\sqrt{2}-5\sqrt{3})$

10. If $2s = a + b + c$ and $A = \begin{bmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{bmatrix}$ then det A = ...

(a) $2s^2 (s - a) (s - b) (s - c)$ (b) $2s^3 (s - a) (s - b) (s - c)$

(c) $2s (s - a)^2 (s - b)^2 (s - c)^2$ (d) $2s^2 (s - a)^2 (s - b)^2 (s - c)^2$

11. The homogeneous system of equations

$$\begin{bmatrix} 2 & \alpha+\beta+\gamma+\delta & \alpha\beta+\gamma\delta \\ \alpha+\beta+\gamma+\delta & \alpha(\alpha+\beta)(\gamma+\delta) & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) \\ \alpha\beta+\gamma\delta & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) & 2\alpha\beta\gamma\delta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

has non - trivial solutions only if...

(a) $\alpha + \beta + \gamma + \delta = 0$ (b) for any $\alpha, \beta, \gamma, \delta$

(c) $\alpha\beta + \gamma\delta = 0$ (d) $\alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta)$

12. Let $A=\begin{bmatrix}4 & 4k & k\\ 0 & k & 4k\\ 0 & 0 & 4\end{bmatrix}$. If $\det(A^2)=16$ then $|k|$ is ...

(a) 1 (b) $\frac{1}{4}$ (c) 4 (d) 4^2

13. If 1, ω,ω^2 are cube roots of unity, then $\begin{vmatrix}a & a^2 & a^3-1\\ a^{\omega} & a^{2\omega} & a^{3\omega}-1\\ a^{\omega^2} & a^{2\omega^2} & a^{3\omega^2}-1\end{vmatrix}=$ ...

(a) 0 (b) a (c) a^2 (d) a^3

14. If a_1, a_2, a_3 ... are in GP , then

$$\begin{vmatrix}\log a_n & \log a_{n+1} & \log a_{n+2}\\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5}\\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8}\end{vmatrix}= ...$$

(a) 0 (b) 1 (c) 2 (d) 4

15. If $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q=PAP^T$, then $P^TQ^{2013}P=$ ...

(a) $\begin{bmatrix}1 & 2013\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}4+2013\sqrt{3} & 6039\\ 2012 & 4-2013\sqrt{3}\end{bmatrix}$

(c) $\frac{1}{4}\begin{bmatrix}2+\sqrt{3} & 1\\ -1 & 2-\sqrt{3}\end{bmatrix}$ (d) $\frac{1}{4}\begin{bmatrix}2012 & 2-\sqrt{3}\\ 2+\sqrt{3} & 2012\end{bmatrix}$

16. $\bar{A}=\begin{bmatrix}-1 & 2-3i & 3+4i\\ 2+3i & 5 & 1+i\\ 3-4i & 1-i & 4\end{bmatrix}$, then det A is ...

(a) Purely real (b) purely imaginary (c) complex number (d) 0

17. The value of $\begin{vmatrix}\log_3 1024 & \log_3 3\\ \log_3 8 & \log_3 9\end{vmatrix}\times\begin{vmatrix}\log_2 3 & \log_4 3\\ \log_3 4 & \log_3 4\end{vmatrix}=$ ...

(a) 6 (b) 9 (c) 10 (d) 12

18. If $A = \begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix}$, $i = \sqrt{-1}$, then $A^n = I$ where I is unit matrix when n = ...

(a) 4p + 1 (b) 4p + 3 (c) 4p (d) 4p + 2

19. If $A = \begin{bmatrix} k & 3 \\ 3 & k \end{bmatrix}$ and $|A^3| = 343$, then find the value of k = ...

(a) $\pm$ 1 (b) $\pm$ 2 (c) $\pm$ 3 (d) $\pm$ 4

20. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then $A^n + (n - 1) I$ = ...

(a) $2^{n-1}A$ (b) - nA (c) nA (d) (n + 1) A

21. If $\begin{vmatrix} x^2 + x & x+1 & x-2 \\ 2x^2 + 3x - 1 & 3x & 2x-3 \\ x^2 + 2x + 3 & 2x-1 & 2x-1 \end{vmatrix} = 24x + B$ then B = ...

(a) -12 (b) 12 (c) 24 (d) -8

22. $\begin{vmatrix} \tan^2 x & -\sec^2 x & 1 \\ -\sec^2 x & \tan^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} = ...$

(a) $12 \tan^2 x - 10 \sec^2 x$ (b) $12 \sec^2 x - 10 \tan^2 x + 2$

(c) 0 (d) $\tan^2 x \cdot \sec^2 x$

23. If $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cos x \operatorname{cosec}^2 x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \operatorname{cosec}^2 x \end{vmatrix}$, then $\int_0^{\frac{\pi}{2}} f(x)\, dx = ...$

(a) $\frac{1}{3} - \frac{\pi}{3}$ (b) $\frac{1}{3} - \frac{\pi}{4}$ (c) $\frac{2}{3} + \frac{\pi}{3}$ (d) $\frac{4}{3} - \frac{\pi}{4}$

24. If $f(x) = \begin{vmatrix} x & e^{x^2} & \sec x \\ \sin x & 2 & \cos x \\ \operatorname{cosec} x & x^2 & 5 \end{vmatrix}$, then the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\, dx = ...$

(a) 0 (b) $5e^{\pi}$ (c) $1 - \frac{\pi}{2}$ (d) 34

25. If $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a, b, c R^+, $abc = 1$ and $|A| > 0$, $A^TA = I$, then $a^3 + b^3 + c^3 = ...$

(a) 12 (b) 4 (c) -8 (d) 28

26. If $f(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ p & p^2 & p^3 \end{vmatrix}$, then $\frac{d^n}{dx^n}\left(f(x)\right)$ at $x = 0$ is ... where p is constant.

(a) p (b) $p + p^2$

(c) p^3 (d) Independent of p.

27. The value of the determinant $\begin{vmatrix} \cos^2\left(\frac{\pi}{2}+x\right) & \cos^2\left(\frac{3\pi}{2}+x\right) & \cos^2\left(\frac{5\pi}{2}+x\right) \\ \cos\left(\frac{\pi}{2}+x\right) & \cos\left(\frac{3\pi}{2}+x\right) & \cos\left(\frac{5\pi}{2}+x\right) \\ \cos\left(\frac{\pi}{2}-x\right) & \cos\left(\frac{3\pi}{2}-x\right) & \cos\left(\frac{5\pi}{2}-x\right) \end{vmatrix}$ is ...

(a) 0 (b) $\cos^2\left(3x - \frac{9\pi}{2}\right)$

(c) $\sin^2\left(\frac{3\pi}{2} + x\right)$ (d) $\cos^2\left(\frac{15\pi}{2} - x\right)$

28. If $\Delta(x) = \begin{vmatrix} x^2 - 5x + 3 & 2x - 5 & 3 \\ 3x^2 + x + 4 & 6x + 1 & 9 \\ 7x^2 - 6x + 9 & 14x - 6 & 21 \end{vmatrix} = ax^3 + bx^3 + cx + d$, then d = ...

(a) 156 (b) 187 (c) 119 (d) 141

29. If $A = \begin{bmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{bmatrix}$, $a, b, c \in R$, $abc = 1$ and $AA^T = 64I$ and $|A| > 0$, then $\left(a^3 + b^3 + c^3\right) = ...$

(a) 343 (b) 729 (c) 256 (d) 512

30. Let P be a non-singular matrix and $1 + P + P^2 + ... + P^n = O$, (O denotes the null matrix) then $P^{-1} = ...$

(a) 0 (b) P (c) P^n (d) I

31. The matrix $\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$ is singular if...

(a) $a - b = 0$ (b) $a + b = 0$

(c) $a + b + c = 0$ (d) $a = 0$

32. $\begin{vmatrix} x+1 & x+3 & x+4 \\ x+4 & x+6 & x+8 \\ x+8 & x+10 & x+14 \end{vmatrix} = \ldots$

(a) 2 (b) -2 (c) 4 (d) -4

33. If a, b, c are positive and not all equal, then the value of determinant

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is ...

(a) > 0 (b) ≥ 0 (c) < 0 (d) ≤ 0

34. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = \ldots$

(a) 0 (b) $(a^2 - bc)(b^2 - ca)(c^2 - ab)$

(c) $(a - b)(b - c)(c - a)$ (d) -1

35. If the equations $y + z = -ax$, $z + x = -by$, $x + y = -cz$ have non trivial solutions, then

$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = \ldots$

(a) 1 (b) 2 (c) -1 (d) -2

36. If the equations $a(y + z) = x$, $b(z + x) = y$, $c(x + y) = z$ have non trivial solutions, then

$\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = \ldots$

(a) 1 (b) 2 (c) -1 (d) -2

37. If the equations $x - 2y + 3z = 0$, $-2x + 3y + 2z = 0$, $-8x + \lambda y = 0$ have non-trivial solution then $\lambda = ...$

(a) 18 (b) 13 (c) -10 (d) 4

38. If the equations $x + 3y + z = 0$, $2x - y - z = 0$, $kx + 2y + 3z = 0$ have non-trivial solution then k = ...

(a) $\frac{13}{2}$ (b) $\frac{9}{2}$ (c) $-\frac{15}{2}$ (d) $-\frac{13}{2}$

39. If the equations $ax + by + cz = 0$, $4x + 3y + 2z = 0$, $x + y + z = 0$ have non-trivial solution, then a, b, c are in...

(a) A.P. (b) G.P.

(c) Increasing sequence (d) decreasing sequence.

40. If the system of equations $x + ay = 0$, $az + y = 0$, $ax + z = 0$ has infinite number of solutions then $a = ...$

(a) 0 (b) 1 (c) -1 (d) -2

41. The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is ...

(a) {1, 2} (b) {-1, -2} (c) {1, -2} (d) {-1, 2}

42. The equations $x + 2y + 3z = 1$, $2x + y + 3z = 2$, $5x + 5y + 9z = 4$ have....

(a) no solution (b) unique solution

(c) Infinity solutions (d) can not say anything

43. If $A = \begin{bmatrix} 2 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then $A^{-1} = ...$

(a) A (b) A^2 (c) A^3 (d) A^4

• Read the following paragraph carefully and answer the following questions No. 44 to 46.

$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ if U_1, U_2 and U_3 are column matrices satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and U is 3×3 matrix whose columns are U_1, U_2, U_3, then

44. The value of | U | is ...

(a) 3 (b) -3 (c) $\frac{3}{2}$ (d) 2

45. The sum of the elements of U^{-1} is ...

(a) -1 (b) 0 (c) 1 (d) 3

46. The value of determinant of $[3 \;\; 2 \;\; 0] \; U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is ...

(a) 5 (b) $\frac{5}{2}$ (c) 4 (d) $\frac{3}{2}$

47. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$, then ...

(a) $a = 2$, $c = -\frac{1}{2}$ (b) $a = 1$, $c = -1$

(c) $a = -1$, $c = 1$ (d) $a = \frac{1}{2}$, $c = \frac{1}{2}$

48. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then *adj* A = ...

(a) A (b) A^T (c) 3A (d) $3A^T$

49. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$, then $A^3 = ...$

(a) I (b) A^T (c) O (d) A^{-1}

50. If $A = \begin{bmatrix} \frac{-1+i\sqrt{3}}{2i} & \frac{-1-i\sqrt{3}}{2i} \\ \frac{1+i\sqrt{3}}{2i} & \frac{1-i\sqrt{3}}{2i} \end{bmatrix}$, $i = \sqrt{-1}$ and $f(x) = x^2 + 2$ then f(A) = ...

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\left(\frac{3-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\left(\frac{5-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\left(2+i\sqrt{3}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

51. If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$ is an idempotent matrix, then x is equal to ...

(a) - 1 (b) - 5 (c) - 4 (d) - 3

52. Let a, b, c be positive real numbers, the following system of equations in x, y and z

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, \quad -\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, \quad -\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$ has ...

(a) unique solutin (b) no solution

(c) finitely many solutions (d) Infinitely many solutions.

53. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $A^n = \ldots$

(a) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ (b) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$

(c) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$ (d) $\begin{bmatrix} 2n+1 & -4n \\ n & 1-2n \end{bmatrix}$

54. Suppose a matrix A satisfies $A^2 - 5A + 7I = O$. If $A^5 = aA + bI$ then the value of $2a - 3b$ must be...

(a) 4135 (b) 1435 (c) 1453 (d) 3145

55. In a ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$, then the value of $64(\sin^2A + \sin^2B + \sin^2C)$ must be...

(a) 64 (b) 144 (c) 128 (d) 0

56. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then $A^{2013} = \ldots$

(a) $2^{2012}A$ (b) $2^{1006}A$ (c) $-2^{2013}A$ (d) I

57. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then $A^{2013} = \ldots$

(a) $3^{2013}A$ (b) $-3^{2012}I$ (c) $3^{2011}A$ (d) $3^{1006}A$

58. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and $B = (adj\ A)$, $C = 5A$ then $\frac{|adj\ B|}{|c|} = ...$

(a) 5 (b) 1 (c) 3 (d) $\frac{1}{5}$

59. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$, then $\left|\left(adj\left(adj\left(adj\left(adj\ A\right)\right)\right)\right)\right| = ...$

(a) 1 (b) 2 (c) 2^4 (d) 2^{12}

60. If $A_r = \begin{vmatrix} r & r-1 \\ r-1 & r \end{vmatrix}$, where r is a natureal number then the value of

$\sqrt{\left(\sum_{r=1}^{2013} A_r\right)}$ is ...

(a) 1 (b) 40 (c) 2012 (d) 2013

61. If z is a complex number and a_1, a_2, a_3, b_1, b_2, b_3 are all real, then

$$\begin{vmatrix} a_1z + b_1\bar{z} & a_2z + b_2\bar{z} & a_3z + b_3\bar{z} \\ b_1z + a_1\bar{z} & b_2z + a_2\bar{z} & b_3z + a_3\bar{z} \\ b_1z + a_1 & b_2z + a_2 & b_3z + a_3 \end{vmatrix} = ...$$

(a) $|\bar{z}|^2$ (b) $(a_1a_2a_3 + b_1b_2b_3)^2\ |z|^2$

(c) 3 (d) 0

62. If $D = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$, then maximum value of D is...

(a) 9 (b) 1 (c) 10 (d) 16

63. $$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} \cos^2\phi & \cos\theta\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

provided $\theta - \phi = ...,\ n \in Z$

(a) $n\pi$ (b) $(2n+1)\frac{\pi}{2}$ (c) $n\frac{\pi}{2}$ (d) $2n\pi$

64. If P, Q, R represent the angles of an acute angled triangle, no two of them being equal then the value of $\begin{vmatrix} 1 & 1+\cos P & \cos P\,(1+\cos P) \\ 1 & 1+\cos Q & \cos Q\,(1+\cos Q) \\ 1 & 1+\cos R & \cos R\,(1+\cos R) \end{vmatrix}$ is ...

(a) positive (b) 0 (c) negative (d) can not be determined

65. If $0 \le [x] < 2$, $-1 \le [y] < 1$, $1 \le [z] < 3$ ($[\cdot]$ denotes the greatest integer function) then the maximum value of determinant $D = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is ...

(a) 6 (b) 2 (c) 4 (d) 8

66. If $A = \begin{bmatrix} 1 & \tan\alpha \\ -\tan\alpha & 1 \end{bmatrix}$ then $A^T A^{-1} = \ldots$

(a) $\begin{bmatrix} -\cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$ (b) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

(c) $\begin{bmatrix} \cos 2x & \sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$ (d) $\begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$

67. $f(x) = \begin{bmatrix} \cos x & 0 & \sin x \\ 0 & 1 & 0 \\ -\sin x & 0 & \cos x \end{bmatrix}$, $g(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(i) $f(x) \,.\, g(y) = \ldots$

(a) $f(xy)$ (b) $f\left(\frac{x}{y}\right)$ (c) $f(x + y)$ (d) $f(x - y)$

(ii) Which of the following is correct ?

(a) $[f(x)]^{-1} = \frac{1}{f(x)}$ (b) $[f(x)]^{-1} = -f(x)$

(c) $[f(x)]^{-1} = f(-x)$ (d) $[f(x)]^{-1} = -f(-x)$

(iii) $[f(x)\, g(y)]^{-1} = \ldots$

(a) $f(x^{-1})\, g(y^{-1})$ (b) $f(y^{-1})\, g(x^{-1})$

(c) $f(-x)\, g(-y)$ (d) $g(-y)\, f(-x)$

68. If $D_1 = \begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix}$ and $D_2 = \begin{vmatrix} x & a \\ a & x \end{vmatrix}$, then ...

(a) $D_1 = 3(D_2)^{\frac{3}{2}}$ (b) $D_1 = 3{D_2}^2$

(c) $\frac{d}{dx}(D_1) = 3{D_2}^2$ (d) $\frac{d}{dx}(D_1) = 3D_2$

69. If α, β are the roots of the equation $x^2 + bx + c = 0$, then

$$\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^2+\beta^2 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix} = \ldots$$

(a) $(1 + b + c)^2 (b^2 - 4c)$ (b) $(1 - b - c)^2 (b^2 - 4c)$

(c) $(1 - b + c)^2 (b^2 - 4c)$ (d) $(1 + b - c)^2 (b^2 - 4c)$

70. If $x^a y^b = e^m$, $x^c y^d = e^n$, $D_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $D_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$ and $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then

$x = \ldots$ and $y = \ldots$

(a) $\log\left(\frac{D_1}{D}\right)$, $\log\left(\frac{D_2}{D}\right)$ (b) $\frac{D_1}{D}$, $\frac{D_2}{D}$ (c) $e^{\frac{D_1}{D}}, e^{\frac{D_2}{D}}$ (d) $\frac{D_2}{D_1}$, $\frac{D}{D_1}$

71. The value of determinant $\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}\,i & \sqrt{27}+2i \end{vmatrix}$ is a

(a) real number (b) rational number

(c) irrational number (d) complex number

72. Match the following columns :

Column I	Column II
1. A is a square matrix such that $A^2 = A$	A. A is a Nilpotent matrix
2. A is a square matreix such that $A^m = O$	B. A is an Involutary matrix
3. A is a square matrix such that $A^2 = I$	C. A is a symmetric matrix
4. A is a square matrix such that $A^T = A$	D. A is an dempotent matrix

(a) 1-D, 2-A, 3-C, 4-B (b) 1-D, 2-A, 3-B, 4-C

(c) 1-A, 2-C, 3-D, 4-B (d) 1-B, 2-D, 3-C, 4-A

73. If $f(x)=\begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$, then $\int_0^{\frac{\pi}{2}} \left(f(x)+f'(x)\right)\,dx = ...$

(a) 1 (b) π (c) 0 (d) $\frac{1}{3}-\frac{\pi}{2}$

74. If $f(x)=\begin{vmatrix} x & \sin x & \cos x \\ x^2 & -\tan x & -x^3 \\ 2x & \sin 2x & 5x \end{vmatrix}$, then $\lim_{x\to 0} \frac{f'(x)}{x} = ...$

(a) 0 (b) 1 (c) 2 (d) 4

75. The set of natural numbers N is partitioned into arrays of rows and columns in the form of matrices as $M_1=[1]$, $M_2=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $M_3=\begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$, ... , $M_n=[...]$ and so on. Find the sun of the elements of the diagonal in M_6.

(a) 144 (b) 441 (c) 321 (d) 461

76. Let $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = ax^5+bx^4+cx^3+dx^2+ex+f$, then match the following columns:

Column I	Column II
1. The Value of f = ...	A . 0
2. The value of e = ...	B. 1
3. The value of a + c = ...	C. -1
4. The value of b + d = ...	D. 3

(a) 1-C, 2-D, 3-A, 4-B (b) 1-A, 2-B, 3-B, 4-C

(c) 1-B, 2-D, 3-C, D-B (d) 1-D, 2-C, 3-D, 4-A

77. If $f(x)=\begin{vmatrix} (1+3x)^m & (1+5x)^m & 1 \\ 1 & (1+3x)^m & (1+5x)^n \\ (1+5x)^n & 1 & (1+3x)^m \end{vmatrix}$, a, b being positive integers, then sum of constant term and coefficient of x is equal to ...

(a) 5 (b) - 8 (c) 1 (d) 0

78. If maximum and minimum value of the determinant

$$\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$$ are M and m respectively, then match the following columns.

Column I	Column II
1. $M^2 + m^{2013} =$	A. always an odd for $k \in N$
2. $M^3 - m^3 =$	B. Being three sides of triangle
3. $M^{2k} - m^{2k} =$	C. 10
4. 2M - 3m, M + m, M + 2m	D. 4
	E. Always an even for $k \in N$
	F. does not being three sides of triangle.
	G. 26

(a) 1-D, 2-G, 3-A, 4-B (b) 1-G, 2-D, 3-A, 4-E

(c) 1-C, 2-G, 3-E, 4-B (d) 1-D, 2-C, 3-E, 4-F

79. If $[x]$ is the greatest integer less than or equal to x, then the determinant's value of the matrix.

$$\begin{bmatrix} [e] & [\pi] & [\pi^2-6] \\ [\pi] & [\pi^2-6] & [e] \\ [\pi^2-6] & [e] & [\pi] \end{bmatrix}$$ is ...

(a) 8 (b) 0 (c) 1 (d) -8

80. When the determinant $f(x) = \begin{vmatrix} \cos 2x & \sin^2 x & \cos 4x \\ \sin^2 x & \cos 2x & \cos^2 x \\ \cos 4x & \cos^2 x & \cos 2x \end{vmatrix}$ is expanded in powers of $\sin x$, then the constant term in that expansion is...

(a) 0 (b) -1 (c) 2 (d) 1

81. The value of the determinant $\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}$ is ...

(a) 0 (b) $2\sin\theta$ (c) $-\sin 2\theta$ (d) $-2\cos\theta$

82. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then $A^2 = B$ for

(a) $\alpha = 4$ (b) $\alpha = 1$ (c) $\alpha = -1$ (d) no α

83. If $A = \begin{bmatrix} \alpha & 0 \\ 2 & 3 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A^2 = 9I$ for

(a) $\alpha = 4$ (b) $\alpha = 3$ (c) $\alpha = -3$ (d) no α

84. If $A = \begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix}$, then $I + 2A + 3A^2 + ... \infty = ...$

(a) $\begin{bmatrix} 9 & 1 \\ -9 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 1 \\ -9 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 2 \\ -18 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 2 \\ -5 & -18 \end{bmatrix}$

85. If M is a 3 × 3 matrix, where $M^TM = I$ and det M = 1, then det(M - I) = ...

(a) 0 (b) -1 (c) 4 (d) -3

86. Let λ and α be real. The system of equations

$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$-x + (\sin\alpha)y - (\cos\alpha)z = 0$ has no trivial solution.

(i) The the set of all values of λ is

(a) $\left[-\sqrt{3}, \sqrt{3}\right]$ (b) $\left[-\sqrt{2}, \sqrt{2}\right]$ (c) [-1, 1] (d) $\left[0, \frac{\pi}{2}\right]$

(ii) For $\lambda = 1$, $\alpha = ...$

(a) $n\pi, n\pi - \frac{\pi}{4}$ (b) $2n\pi, n\pi - \frac{\pi}{4}$ (c) $n\pi, n\pi + \frac{\pi}{4}$ (d) $-\pi, -\frac{3\pi}{4}$

87. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $A^2 = 8A + kI_2$, then $k = ...$

(a) 1 (b) -1 (c) 7 (d) -7

88. The identity element in the group $M = \left\{ \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} / x \in R,\ x \neq 0 \right\}$ with respect to matrix multiplicaiton is ...

(a) $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

89. The inverse element of $\begin{bmatrix} y & y & y \\ y & y & y \\ y & y & y \end{bmatrix}$ in group

$$M = \left\{ \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} / x \in R,\ x \neq 0,\ I = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right\} \text{ w.r.t.}$$

matrix multiplication is...

(a) $\begin{bmatrix} \frac{1}{y} & \frac{1}{y} & \frac{1}{y} \\ \frac{1}{y} & \frac{1}{y} & \frac{1}{y} \\ \frac{1}{y} & \frac{1}{y} & \frac{1}{y} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{3y} & \frac{1}{3y} & \frac{1}{3y} \\ \frac{1}{3y} & \frac{1}{3y} & \frac{1}{3y} \\ \frac{1}{3y} & \frac{1}{3y} & \frac{1}{3y} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{6y} & \frac{1}{6y} & \frac{1}{6y} \\ \frac{1}{6y} & \frac{1}{6y} & \frac{1}{6y} \\ \frac{1}{6y} & \frac{1}{6y} & \frac{1}{6y} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \\ \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \\ \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \end{bmatrix}$

90. If $A = \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$ and $\phi(x) = (1+x)(1-x)^{-1}$, then $\phi(A) = ...$

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

91. Costruct an orthogonal matrix using the skew-symmetric matrix $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

(a) $\begin{bmatrix} -\frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$ (d) $\begin{bmatrix} -\frac{4}{5} & -\frac{3}{5} \\ -\frac{3}{5} & -\frac{4}{5} \end{bmatrix}$

92. Construct an orthogonal matrix using the skew-symmetric matrix

$$A = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -5 \\ -1 & 5 & 0 \end{bmatrix}$$

(a) $\frac{1}{31}\begin{bmatrix} 21 & 6 & 22 \\ 14 & 21 & 6 \\ 18 & 14 & 21 \end{bmatrix}$ (b) $\frac{1}{31}\begin{bmatrix} 21 & 6 & 22 \\ 14 & -27 & -6 \\ 18 & 14 & -21 \end{bmatrix}$

(c) $\frac{1}{31}\begin{bmatrix} 21 & 6 & 22 \\ 6 & 14 & 18 \\ 22 & 18 & 7 \end{bmatrix}$ (d) $\frac{1}{31}\begin{bmatrix} 21 & -6 & 22 \\ 6 & 14 & -18 \\ -22 & 18 & 7 \end{bmatrix}$

93. If $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, then $A^3 - 7A^2 + 10A = ...$

(a) $5I - A$ (b) $5I + A$ (c) $A - 5I$ (d) $7I$

94. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $8A^{-4} = ...$

(a) $145 A^{-1} - 27I$ (b) $27I - 145 A^{-1}$

(c) $29A^{-1} + 9I$ (d) $145A^{-1} + 27I$

95. The system of equations

$x_1 + 2x_2 + 3x_3 = \lambda x_1$,

$3x_1 + x_2 + 2x_3 = \lambda x_2$,

$2x_1 + 3x_2 + x_3 = \lambda x_3$ can possess a non-trivial solution then $\lambda = ...$

(a) 1 (b) 2 (c) 3 (d) 6

96. Solution of the system of linear equations (α constant)

$x \sec^2\alpha - y \tan^2\alpha + z = 2.$

$x \cos^2\alpha + y \sin^2\alpha = 1$

$x + z = 2$ is $(x, y, z) = ...$

(a) (1, 1, 1) (b) (1, 2, 2) (c) (2, 1, 2) (d) (1, 0, 1)

97. For what value of k the following system of linear equations

$x + 2y - z = 0$,

$3x + (k + 7)y - 3z = 0$,

$2x + 4y + (k - 3)z = 0$ possesses a non-triuvial solution.

(a) 1 (b) 0 (c) 2 (d) -2

98. The correct match of the following columns is given by

Column I C olumn II

1.	Leibnitz	A.	$e^{i\theta}$
2.	Euler	B.	Mathematical logic
3.	Cayley - Hamilton	C.	Calculus
4.	George Boole	D.	$(e^{i\theta})^n = e^{i(n\theta)}$
5.	De-moivre	E.	Theory of Matices

(a) 1-D, 2-A, 3-E, 4-b, 5-A (b) 1-B, 2-D, 3-A, 4-C, 5-E

(c) 1-C, 2-A, 3-D, 4-B, 5-E (d) 1-C, 2-A, 3-E, 4-B, 5-D

99. Let $A=\begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -5 & -2\end{bmatrix}$. If q is the angle between two non-zero column vectors X such that AX = λX for some scalar λ, then tanθ = ...

(a) $\frac{7}{\sqrt{202}}$ (b) $\frac{\sqrt{3}}{19}$ (c) $\sqrt{\frac{3}{202}}$ (d) $\frac{7}{19}$

100. Let the 3-digit numbers A28, 3B9 and 62C, where A, B, C are integers between 0 and 9, be divisible by a fixed interger k, then the determinant $\begin{vmatrix}A & 3 & 6\\ 8 & 9 & C\\ 2 & B & 2\end{vmatrix}$ is divisible by ...

(a) 3k (b) k^3 (c) k (d) $\frac{k}{3}$

101. If $A=\begin{bmatrix}-2 & 3\\ -1 & 1\end{bmatrix}$ then $I + A + A^2 + \ldots\infty = \ldots$

(a) $\begin{bmatrix}0 & 3\\ 1 & 3\end{bmatrix}$ (b) $\frac{1}{3}\begin{bmatrix}0 & -3\\ -1 & 3\end{bmatrix}$ (c) $\frac{1}{3}\begin{bmatrix}0 & 3\\ -1 & 3\end{bmatrix}$ (d) undefined.

102. If $A=\begin{bmatrix}-2 & 3\\ -1 & 1\end{bmatrix}$, then $A^3 = \ldots$

(a) I (b) O (c) -A (d) A + I

103. If $\begin{vmatrix}\binom{x}{r} & \binom{x+1}{r+1} & \binom{x+2}{r+2}\\ \binom{y}{r} & \binom{y+1}{r+1} & \binom{y+2}{r+2}\\ \binom{z}{r} & \binom{z+1}{r+1} & \binom{z+2}{r+2}\end{vmatrix} = \lambda\begin{vmatrix}\binom{x}{r} & \binom{x}{r+1} & \binom{x}{r+2}\\ \binom{y}{r} & \binom{y}{r+1} & \binom{y}{r+2}\\ \binom{z}{r} & \binom{z}{r+1} & \binom{z}{r+2}\end{vmatrix}$, then λ is ...

(a) 0 (b) 1 (c) -1 (d) 2

104. Investigate the values λ and μ for the system

$x + 2y + 3z = 6$

$x + 3y + 5z = 9$

$2x + 5y + \lambda z = \mu$

and correct match the following columns.

Column I		Column II	
1.	$\lambda = 8,\ \mu \neq 15$	A.	Infinity of solutions
2.	$\lambda \neq 8,\ \mu \in R$	B.	No solutions
3.	$\lambda = 8,\ \mu = 15$	A.	Unique solution

(a) 1-A, 2-B, 3-C (B) 1-B, 2-C, 3-A

(c) 1-C, 2-A, 3-B (d) 1-C, 2-B, 3-A

105. The value of α, β, γ when $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal are ...

(a) $\pm\frac{1}{\sqrt{2}},\ \pm\frac{1}{\sqrt{6}},\ \pm\frac{1}{\sqrt{2}}$ (b) $\pm\frac{1}{\sqrt{3}},\ \pm\frac{1}{\sqrt{2}},\ \pm\frac{1}{\sqrt{6}}$

(c) $\pm\frac{1}{\sqrt{2}},\ \pm\frac{1}{\sqrt{6}},\ \pm\frac{1}{\sqrt{3}}$ (d) $\pm\frac{1}{\sqrt{2}},\ \pm\frac{1}{\sqrt{2}},\ \pm\frac{1}{\sqrt{2}}$

Hint

1. $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0 \Rightarrow k = \dfrac{33}{2}$

Put value of k in given equations and solved them

$\Rightarrow \dfrac{x}{15} = \dfrac{y}{-2} = \dfrac{z}{6} \Rightarrow \dfrac{xy}{z^2} = \dfrac{-30}{36} = \dfrac{-5}{6}$

2. $D = \begin{vmatrix} n & n! & 1 \\ n+1 & (n+1)! & 1 \\ n+2 & (n+2)! & 1 \end{vmatrix}$

$= n!\begin{vmatrix} n & 1 & 1 \\ n+1 & n+1 & 1 \\ n+2 & (n+2)(n+1) & 1 \end{vmatrix}$ $\quad \because C_2\left(\dfrac{1}{n!}\right)$

$= n!\begin{vmatrix} n & 1 & 1 \\ 1 & n & 0 \\ 1 & (n+1)^2 & 0 \end{vmatrix}$ $\quad \because R_{23}(-1), R_{12}(-1)$

$= \left(n^2 + n + 1\right)n!$ $\quad \because$ Expanding along C_3

3. The 2[nd] det $D_2 = \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^n a & -(-1)^n b & (-1)^n c \end{vmatrix}$

$= (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}$

$= (-1)^n D_1$ $\quad \because R_{13}, R_{23}$ and taking transpose

$\therefore \left(1 + (-1)^n\right) D_1 = 0$ For any odd integer

$\because D_1 \neq o$ since $b(a+c) \neq o$

4. Given determinant is a product of two eleminant

$$\begin{vmatrix} \sin x & \cos x & 0 \\ \sin y & \cos y & 0 \\ \sin z & \cos z & 0 \end{vmatrix} \begin{vmatrix} \cos p & \cos q & \cos r \\ \sin p & \sin q & \sin r \\ 0 & 0 & 0 \end{vmatrix} = 0 \times 0 = 0$$

5. $$\begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} = 0$$ $\because C_{21}(1), C_{31}(1)$

$\therefore (3x-2)(3x-11)^2 = 0$ $\because C_1\left(\frac{1}{3x-2}\right)$ and expanding along R_1

$\therefore x = \frac{2}{3}, \frac{11}{3}$

6. Let a = 1, b = - 1, c = 2

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 9 & 1 \\ 4 & 4 & 0 \end{vmatrix} = k(-2)(2)^3$$

$\therefore -32 = -16k$

$\therefore k = 2$

7. Let a = 1, b = -1, c = 2

$$\begin{vmatrix} 1 & 2 & 2 \\ 1 & 5 & 1 \\ -1 & -1 & -5 \end{vmatrix} = k(1)(-1)(2)$$

$\therefore -8 = -2k$

$\therefore k = 4$

8. $$\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\therefore (1+abc)\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}=0$$

$$\therefore abc=-1 \text{ since } \begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}\neq 0$$

9. $$D=\left(\sqrt{5}\right)\left(\sqrt{5}\right)\begin{vmatrix}\sqrt{11}+\sqrt{3} & 2 & 1\\ \sqrt{15}+\sqrt{22} & \sqrt{5} & \sqrt{2}\\ 3+\sqrt{55} & \sqrt{3} & \sqrt{5}\end{vmatrix} \qquad \because C_2\left(\frac{1}{\sqrt{5}}\right), C_3\left(\frac{1}{\sqrt{5}}\right)$$

$$=5\begin{vmatrix}-\sqrt{3} & 2 & 1\\ 0 & \sqrt{5} & \sqrt{2}\\ 0 & \sqrt{3} & \sqrt{5}\end{vmatrix} \qquad \because C_{21}\left(\sqrt{-3}\right), C_{31}\left(-\sqrt{11}\right)$$

$$=5\left(3\sqrt{2}-5\sqrt{3}\right) \qquad \because \text{ expanding along } R_1$$

10. $$\text{If } s=0, \det A = a^2b^2c^2\begin{vmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{vmatrix}=0$$

$\therefore S^2$ is a factor of det A since 3 rows are identical

Let s=a

$$\therefore \det A=\begin{vmatrix}a^2 & 0 & 0\\ (a-b)^2 & b^2 & (a-b)^2\\ (a-c)^2 & (a-c)^2 & c^2\end{vmatrix}=\begin{vmatrix}a^2 & 0 & 0\\ c^2 & b^2 & c^2\\ b^2 & b^2 & c^2\end{vmatrix}=0 \qquad \because b+c=a$$

$\therefore$ s - a is a factor of det A

Similarly (s - b) and (s - c) are also factors.

but det A is a sixth degree polynomial

$\therefore$ The sixthe factor is of the form k (a+b+c).

$$\therefore \det A = k\left(a+b+c\right)s^2\left(s-a\right)\left(s-b\right)\left(s-c\right)$$

Let $a=b=0,\ c=2 \Rightarrow s=1$

$$\therefore \det A = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 4 \end{vmatrix} = -2k \Rightarrow k = 1$$

$$\therefore \det A = 2s^3(s-a)(s-b)(s-c)$$

11. The determinant of coefficient matrix is the product of two determinant.

$$\begin{vmatrix} 1 & 1 & 0 \\ \alpha+\beta & \gamma+\delta & 0 \\ \alpha\beta & \gamma\delta & 0 \end{vmatrix} \begin{vmatrix} 1 & \gamma+\delta & \gamma\delta \\ 1 & \alpha+\beta & \alpha\beta \\ 0 & 0 & 0 \end{vmatrix} = 0 \times 0 = 0$$

for any $\alpha, \beta, \gamma, \delta$

12. $\det\left(A^2\right) = (\det A)^2 = \begin{vmatrix} 4 & 4k & k \\ 0 & k & 4k \\ 0 & 0 & 4 \end{vmatrix}^2 = 16$

$$\Rightarrow (16k)^2 = 16$$

$$\Rightarrow k^2 = \frac{1}{16}$$

$$\therefore |k| = \frac{1}{4}$$

13. $D = \begin{vmatrix} a & a^2 & a^3 \\ a^\omega & a^{2\omega} & a^{3\omega} \\ a^{\omega^2} & a^{2\omega^2} & a^{3\omega^2} \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ a^\omega & a^{2\omega} & 1 \\ a^{\omega^2} & a^{2\omega^2} & 1 \end{vmatrix}$

$$= a\, a^\omega\, a^{\omega^2} \begin{vmatrix} 1 & a & a^2 \\ 1 & a^\omega & a^{2\omega} \\ 1 & a^{\omega^2} & a^{2\omega^2} \end{vmatrix} + \begin{vmatrix} 1 & a^2 & a \\ 1 & a^{2\omega} & a^\omega \\ 1 & a^{2\omega^2} & a^{\omega^2} \end{vmatrix}$$

$=0 \qquad \because 1 + \omega + \omega^2 = 0$

14. Use $a_n = a_1 r^{n-1}$ $\quad \therefore \log a_n = \log a_1 + (n-1)\log r$

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ 3\log r & 3\log r & 3\log r \\ 6\log r & 6\log r & 6\log r \end{vmatrix} = 0$$

$\because R_{12}(-1), R_{13}(-1)$ and then $R_2 = R_3$

15. Clearly P is an orthogonal matrix $\quad \therefore \; PP^T = P^T P = I$

$Q^{2013} = (PAP^T)(PAP^T) \ldots (PAP^T) \quad (2013 \text{ times})$

$= PA^{2013} P^T$

$P^T Q^{2013} P = A^{2013}$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \; A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \ldots, \; A^{2013} = \begin{bmatrix} 1 & 2013 \\ 0 & 1 \end{bmatrix}$$

16. $\det \overline{A} = \det (\overline{A})^T = \det A$

$\Rightarrow \overline{\det A} = \det \overline{A} = \det A$

$\Rightarrow \det A$ is Purely real

17. $$D = \begin{vmatrix} \log_3 2^{10} & \log_{2^3} 3 \\ \log_3 2^3 & \log_{2^2} 3^2 \end{vmatrix} \begin{vmatrix} \log_2 3 & \log_{2^2} 3 \\ \log_3 2^2 & \log_3 2^2 \end{vmatrix}$$

$$= \begin{vmatrix} 10\log_3 2 & \frac{1}{3}\log_2 3 \\ 3\log_3 2 & \frac{2}{2}\log_2 3 \end{vmatrix} \begin{vmatrix} \log_2 3 & \frac{1}{2}\log_2 3 \\ 2\log_3 2 & 2\log_3 2 \end{vmatrix}$$

$= (10-1)(2-1) = 9$

18. $A = iI$

$A^n = i^n I^n = i^n I = I \;$ if $n = 4p$

19. $|A^3| = 7^3$

$\Rightarrow |A|^3 = 7^3 \Rightarrow |A| = 7$

$\Rightarrow k^2 - 9 = 7 \Rightarrow k = \pm 4$

20. $$A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, \ldots, \; A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$$

$$\therefore A^n + (n-1) I = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix} = nA$$

21. Putting x=0, we get

$$\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 3 & -1 & -1 \end{vmatrix} = B \Rightarrow B = -12$$

22. $D = \begin{vmatrix} 1 & 1 & 2 \\ -\sec^2 x & \tan^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} \qquad \because R_{21}(1)$

$$= \begin{vmatrix} 1 & 2 & 2 \\ -\sec^2 x & 1 & 1 \\ -10 & 2 & 2 \end{vmatrix} \qquad \because C_{12}(1)$$

$= 0 \qquad \because C_2 = C_3$

23. $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cos x \cdot \operatorname{cosec}^2 x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ \sin^2 x & 0 & 0 \end{vmatrix} \qquad \because C_{23}(-1)$

$= \cos x - \sin^2 x - \cos^3 x$

$= \sin^2 x \cdot \cos x - \sin^2 x$

$$\int_0^{\frac{\pi}{2}} f(x)dx = \int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx - \int_0^{\frac{\pi}{2}} \sin^2 x \, dx \qquad (\because \text{ taking } \sin x = t \text{ substitution})$$

$$= \left[\frac{t^3}{3}\right]_0^1 - \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2} = \frac{1}{3} - \frac{\pi}{4}$$

24. since f (- x) = - f (x), odd function.

$$\therefore \int_{-\pi/2}^{\pi/2} f(x)dx = 0$$

25. since $A^T A = I \Rightarrow A^2 = I \ (\because \ A^T = A \text{ clearly})$

$$\therefore \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} a & b & c \\ b & c & a \\ a & b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore a^2+b^2+c^2=1,\ ab+bc+ca=o$

$\therefore (a+b+c)^2=1 \Rightarrow a+b+c=1$

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\therefore a^3+b^3+c^3=3(1)+1=4 \qquad (\because abc=1)$

26. $\frac{d^n}{dx^n}f(x)=\begin{vmatrix} n! & \sin\left(x+\frac{n\pi}{2}\right) & \cos\left(x+\frac{n\pi}{2}\right) \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ p & p^2 & p^3 \end{vmatrix}$

$\therefore \frac{d^n}{dx^n}f(0)=f^n(0)$

$=0 \quad \because R_1=R_2$

27. By $R_{32}(1)$ we get all entries of R_2 is zero

$\therefore D=0$

28. $\Delta^1(x)=\begin{vmatrix} 2x-5 & 2x-5 & 3 \\ 6x+1 & 6x+1 & 9 \\ 14x-6 & 14x-6 & 21 \end{vmatrix}+\begin{vmatrix} x^2-5x+3 & 2 & 3 \\ 3x^2+x+4 & 6 & 9 \\ 7x^2-6x+9 & 14 & 21 \end{vmatrix}$

$=0(\because C_1=C_2)+0(\because C_2=C_3)=0$

$\therefore \Delta(x)$ is a constant

$\therefore a=0,\ b=0,\ c=0$

Put x $=0$ both sides, we get

$d=\begin{vmatrix} 3 & -5 & 3 \\ 4 & 1 & 9 \\ 9 & -6 & 21 \end{vmatrix}=141$

29. Clearly $A^T=A \Rightarrow AA^T=64I$

$\Rightarrow A^2=64I$

$\Rightarrow |A|=8$

$$\Rightarrow\begin{vmatrix}3a & b & c\\ b & 3c & a\\ c & a & 3b\end{vmatrix}=8$$

$$\Rightarrow 29abc-3\left(a^3+b^3+c^3\right)=8$$

$$\Rightarrow a^3+b^3+c^3=7 \qquad \because\ abc=1$$

$$\Rightarrow \left(a^3+b^3+c^3\right)^3=343$$

30. $P^{-1}\left(1+P+P^2+...+P^n\right)=P^{-1}.O$

$\therefore\ P^{-1}+I+IP+...+IP^{n-1}=O$

$\therefore\ P^{-1}+I\left(1+P+...+P^{n-1}\right)=O$

$\therefore\ P^{-1}+I\left(-P^n\right)=O$

$\therefore P^{-1}=P^n$

31. determinant $\begin{vmatrix}a+b+c & a+b+c & a+b+c\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$ $\qquad \because R_{21}(1),R_{31}(1)$

$=\left(a+b+c\right)\begin{vmatrix}1 & 0 & 0\\ 2b & -\left(a+b+c\right) & 0\\ 2c & 0 & -\left(a+b+c\right)\end{vmatrix}$ $\qquad \because R_1\left(\dfrac{1}{a+b+c}\right),C_{12}(-1),C_{13}(-1)$

$=\left(a+b+c\right)^3$ $\qquad \because C_2\left(\dfrac{1}{a+b+c}\right),C_3\left(\dfrac{1}{a+b+c}\right)$ and expanding along R_1

$=0$ if $a+b+c=0$

32. $D=\begin{vmatrix}x+1 & x+3 & x+4\\ 3 & 3 & 4\\ 4 & 4 & 6\end{vmatrix}$ $\qquad \because R_{23}(-1),R_{12}(-1)$

$=\begin{vmatrix}x+1 & 2 & 1\\ 3 & 0 & 1\\ 4 & 0 & 2\end{vmatrix}$ $\qquad \because C_{23}(-1),C_{12}(-1)$

$= -4\ \because$ expanding along C_2

33. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

$= -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= -\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right] < 0 \;\therefore$ negative

34. $D = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

$= (a-b)(b-c)(c-a) - (a-b)(b-c)(c-a)$

$= 0$

35. equations $\begin{matrix} ax+y+z=0 \\ x+by+z=0 \\ x+y+cz=0 \end{matrix}$

$\therefore \text{since} \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$\therefore \begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 0 & 1-b & c-1 \end{vmatrix} = 0 \qquad \because R_{23}(-1), R_{12}(-1)$

$\therefore a(b-1)(c-1) - 1(1-a)(c-1) + 1(1-b)(1-b) = 0$

$\therefore \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0 \quad \because$ dividing both sides by $(1-a)(1-b)(1-c) \neq 0$

$\therefore \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

36. determinant of coefficient matrix $\begin{vmatrix} -1 & a & a \\ b & -1 & b \\ c & c & -1 \end{vmatrix} = 0$

$$\therefore \begin{vmatrix} -1 & 1+a & 1+a \\ b & -1-b & 0 \\ c & 0 & -1-c \end{vmatrix} = 0 \quad \because C_{12}(-1), C_{13}(-1)$$

$$\therefore \begin{vmatrix} \frac{-1}{1+a} & 1 & 1 \\ \frac{b}{1+b} & -1 & 0 \\ \frac{c}{1+c} & 0 & -1 \end{vmatrix} = 0 \qquad \because R_1\left(\frac{1}{1+a}\right), R_2\left(\frac{1}{1+b}\right), R_3\left(\frac{1}{1+c}\right)$$

$$\therefore \frac{-1}{1+a} + \frac{b}{1+b} + \frac{c}{1+c} = 0$$

$$\therefore \frac{-1}{1+a} + 1 - \frac{1}{1+b} + 1 - \frac{1}{1+c} = 0$$

$$\therefore \frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = 2$$

37. $\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & 2 \\ -8 & \lambda & 0 \end{vmatrix} = 0$

$\Rightarrow -2\lambda + 32 - 6\lambda + 72 = 0$

$\Rightarrow -8\lambda + 104 = 0$

$\Rightarrow \lambda = 13$

38. $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ k & 2 & 3 \end{vmatrix} = 0$

$\Rightarrow -1 - 18 - 3k + 4 + k = 0$

$\Rightarrow k = -\frac{15}{2}$

39. $\begin{vmatrix} a & b & c \\ 4 & 3 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow a - 2b + c = 0 \Rightarrow a, b, c$ are in AP

40. $\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1 - a\left(-a^2\right) = 0 \Rightarrow a^3 + 1 = 0 \Rightarrow a = -1$

41. clearly if $x = -1 \Rightarrow R_2 = R_3$ and $D = 0$

if $x = 2 \Rightarrow R_1 = R_3$ and $D = 0$

$\therefore$ solution set $\{-1, 2\}$

42. $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix} \neq 0 \Rightarrow$ unique solution

43. $|A| = 1 \neq 0, A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$ and $A^4 = I$

$\therefore A^4 \cdot A^{-1} = IA^{-1} \Rightarrow A^3 = A^{-1}$

44. Let $U_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, then $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ 2x + y \\ 3x + 2y + z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow x = 1, y = -2, z = 1$

$\therefore U_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, similarly $U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}, U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

$\therefore U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & 4 & -3 \end{bmatrix} \Rightarrow |U| = 3$

45. $U^{-1} = \frac{1}{|U|} \, adj \, U = \frac{1}{3}\begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$

Sum of elements of $U^{-1} = \frac{1}{3}[-1 - 2 + 0 - 7 - 5 - 3 + 9 + 6 + 3] = 0$

46. $\begin{bmatrix}3 & 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2 & 2\\ -2 & -1 & -1\\ 1 & 4 & -3\end{bmatrix}\begin{bmatrix}3\\2\\0\end{bmatrix}=\begin{bmatrix}-1 & 4 & 4\end{bmatrix}\begin{bmatrix}3\\2\\0\end{bmatrix}=\begin{bmatrix}5\end{bmatrix}$

det [5] = 5

47. $|A|=8+2(a-6)=2a-4$

cofactor of $a_{12}(=1)$ is 8 in $|A|$

In A^{-1} now $A_{21}=\frac{8}{|A|}=-4\Rightarrow\frac{8}{2a-4}=-4\Rightarrow\frac{2}{2a-4}=-1$

$\Rightarrow 2=-2a+4\Rightarrow 2a=2\Rightarrow a=1 \qquad \therefore |A|=-2$

In $A^{-1}, A_{23}=C=\frac{\text{cofator a in}|A|}{|A|}=\frac{2}{-2}=-1$

48. Matrix of confactors of elements of $A = A^c$

$A^c=\begin{bmatrix}-3 & -6 & -6\\ 6 & 3 & -6\\ 6 & -6 & 3\end{bmatrix}\Rightarrow adj\,A=\left(A^c\right)^T=\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix}=3A^T$

49. $A^2=\begin{bmatrix}0 & 0 & 0\\ 3 & 3 & 9\\ -1 & -1 & -3\end{bmatrix}$ and $A^3=O$

50. Let $\omega=\frac{-1+i\sqrt{3}}{2}$ and $\omega^2=\frac{-1-i\sqrt{3}}{2}$

$\therefore \omega^3=1$ and $1+\omega+\omega^2=0$

$\therefore A=\begin{bmatrix}\frac{\omega}{i} & \frac{\omega^2}{i}\\ \frac{-\omega^2}{i} & \frac{-\omega}{i}\end{bmatrix}=\frac{\omega}{i}\begin{bmatrix}1 & \omega\\ -\omega & -1\end{bmatrix}$

$\therefore A^2=\frac{\omega^2}{i^2}\begin{bmatrix}1 & \omega\\ -\omega & -1\end{bmatrix}\begin{bmatrix}1 & \omega\\ -\omega & -1\end{bmatrix}=-\omega^2\begin{bmatrix}1-\omega^2 & 0\\ 0 & 1-\omega^2\end{bmatrix}$

$$=\begin{bmatrix}-\omega^2+\omega^4 & 0\\ 0 & -\omega^2+\omega^4\end{bmatrix}=\begin{bmatrix}1+2\omega & 0\\ 0 & 1+2\omega\end{bmatrix}$$

$$f(A)=A^2+2I=\begin{bmatrix}1+2\omega & 0\\ 0 & 1+2\omega\end{bmatrix}+\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

$$=(3+2\omega)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\left\{3+2\left(\frac{-1+i\sqrt{3}}{2}\right)\right\}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\left(2+i\sqrt{3}\right)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

51. $A^2=A$

$$\therefore\begin{bmatrix}2 & -2 & -16-4x\\ -1 & 3 & 16+4x\\ 4+x & -8-2x & -12+x^2\end{bmatrix}=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & x\end{bmatrix}$$

By comparing Compoent, we have $4+x=1\Rightarrow x=-3$

52. $$\begin{vmatrix}\frac{1}{a^2} & \frac{-1}{b^2} & \frac{-1}{c^2}\\ \frac{-1}{a^2} & \frac{1}{b^2} & \frac{-1}{c^2}\\ \frac{-1}{a^2} & \frac{-1}{b^2} & \frac{1}{c^2}\end{vmatrix}=\frac{1}{a^2b^2c^2}\begin{vmatrix}1 & -1 & -1\\ -1 & 1 & -1\\ -1 & -1 & 1\end{vmatrix}=\frac{-4}{a^2b^2c^2}\neq 0$$

$\therefore$ Unique solution

53. $$A^2=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}=\begin{bmatrix}5 & -8\\ 2 & -3\end{bmatrix}$$

For n = 2 , A^2 = (d) Ans.

54. $A^3=A\cdot A^2=A(5A-7I)=5A^2-7A$

$$=5(5A-7I)-7A=25A-35I-7A=18A-35I$$

Similarly $A^4=A\cdot A^3$

$$=A(18A-35I)$$

$$=18(5A-7I)-35A$$

$$=55A-126I$$

$$A^5=149A-385I=aA+bI$$

$$\therefore a=149, b=-385$$

$$\therefore 2a-3b=2(149)-3(-385)=1453$$

55. On expanding

$$\therefore 1(c^2-ab)-a(c-a)+b(b-c)=0$$

$$\therefore a^2+b^2+c^2-ab-bc-ca=0$$

$$\therefore \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]=0$$

Provided a = b = c

$$\Rightarrow A=B=C=\frac{\pi}{3}$$

$$\therefore 64\left(\sin^2\frac{\pi}{3}+\sin^2\frac{\pi}{3}+\sin^2\frac{\pi}{3}\right)=64\left(\frac{3}{4}+\frac{3}{4}+\frac{3}{4}\right)=144$$

56. $A^2=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}=2\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}=2A$

$$A^3=A\cdot A^2=A\cdot 2A=2A^2=2(2A)=2^2\cdot A=2^{3-1}A$$

$$A^{2013}=2^{2012}A$$

57. $A^2=3A$

$$A^3=A^2\cdot A=3A\cdot A=3\cdot 3A=3^2A=3^{3-1}A$$

$$A^{2013}=3^{2012}A$$

58. $|A|=1(0+3)+1(0+6)+1(0-4)=5$

$$\frac{|adjB|}{|c|}=\frac{|adj(adjA)|}{|5A|}=\frac{|A|^{(3-1)^2}}{5^3|A|}=\frac{|A|^4}{5^4}=1$$

59. $|A|=\cos^2\alpha+\sin^2\alpha=1$

$$adj(adjA)=|A|^{n-2}A=(1)^{3-2}A=A$$

$$\therefore\left|\left(adj\left(adj\left(adj\left(adj\,A\right)\right)\right)\right)\right|=|adj(adjA)|$$

$$=|A|^{(n-1)^2}$$

$$=|A|^4=1$$

60. $A_r = r^2-(r-1)^2 = 2r-1$

$$\sum A_r = \sum(2r-1) = 2\frac{r}{2}(r+1)-r = r^2$$

$$\sum_{r=1}^{2013} A_r = (2013)^2 \Rightarrow \sqrt{\left(\sum_{r=1}^{2013} A_r\right)} = 2013$$

61. Given determinant is a product of two determinant

$$\begin{vmatrix} z & \bar{z} & 0 \\ \bar{z} & z & 0 \\ 1 & z & 0 \end{vmatrix} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 0 & 0 & 0 \end{vmatrix} = 0\times 0 = 0$$

62. $D = 1(1-3\sin\theta\cos\theta)-3\cos\theta(\sin\theta-3\cos\theta)+1(\sin^2\theta-1)$

$$=1-6\sin\theta\cos\theta+8\cos^2\theta$$

$$=(3\cos\theta-\sin\theta)^2=(a\cos\alpha+b\sin\alpha)^2$$

$$\therefore -\sqrt{a^2+b^2}\le\sqrt{D}\le\sqrt{a^2+b^2}$$

$$\therefore -\sqrt{9+1}\le(3\cos\theta-\sin\theta)\le\sqrt{9+1}$$

$$\therefore 0\le(3\cos\theta-\sin\theta)^2\le 10$$

$\therefore$ Range = [0,10]

63. Product =

$$\begin{bmatrix} \cos^2\theta\cos^2\phi+\cos\theta\sin\theta\times\cos\phi\sin\phi & \cos^2\theta\cos\phi\sin\phi+\sin^2\phi\cos\theta\sin\theta \\ \cos\theta\cdot\sin\theta\cos^2\phi+\sin^2\theta\cdot\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi+\sin^2\phi\sin^2\theta \end{bmatrix}$$

$$=\begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \sin\theta\cos\phi\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ if } \theta-\phi=(2n+1)\frac{\pi}{2} \qquad \because \cos(\theta-\phi)=0$$

64. $D=\begin{vmatrix}1 & \cos P & \cos^2 P\\ 1 & \cos Q & \cos^2 Q\\ 1 & \cos R & \cos^2 R\end{vmatrix}= -(\cos P-\cos Q)(\cos Q-\cos R)(\cos R-\cos P)$

if $P<Q<R \quad \Rightarrow D>0$

if $P>Q>R \quad \Rightarrow D<0$ can not be determined.

65. $D=\begin{vmatrix}1 & [y] & [z]\\ 1 & [y]+1 & [z]\\ 1 & [y] & [z]+1\end{vmatrix}\left([x]+[y]+[z]+1\right)\because C_{21}(1), C_{31}(1)$ and $C_1\left(\frac{1}{[x]+[y]+[z]+1}\right)$

$\begin{vmatrix}1 & [y] & [z]\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}\left([x]+[y]+[z]+1\right) \because R_{12}(-1), R_{13}(-1)$

$=\left([x]+[y]+[z]+1\right)$

$\therefore$ Maximum value of D= 1+0+2+1=4

66. $A^T A^{-1}=\begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}\left(\frac{1}{1+\tan^2 x}\right)\begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}$

$$=\frac{1}{1+\tan^2 x}\begin{bmatrix}1-\tan^2 x & -2\tan x\\ 2\tan x & 1-\tan^2 x\end{bmatrix}=\begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix}$$

67. (i) $f(x)f(y)=\begin{bmatrix}\cos x & 0 & \sin x\\ 0 & 1 & 0\\ -\sin x & 0 & \cos x\end{bmatrix}\begin{bmatrix}\cos y & 0 & \sin y\\ 0 & 1 & 0\\ -\sin y & 0 & \cos y\end{bmatrix}$

$$=\begin{bmatrix}\cos(x+y) & 0 & \sin(x+y)\\ 0 & 1 & 0\\ -\sin(x+y) & 0 & \cos(x+y)\end{bmatrix}=f(x+y)$$

(ii) $|f(x)|=1\neq 0, \operatorname{adj}(f(x))=\begin{bmatrix}\cos x & 0 & -\sin x\\ 0 & 1 & 0\\ \sin x & 0 & \cos x\end{bmatrix}=f(-x)$

$\therefore [f(x)]^{-1}=\frac{1}{|f(x)|}\operatorname{adj}(f(x))=\frac{1}{1}f(-x)=f(-x)$

(iii) $\left[f(x)g(y)\right]^{-1}=\left[g(y)\right]^{-1}\left[f(x)\right]^{-1}=g(-y)f(-x)$ $\quad\because$ (ii)

68. $D_1=x^3-3a^2x+2a^3,\ D_2=x^2-a^2$

$$\frac{d}{dx}(D_1)=3x^2-3a^2=3\left(x^2-a^2\right)=3D_2$$

69. $D=\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}\begin{vmatrix}1&1&1\\1&\alpha&\alpha^2\\1&\beta&\beta^2\end{vmatrix}$ $\quad\because$ D is a product of two deferminant

$=\{(1-\alpha)(\alpha-\beta)(\beta-1)\}\{(1-\alpha)(\alpha-\beta)(\beta-1)\}$

$=(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

since $x^2+bx+c=(x-\alpha)(x-\beta)$

$\therefore 1+b+c=(1-\alpha)(1-\beta)$

& $\alpha+\beta=-b,\alpha\beta=c$

$\therefore(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=(-b)^2-4c=b^2-4c$

$\therefore D=(1+b+c)^2\left(b^2-4c\right)$

70. Linear equations

$a\log x+b\log y=m$

$c\log x+d\log y=n$

$\therefore$ By cramer's rule : $\quad \log x=\dfrac{D_1}{D},\log y\dfrac{D_2}{D}$

$$\therefore x=e^{\frac{D_1}{D}},y=e^{\frac{D_2}{D}}$$

71. $D=\sqrt{6}\begin{vmatrix}1&2i&3+\sqrt{6}\\\sqrt{2}&\sqrt{3}+2\sqrt{2}i&3\sqrt{2}+\sqrt{6}i\\\sqrt{3}&\sqrt{2}+2\sqrt{3}i&3\sqrt{3}+2i\end{vmatrix}$ $\quad\because C_1\left(\dfrac{1}{\sqrt{6}}\right)$

$=\sqrt{6}\begin{vmatrix}1&0&\sqrt{6}\\\sqrt{2}&\sqrt{3}&\sqrt{6}i\\\sqrt{3}&\sqrt{2}&2i\end{vmatrix}$ $\quad\because C_{12}(-2i),C_{13}(-3)$

$$=\sqrt{6}\begin{vmatrix} 1 & 0 & \sqrt{6} \\ \sqrt{2} & \sqrt{3} & 0 \\ \sqrt{3} & \sqrt{2} & 0 \end{vmatrix} \quad \because C_{23}\left(-\sqrt{2}i\right)=0-0+\sqrt{6}.\sqrt{6}(2-3)=-6 \text{ a real number.}$$

72. Match the following colums.

Column I		Column II	
1.	A is a square matrix such that $A^2 = A$	D.	A is a an Idempotent matrix
2.	A is a square matrix such that $A^m = O$	A.	A is a nilpotent matrix
3.	A is a square matrix such that $A^2 = I$	B.	A is an Involutanry matrix
4.	A is a square matrix such that $A^T = A$	C.	A is a symmetric materix

73. $$f(x)=\begin{vmatrix} 2 & 0 & -\sin x \\ 0 & 2 & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix} \quad \because C_{31}(-2\sin x),\ C_{32}(2\cos x)$$

$$=2\left(0+\cos^2 x\right)-0-\sin x\left(0-2\sin x\right)=2$$

$$f(x)=2,\quad f^1(x)=2 \qquad \therefore I=\int_0^{\pi/2}\left(f(x)+f^1(x)\right)dx$$

$$=2[x]_0^{\pi/2}=\pi$$

74. $$\frac{f^1(x)}{x}=\begin{vmatrix} 1 & \cos x & -\sin x \\ x & \frac{-\tan x}{x} & -x^2 \\ 2x & \sin 2x & 5x \end{vmatrix}+\begin{vmatrix} x & \sin x & \cos x \\ 2x & -\sec^2 x & -3x^2 \\ 2 & \frac{\sin 2x}{x} & 5 \end{vmatrix}+\begin{vmatrix} x & \sin x & \cos x \\ x & \frac{-\tan x}{x} & -x^2 \\ 2 & 2\cos 2x & 5 \end{vmatrix}$$

$$\therefore \lim_{x\to 0}\frac{f^1(x)}{x}=\begin{vmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix}+\begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 2 & 2 & 5 \end{vmatrix}+\begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 2 & 2 & 5 \end{vmatrix}$$

$$=0+2+2=4$$

75. Consider $S_n = 1+2+6+15+\ldots+t_{n-1}+t_n$,

Where $t_n =$ element a_{11} of M_n

1st We find a_{11} of M_n

$$S_n = 1 + 2 + 6 + 15 + \ldots + t_{n-1} + t_n$$
$$S_n = \quad 1 + 2 + 6 + \ldots + t_{n-2} + t_{n-1} + t_n$$
$$- \quad - \quad - \quad - \quad - \quad - \quad - \quad -$$

$$0 = 1 + 1 + 4 + 9 + \ldots + (t_n - t_{n-1}) - t_n$$

$$\therefore t_n = 1 + \left(1 + 2^2 + 3^2 + 4^2 + \ldots + (n-1)^2\right)$$

$$= 1 + \frac{n-1}{6}\left((n-1)+1\right)\left(2(n-1)+1\right)$$

$$t_n = 1 + \frac{n(n-1)(2n-1)}{6}$$

It is clear in nxn matrix that distance of consecutive diagonal element is $(n+1)$

$\therefore$ First term $= 1 + \dfrac{n(n-1)(2n-1)}{6}$, difference $= n + 1$

$\therefore$ Sum of diagonal element of M_n $= \dfrac{n}{6}\left[2\left(1 + \dfrac{n(n-1)(2n-1)}{6}\right) + (n-1)(n+1)\right]$

$$= \frac{n}{6}\left[2n^3 + n + 3\right]$$

For n=6, Sum of diagonal element =441

76. Let x=0 both sides $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ =0+0+0+0+0+f $\quad \therefore$ f = 1

Differentiate both sides and Put x = 0.

$$\therefore \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = e \qquad \therefore e = 3$$

Put x = 1 both sides

$$\therefore \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = a + b + c + d + e + f$$

Put x = - 1 both sides

$$\therefore \begin{vmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} = -a + b - c + d - e + f$$

$\therefore$ 4 = a + b + c + d + 3 + 1

$\therefore$ a + b + c + d = 0 ...... (1)

$\therefore$ 200 = -a + b - c + d - 3 + 1

$\therefore$ - a + b - c + d = 2 .... (ii)

by (i) & (ii)

a + c = - 1, b + d = 1

77. Put x = 0, constant term = f (0) = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ = 0

Differentiate and Put x = 0,

$$\therefore \text{ coefficient of } x = f'(0) = \begin{vmatrix} 3m & 5n & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$+\begin{vmatrix} 1 & 1 & 1 \\ 0 & 3m & 5n \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 5n & 0 & 3m \end{vmatrix}$$

$\therefore$ f' (0) = 0.

$\therefore$ Required sum = f (0) + f ' (0) = 0 + 0 = 0

78. $D = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$ $\quad \because R_{31}(-1), R_{32}(-1)$

$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \sin^2 x & \cos^2 x & 2+\sin 2x \end{vmatrix}$ $\quad \because C_{13}(1),\ C_{23}(1)$

$= 2 + \sin 2x$

Now $-1 \le \sin 2x \le 1$

$1 \le 2 + \sin 2x \le 3$ $\quad \therefore$ M=3, m=1

1. $M^2 + m^{2013} = 10$,

2. $M^3 - m^3 = 26$

3. $M^{2k} - m^{2k} = \text{odd} - 1 = \text{even}$ always

4. 2 M - 3 m = 3, M + m = 4 , M + 2 m = 5 are become three sides of triangle.

79. Since

$2 < e < 3 \Rightarrow [e] = 2$

$3 < \pi < 4 \Rightarrow [\pi] = 3$

$3<\pi^2-6<4 \Rightarrow [\pi^2-6]=3$

$\therefore \det\begin{vmatrix} 2 & 3 & 3 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix} = -8$

80. Constant term $= f(0) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1$

81. $D = \begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ -\sin\theta & -\cos\theta & -\sin 2\theta \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}$ $\quad \because R_{32}(1).$

$= 0 \quad \because R_1 = R_2$

since $\sin\left(\theta+\frac{2\pi}{3}\right)+\sin\left(\theta-\frac{2\pi}{3}\right)=2\sin\theta\cos\frac{2\pi}{3}=2\sin\theta\left(\frac{-1}{2}\right)=-\sin\theta$

similarly $\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta-\frac{2\pi}{3}\right)=-\cos\theta$ and

$\sin\left(2\theta+\frac{4\pi}{3}\right)+\sin\left(2\theta-\frac{4\pi}{3}\right)=-\sin 2\theta$

82. $A^2 = B$

$\therefore \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\therefore \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\therefore \alpha^2 = 1, \alpha+1=5$

There is no α given in option satisfies the obtain equation.

$\therefore$ no $= \alpha$

83. $A^2 = 9I \Rightarrow \begin{bmatrix} \alpha & 0 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 2 & 3 \end{bmatrix} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ 2\alpha+6 & 9 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$\Rightarrow \alpha^2 = 9, 2\alpha + 6 = 0$

$\Rightarrow \alpha = -3$ satisfies above both aquations.

84. $A^2 = \begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$\therefore$ $I+2A+3A^2+\ldots\infty = I+2A+0+\ldots\infty$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ -18 & -6 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ -18 & -5 \end{bmatrix}$$

85. $\det(M\text{-}I) = \det(M-I)^T$

$= \det(M^T - I)$

$= \det(M^T - M^T M)$

$= \det(M^T(I-M)$

$= \det M^T \cdot \det(I-M)$

$= \det M \cdot \det(-(M-I))$

$= (-1)^3 \det(M-I)$

det (M-I) = - det (M-I)

$\therefore$ det (M-I) = 0

86. $\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$ on expanding along C_1

$\therefore \lambda = \cos 2\alpha + \sin 2\alpha$

Compare with $f(\alpha) = a\cos\alpha + b\sin\alpha$

whose range is $[-\sqrt{a^2+b^2}, \sqrt{a^2+b}\ ]$

(i) $\therefore$ Range of λ is $[-\sqrt{2}, \sqrt{2}]$ ($\because$ Here a = 1, b = 1)

(ii) For $\lambda = 1$

$\cos 2\alpha + \sin 2\alpha = 1$

Dividing both sides by $\sqrt{2}$

$$\therefore \frac{1}{\sqrt{2}}\cos 2\alpha + \frac{1}{\sqrt{2}}\sin 2\alpha = \frac{1}{\sqrt{2}}$$

$$\therefore \cos\left(2\alpha - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$\therefore 2\alpha - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

$\Rightarrow \alpha = n\pi, n\pi + \frac{\pi}{4}$

87. $|A - \lambda I| = 0$

$\therefore \begin{vmatrix} 1-\lambda & O \\ -1 & 7-\lambda \end{vmatrix} = 0 \quad \Rightarrow \lambda^2 - 8\lambda + 7 = O$

$\Rightarrow A^2 - 8A + 7I = O$

$\Rightarrow A^2 = 8A - 7I$

$\Rightarrow k = -7$

88. Let $\begin{bmatrix} k & k & k \\ k & k & k \\ k & k & k \end{bmatrix}$ be the identity element, then

$$\therefore \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}\begin{bmatrix} k & k & k \\ k & k & k \\ k & k & k \end{bmatrix} = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$$

$$\therefore \begin{bmatrix} 3kx & 3kx & 3kx \\ 3kx & 3kx & 3kx \\ 3kx & 3kx & 3kx \end{bmatrix} = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$$

$\therefore 3kx = x$

$\therefore (3k-1)x = 0$

$k = \frac{1}{3}$ ($\because x \mp 0$)

$\therefore$ Required identity elenent $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

89. $AB = I \Rightarrow \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}\begin{bmatrix} y & y & y \\ y & y & y \\ y & y & y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3xy & 3xy & 3xy \\ 3xy & 3xy & 3xy \\ 3xy & 3xy & 3xy \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$\Rightarrow 3xy = \frac{1}{3}$$

$$\Rightarrow y = \frac{1}{9x} \qquad \text{or} \qquad x = \frac{1}{9y}$$

The required inverse of $\begin{bmatrix} y & y & y \\ y & y & y \\ y & y & y \end{bmatrix}$ is $\begin{bmatrix} \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \\ \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \\ \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \end{bmatrix}$

90. $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix}$

$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}, \qquad |I - A| = 4 \neq 0$

$\therefore (I - A)^{-1} = \frac{1}{|I - A|} adj(I - A) = \frac{1}{4} \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix}$

$\therefore \phi(A) = (I + A)(I - A)^{-1}$

$= \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix} \frac{1}{4} \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 0 & 4 \\ -4 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$

91. Construct an orthogonal matrix using the skew-symmetric matrix $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, |I - A| = 5 \neq 0$

$(I - A)^{-1} = \frac{1}{|I - A|} adj(I - A) = \frac{1}{5} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

$\therefore$ Orthogonal matrix $\phi(A) = (I + A)(I - A)^{-1}$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \frac{1}{5} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix}$$

92. $I + A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & -5 \\ -1 & 5 & 1 \end{bmatrix}, I - A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 5 \\ 1 & -5 & 1 \end{bmatrix}, |I - A| = 31 \neq 0$

$$(I - A)^{-1} = \frac{1}{|I - A|}\left(\text{adj}(I - A)\right) = \frac{1}{31} \begin{bmatrix} 26 & 3 & 11 \\ 7 & 2 & -3 \\ 9 & 7 & 5 \end{bmatrix}$$

Required Orthogonal matrix= $\phi(A) = (I + A)(I - A)^{-1}$

$$= \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & -5 \\ -1 & 5 & 1 \end{bmatrix} \frac{1}{31} \begin{bmatrix} 26 & 3 & 11 \\ 7 & 2 & -3 \\ 9 & 7 & 5 \end{bmatrix} = \frac{1}{31} \begin{bmatrix} 21 & 6 & 22 \\ 14 & -27 & -6 \\ 18 & 14 & -21 \end{bmatrix}$$

93. The Charateristic eqution is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0 \quad \Rightarrow \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$$

$$\Rightarrow A^3 - 7A^2 + 11A - 5I = 0$$

$$\Rightarrow A^3 - 7A^2 + 10A = 5I - A$$

94. The Characteristic equation of A is $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0 \quad \Rightarrow \lambda^2 - 5\lambda - 2 = 0$$

$$\Rightarrow A^2 - 5A - 2I = O$$

$$\Rightarrow I - 5A^{-1} - 2A^{-2} = O$$

$$\Rightarrow A^{-2} = \frac{1}{2}\left[I - 5A^{-1}\right]$$

$$\Rightarrow A^{-4} = \frac{1}{4}\left(I - 5A^{-1}\right)^2$$

$$\Rightarrow A^{-4} = \frac{1}{4}\left(I - 10A^{-1} + 25A^{-2}\right)$$

$$\Rightarrow A^{-4} = \frac{1}{4}\left[I - 10A^{-1} + \frac{25}{2}\left(I - 5A^{-1}\right)\right]$$

$$\Rightarrow 8A^{-4} = 27I - 145A^{-1}$$

95. $(1-\lambda)x_1 + 2x_2 + 3x_3 = 0$

$3x_1 + (1-\lambda)x_2 + 2x_3 = 0$

$2x_1 + 3x_2 + (1-\lambda)x_3 = 0$

$$\therefore \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\left((1-\lambda)^2 - 6\right) - 2\left(3 - 3\lambda - 4\right) + 3(9 - 2 + 2\lambda) = 0$$

$$\Rightarrow (1-\lambda)(\lambda^2 - 2\lambda - 5) - 2(-3\lambda - 1) + 3(7 + 2\lambda) = 0$$

$$\Rightarrow \lambda^2 - 2\lambda - 5 - \lambda^3 + 2\lambda^2 + 5\lambda + 6\lambda + 2 + 21 + 6\lambda = 0$$

$$\Rightarrow -\lambda^3 + 3\lambda^2 + 15\lambda + 18 = 0$$

$\Rightarrow \lambda = 6$ Satisfies the equation.

96. determinant of Coefficient matrix is

$$D = \begin{vmatrix} \sec^2\alpha & -\tan^2\alpha & 1 \\ \cos^2\alpha & \sin^2\alpha & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$= \sec^2\alpha \sin^2\alpha + \tan^2\alpha \cdot \cos^2\alpha - \sin^2\alpha$

$= \tan^2\alpha \neq 0$ unique solution exists

$D_x = D_y = D_z = \tan^2\alpha$

$$\therefore x = \frac{D_x}{D} = 1, y = \frac{D_y}{D} = 1, Z = \frac{D_z}{D} = 1$$

$\therefore (x, y, z) = (1, 1, 1)$

97. The determinant of the coefficient matrix is

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & k+7 & -3 \\ 3 & 4 & k-3 \end{vmatrix} = 0 \Rightarrow k^2 - 1 = 0 \qquad (\because \text{By expanding along } C_1)$$

$$\Rightarrow k = \pm 1$$

98. clearly (d).

99. $\begin{vmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -5 & -2-\lambda \end{vmatrix} = 0$

$\Rightarrow (\lambda-1)(\lambda-2)^2 = 0 \Rightarrow \lambda = 1, 2$

For $\lambda = 1 \therefore 3x + 6y + 6z = 0$

$x + 2y + 2z = 0$

$x - 5y - 3z = 0$

By cross multiplication $\frac{x}{4} = \frac{y}{1} = \frac{z}{-3} \quad \therefore x = \begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix}$......(1)

For $\lambda = 2 \therefore 2x + 6y + 6z = 0$

$x + y + 2z = 0$

$-x - 5y - 4z = 0$

$\therefore \frac{x}{6} = \frac{y}{2} = \frac{z}{-4} \therefore x = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$........(2)

By (i) and (ii), $\cos\theta = \frac{(4,1,-3)}{\sqrt{16+1+9}} \cdot \frac{(3,1,-2)}{\sqrt{9+1+4}} = \frac{19}{\sqrt{364}}$

$\therefore \tan\theta = \sqrt{\sec^2\theta - 1} = \frac{\sqrt{3}}{19}$

100. $\begin{vmatrix} A & 3 & 6 \\ 100A+8+20 & 300+9+10B & 600+C+20 \\ 2 & B & 2 \end{vmatrix} = \begin{vmatrix} A & 3 & 6 \\ A28 & 3B9 & 62C \\ 2 & B & 2 \end{vmatrix} = km$

is divisible by k since the 2nd row is divisible by k.

101. $I + A + A^2 + \ldots\infty = (I - A)^{-1} = \begin{bmatrix} 3 & -3 \\ 1 & 0 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 0 & 3 \\ -1 & 3 \end{bmatrix}$

102. $\begin{vmatrix} -2-\lambda & 3 \\ -1 & 1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 + \lambda + 1 = 0$

$\Rightarrow A^2 + A + I = O$

$\Rightarrow A^2 = - A - I$

$\Rightarrow A^3 = -A^2 - A$

$$= -(-A - I) - A$$

$$\Rightarrow A^3 = I$$

103. By the pascal rule $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$

$$\therefore \binom{n+1}{r+1} - \binom{n}{r} = \binom{n}{r+1}$$

$$\therefore \text{LHS} = \begin{vmatrix} \binom{x}{r} & \binom{x}{r+1} & \binom{x+1}{r+2} \\ \binom{y}{r} & \binom{y}{r+1} & \binom{y+1}{r+2} \\ \binom{z}{r} & \binom{z}{r+1} & \binom{z+1}{r+2} \end{vmatrix} \quad \because C_{23}(-1) \text{ and } C_{12}(-1)$$

$$\begin{vmatrix} \binom{x}{r} & \binom{x}{r+1} & \binom{x}{r+2} \\ \binom{y}{r} & \binom{y}{r+1} & \binom{y}{r+2} \\ \binom{z}{r} & \binom{z}{r+1} & \binom{z}{r+2} \end{vmatrix} \quad \because C_{23}(-1)$$

$$\therefore \lambda = 1$$

104. Say $x + 2y + 3z = 6$ .... (1)

$x + 3y + 5z = 9$ .... (2)

$2x + 5y + \lambda z = \mu$........(3)

By (3) - 2(1) ,(3) - 2(2) we get

$y + (\lambda - 6)z = \mu - 12$.......(4)

$-y + (\lambda - 10)z = \mu - 18$......(5)

By (4) +(5) we get $(\lambda - 8)z = \mu - 15$

$\lambda \neq 8, \mu \in R$ unique solution

$\lambda = 8, \mu \neq 15$ no solution

$\lambda = 8, \mu = 15$ Infinity of solutions.

105 $AA^T = I$ since A is orthogonal

$$\therefore \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix} \begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 4\beta^2+\gamma^2 & 2\beta^2-\gamma^2 & -2\beta^2+\gamma^2 \\ 2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 \\ -2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore 4\beta^2 + \gamma^2 = 1$......(i)

$2\beta^2 - \gamma^2 = 0$......(ii)

$\alpha^2 + \beta^2 + \gamma^2 = 1$......(iii)

$\alpha^2 - \beta^2 - \gamma^2 = 0$......(iv)

By solving (i) ,(ii) ,(iii) ,(iv) we get

$\alpha = \pm\frac{1}{\sqrt{2}}, \beta = \pm\frac{1}{\sqrt{6}}, \gamma = \pm\frac{1}{\sqrt{3}}$

ANSWERS

1	(b)	22	(c)	43	(c)	64	(d)	83	(c)
2	(a)	23	(b)	44	(a)	65	(c)	84	(c)
3	(c)	24	(a)	45	(b)	66	(b)	85	(a)
4	(d)	25	(b)	46	(a)	67 (i)	(d)	86 (i)	(b)
5	(c)	26	(d)	47	(b)	67 (ii)	(c)	86 (ii)	(c)
6	(d)	27	(a)	48	(d)	67 (iii)	(c)	87	(d)
7	(a)	28	(d)	49	(c)	68	(d)	88	(a)
8	(c)	29	(a)	50	(d)	69	(a)	89	(d)
9	(d)	30	(c)	51	(d)	70	(c)	90	(b)
10	(b)	31	(c)	52	(a)	71	(a)	91	(a)
11	(b)	32	(d)	53	(d)	72	(b)	92	(b)
12	(b)	33	(c)	54	(c)	73	(b)	93	(a)
13	(a)	34	(a)	55	(b)	74	(d)	94	(b)
14	(a)	35	(a)	56	(a)	75	(b)	95	(d)
15	(a)	36	(b)	57	(c)	76	(c)	96	(a)
16	(a)	37	(b)	58	(b)	77	(d)	97	(a)
17	(b)	38	(c)	59	(a)	78	(c)	98	(d)
18	(c)	39	(a)	60	(d)	79	(d)	99	(b)
19	(d)	40	(c)	61	(d)	80	(b)	100	(c)
20	(c)	41	(d)	62	(c)	81	(a)	101	(c)
21	(a)	42	(b)	63	(b)	82	(d)	102	(a)
								103	(b)
								104	(b)
								105	(c)

●●●

Unit - 4

Permutation and Combination

Important Points

* **Fundamental Principle of counting :-**

If an event can occur in m ways and corresponding to each way another event can occur in p ways and corresponding to them, a third event can occur in r ways, then the total number of occurances of the events is mpr.

* **Factorial :-** The Product of first n natural numbres is known as Factorial. It is denoted by n! or

→ $n! = n.(n-1).(n-2)...3.2.1$

→ $n! = n.(n-1)! = n(n-1)(n-2)!$

→ $0! = 1$

* **Permutations (Arrangements) :-**

- A Permutation is an arrangement in a definite order of a number of distinct objects taking some or all at a time.

→ The number of linear permutations of n different objects taking r at a time where $1 \le r \le n$ r, $n \in N$, is denoted by ${}_nP_r$.

→ If repetitions of objects is not allowed and arrangement is linear, the arrangements also called a linear Permutation.

→ ${}_nP_r = n(n-1)(n-2)....(n-r+1)$

→ ${}_nP_r = \dfrac{n!}{(n-r)!}$ → ${}_nP_r = n \; {}_{(n-1)}P_{(r-1)}$

→ ${}_nP_n = n!$ → ${}_nP_1 = n$ → ${}_nP_2 = n(n-1)$

- Number of permutations of n distinct objects taken r at a time with repetitions allowed is n^r.

* **Permutations when some of the objects are identical (alike or one kind) :-**

If P_1 objects are alike, P_2 objects are alike different from earlier ones...., P_K objects are alike different from earlier ones and $n = P_1 + P_2 + ...P_K$ then the number of are permutations of n things is

$$\frac{n!}{P_1!P_2!....P_K!}$$

* **Circular Permutation :-**

The number of ways of arranging n different objects on a circle is called the number of circular permutations of n objects.

- The number of circular permutations of n different things is $(n-1)!$

→ In circular permutation, anti-clockwise and clockwise order of arrangements are considered as distinct permutations .

→ If anti-clockwise and clockwise order of arrangements are not distinct then the number of circular permutations of n distinct items are $\frac{(n-1)!}{2}$

- Ex. 1 : The number of permutations of 5 persons seated around the round table is (5-1) ! = 4! . Because with respect to the table, the clockwise and anti-clockwise arrangements are distinct.
- Ex. 2 : Arrangements of beads, necklace, arrangements of flowers in a garland etc., then the number of circular permutations of n distinct items is $\frac{(n-1)!}{2}$

→ The number of all permutations of n different objects taken r at a time, when a particular object is to be always included in each arrangement is $r.\,{}_{n-1}P_{r-1}$

→ The number of all permutations of n different objects taken r at a time, when a particular object is never taken in each arrangement is ${}_{n-1}P_r$

→ Number of all permutations of n different objects taken r at a time in which two specified objects always occur together is $2!(r-1).\,{}_{n-2}P_{r-2}$

→ Sum of the number formed by n non zero digits = (sum of the digits) $(n-1)!\left(\frac{10^n-1}{10-1}\right)$

→ The highest power of a prime p occurring in n! is $\left[\frac{n}{p}\right]+\left[\frac{n}{P^2}\right]+\text{..........}+\left[\frac{n}{P^r}\right]$

Where r is the largest positive integer such that $P^r \le n \le P^{r+1}$

- **Combination (selection) :-** The number of ways of selecting r things out of n different things is called r combination number of n things and is denoted by $\binom{n}{r}$ or ${}_nC_r$ or ${}^nC_r$ or C (n, r)

→ $\binom{n}{r}=\frac{{}_nP_r}{r!}=\frac{n!}{(n-r)!r!}. \quad 0<r\le n$

→ $\binom{n}{0}=1 \rightarrow \binom{n}{n}=1 \rightarrow \binom{n}{r}=\binom{n}{n-r}, \quad 0\le r\le n$

→ $\binom{n}{r}+\binom{n}{r-1}=\binom{n+1}{r}$

→ $\binom{n}{x} = \binom{n}{y} \Rightarrow$ (i) x = y or (ii) x + y = n

$\binom{n}{r} = \frac{n}{r}\binom{n-1}{r-1}$

→ $n\binom{n-1}{r-1} = (n-r+1)\binom{n}{r-1}, 1 \le r \le n$

→ If n is even, then the greatest value of $\binom{n}{r}$ is $\binom{n}{n/2}$, $0 \le r \le n$

→ If n is odd, then the greatest value of $\binom{n}{r}$ is $\binom{n}{\frac{n+1}{2}}$ or $\binom{n}{\frac{n-1}{2}}$, $0 \le r \le n$

→ The product of n consecutive integers is divisible by n!

→ The number of ways of selecting one or more items from a group of n distinct items is $2^n - 1$

→ The number of ways of selecting none, one or more items from a group of n distinct items is 2^n

→ The number of ways of selecting r items out of n identical items is 1.

→ The number of ways of selecting one or more (at least one) items out of n identical items is n.

→ The number of ways of selecting none, one or more items out of n identical items is n+1.

(Here 1 is added for the case in no item is selected from the set of n identical items.)

→ The number of ways of selecting none, m items of one kind, n items of another kind and p items of another kind out of m+n+p items is (m+1) (n+1) (p+1).

→ The number of ways of selecting at least one (one or more) items from a collection of m items of one kind, n items of another kind and p items of another kind is (m+1) (n+1) (p+1) -1.

(Here -1 is used for rejecting that one case in which no item is selected.)

→ The number of ways of selecting at least one (one or more) item of each kind from a collection of m items of one kind, n items of another kind and p items of another kind is mnp.

→ The total number of ways of selecting one or more items from p identical items of one kind, q identical items of another kind, r identical items of another kind and n different items is

$$= (p+1)(q+1)(r+1)2^n - 1$$

→ The number of ways in which a selection of at least one item can be made from a collection of n distinct items and m identical items is $2^n(m+1) - 1$

(here -1 is used for rejecting that one case in which no item is selected)

→ Number of ways in which m+n+p items can be divided into unequal groups containing m, n and p items is ${}^{m+n+1}C_m \cdot {}^{n+1}C_n \cdot {}^{p}C_p = \dfrac{(m+n+p)!}{m!n!p!}$

→ Number of ways to distribute m+n+p items among 3 persons in the group containing m, n and p items is $\dfrac{(m+n+p)!}{m!n!p!} \times 3!$

(Here 3! Is for arranging the things in between 3 persons as no two persons are alike)

→ The number of ways in which mn different items can be divided equally into m groups, each containing n objects and the order of the groups is not important, is $= \dfrac{(m\,n)!}{(n!)^m} \times \dfrac{1}{m!}$

→ The number of ways in which mn different items can be distributed equally among m different persons is $= \dfrac{(m\,n)!}{(n!)^m}$

→ The number of diagonals of n sided convex polygon is ${}^nC_2 - n = \dfrac{n(n-3)}{2}$, $n > 3$

→ There are n points in the plane such that no three of them are in the same straight line, then the number of lines that can be formed by joining them is ${}^nC_2$

→ There are n points in the plane such that no three of them are in the same straight line, then the number of triangles that can be formed by joining them is ${}^nC_3$

→ There are n points in the plane such that no three of them are in the same straight line except m of them which are in same straight line then the number of lines that can be formed by joining them is ${}^nC_2 - {}^mC_2 + 1$

→ There are n points in the plane such that no three of them are in the same straight line except m of them which are in same straight line, then the number of triangles that can be formed by joining them is ${}^nC_3 - {}^mC_3$

→ If n points lie on a circle then the number of straight lines formed by joining them is ${}^nC_2$

→ If n points lie on a circle then the number of triangles formed by joining them is ${}^nC_3$

→ If n points lie on a circle then the number of quadrilateral formed by joining them is ${}^nC_4$

QUESTION BANK

1. The least positive integer n for which ${}_{n-1}C_5 + {}_{n-1}C_6 < {}_nC_7$ is _______

   (a) 14 (b) 15 (c) 16 (d) 28

2 . n books are arranged on a shelf so that two particular books are not next to each other, There were 480 arrangements altogether. Then the number of books on the shelf is ________

   (a) 5 (b) 6 (c) 10 (d) 8

3. The sum of all possible numbers greater than 10000 formed by using the digits from { 1,3,5,7,9} is

   (a) 666600 (b) 666660 (c) 66666600 (d) none of these

4. How many words can be formed by taking four different letters of the word MATHEMATICS ?

   (a) 756 (b) 1680 (c) 2454 (d) 18

5. If $a_n = \sum_{r=o}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=o}^{n} \frac{r}{{}^nC_r}$ equals _______

   (a) (n-1) a_n (b) n a_n

   (c) $\frac{1}{2}$ n a_n (d) None of these

6. If ${}^nC_r = {}^nC_{r-1}$ and ${}^nP_r = {}^nP_{r+1}$, then the value of n is

   (a) 3 (b) 4 (c) 2 (d) 5

7. How many different nine digit numbers can be formed from the number 22,33,55,888 by rearranging its digits so that the odd digits occupy even positions ?

   (a) 16 (b) 4 (c) 60 (d) 5

8. The number of arrangements that can be made out of the letter of the word " SUCCESS " so that the all S's do not come together is

   (a) 60 (b) 120 (c) 360 (d) 420

9. If the coefficient of the 5th, 6th and 7th terms of the expansion of $(1+x)^n$ are in A.P then the value of n may be _______

   (a) 5 (b) 6 (c) 7 (d) 8

10. The number of numbers greater than 3000, which can be formed by using the digits 0,1,2,3,4,5 without repetition is _______

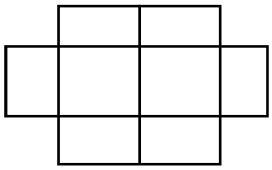
   (a) 1240 (b) 1280 (c) 1320 (d) 1380

11. In a certain test a_i students gave wrong answers to atleast i questions, where i= 1,2,3......k No student gave more than k wrong answers. The total number of wrong answers is _______

   (a) $a_1+a_2+... + a_k$ (b) $a_1+2a_2+... +ka_k$

   (c) $a_1+a_2+... + a_k -k$ (d) $\frac{k}{2}(k+1)$

12. Six x's are to be placed in the squre of the figure given below such that each row contain atleast one X the number of ways this can be done is ________



(a) 18
(b) 22
(c) 26
(d) 30

13. A five digit number divisible by 3 is to be formed using the digit 0,1,2,3,4,5 without repetition, The number of ways this can be done is ________

(a) 216 (b) 184 (c) 256 (d) 225

14. The maximum no. of points into which 4 circles and 4 straight lines intersect is ________

(a) 26 (b) 56 (c) 50 (d) 72

15. The sides AB,BC,CA of a triangle ABC have 3,4 and 5 interior points respectively on them the total no. of triangle that can be constructed by using these points as vertices is ________

(a) 220 (b) 204 (c) 205 (d) 195

16. Seven different teachers are to deliver lectures in seven periods of a class on a particular day. A,B and C are three of the teachers. The no.of ways in which a routine for the day can be made such that A delivers his lacture before B and B before C is ________

(a) 420 (b) 120 (c) 210 (d) none of these

17. If ${}^{189}C_{35} + {}^{189}C_x = {}^{190}C_x$ then x is equal to ________

(a) 34 (b) 35 (c) 36 (d) 37

18. If the different permutations of all the letters of the word EXAMINATION are listed in a dictionary then how many words are there in this list before the first word begins with E ?

(a) 907,200 (b) 970200 (c) 922700 (d) 709002

19. The number of ways in which in a necklace can be formed by using 5 identicle red beads and 6 identicle black beads is

(a) $\frac{11!}{6!\,4!}$ (b) ${}^{11}P_6$ (c) $\frac{10!}{2(6!5!)}$ (d) None of these

20. In an examination a candidate has to pass in each of the four subjects.In how many ways can he fail?

(a) 15 (b) 20 (c) 25 (d)10

21. Ten different letters of english alphabet are given. Out of these letters, words of 5 letters are formed. How many words are formed when at least one letter is repeated ?

(a) 69760 (b) 98748 (c) 96747 (d) 97147

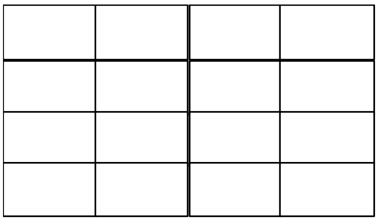
22. If p+q = 1 then $\sum_{r=0}^{n} r \cdot {}^{n}C_r\, p^r q^{n-r}$ is equal to

(a) 1 (b) np (c) npq (d) 0

23. The no. of 10 letter codes that can be formed using the characters x,y,z,r with the restriction that x appars exactly thrice and y appars exactly twice in each such codes is

(a) 60840 (b) 88400 (c) 80640 (d) 64080

24. A rectangle with sides 2m-1 and 2n-1 is divided into squres of unit length by drawing Parallel lines as shown in the diagram then the no.of rectangles possible with odd side length is _______



(a) mn(m+1)(n+1) (b) m^2n^2 (c) $(m+n+1)^2$ (d) 4^{m+n-1}

25. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is _______

(a) 55 (b) 66 (c) 77 (d) 88

26. The value of $^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to _______

(a) $^{47}C_5$ (b) $^{52}C_5$ (c) $^{52}C_4$ (d) $^{53}C_7$

27. If N is the number of positive integral solution of $x_1 x_2 x_3 x_4 = 770$, then the value of N is _______

(a)250 (b) 252 (c) 254 (d) 256

28. If a denotes the no. of permutation of x + 2 things taken all at a time, b the no. of permutation of x things taken 11 at a time and c the no. of permutation of x-11 things taken all at a time such that a=182bc then the value of x is _______

(a) 15 (b) 12 (c) 10 (d) 18

29. In how many ways can 15 members of a school sit along a circular table, when the secretary is to sit on side of the principal and the deputy secretary on the other side ?

(a) $2 \times 12!$ (b) 24 (c) $2 \times 15!$ (d) $2! \times 13!$

30. The total no. of permutations of n(n>1) different things taken not more than r at a time, when each things may be repeated any no. of times is _______

(a) $\frac{n(n^n - 1)}{n-1}$ (b) $\frac{n^r - 1}{n-1}$ (c) $\frac{n(r^n - 1)}{n-1}$ (d) $\frac{n(n-r)}{n-1}$

31. A car will hold 2 in the front seat and 1 in the rear seat.If among 6 persons 2 can drive, than no. of ways in which the car can be filled is _______

(a) 10 (b) 20 (c) 30 (d) 40

32. ABCD is a convex quadrilateral. 3,4,5 and 6 points are marked on the sides AB,BC,CD and DA resp. The no.of triangles with vertices on different sides are _______

(a) 270 (b) 220 (c) 282 (d) 342

33. In chess championship 153 games have been played. If a player with every other player plays only once, then the no. of players are _______

(a) 17 (b) 51 (c) 18 (d) 35

34. Number of points having position vector $a\hat{i}+b\hat{i}+c\hat{k}$, a,b,c $\in$ {1,2,3,4,5}such that $2^a+3^b+5^c$ is divisible by 4 is _______

(a) 140 (b) 70 (c) 100 (d) 75

35. In a certain test there are n questions. In this test 2^k students gave wrong answers to at least n-k question. k=0,1,2... n.If the no.of wrong answers is 4095 then value of n is _______

(a) 11 (b) 12 (c) 13 (d) 15

36. Let E={1,2,3,4}, F = {a,b} then the no. of onto function from E to F is _______

(a) 14 (b) 16 (c) 12 (d) 32

37. A class contain 4 boys and g girls every sunday 5 students including at least 3 boys go for a picnic to doll house, a different group being sent every week. during the picnic the class techer gives each girl in the group a doll . If the total no. of dolls distributed was 85, then value of g is _______

(a) 15 (b) 12 (c) 8 (d) 5

38. The no. of ways in which we can get a sum of the score of 11 by tossing three dices is _______

(a) 18 (b) 27 (c) 45 (d) 56

39. There are 3 set of parallel lines containing p lines, q lines and r lines resp. The greatest no. of parallelograms that can be formed by the system _______

(a) pqr + (p-1) (q-1) (r-1)

(b) $\frac{1}{4}$ { pqr + (p-1) (q-1) (r-1)}

(c) $\frac{1}{4}$ pqr (p+1) (q+1) (r+1)

(d) None of these

40. If a polygon has 90 diagonals, the no. of its sides is given by _______

(a) 12 (b) 11 (c) 10 (d) 15

41. If N is the no. of ways of dividing 2^n people into n couples then

(a) $2^n N=(2n)!$

(b) N(n!)=(1.3.5...(2n-1))

(c) $N={}^{2n}C_n$

(d) none of these

42. If 500! $=2^m\times$ an integer, then _______

(a) m= 494

(b) m= 496

(c) It is equivalent to no. of n is 400! is $=2^n\times$ an integer

(d) $m={}^{500}C_2$

43. If $C_r={}^{2n+1}C_r$ then, $C_0^2-C_1^2+C_2^2.........+(-1)^{2n+1}C_{2n+1}^2$ is equal to _______

(a) $({}^{2n+1}C_n-{}^{2n+1}C_{n+1})^2$

(b) ${}^{2n}C_n$

(c) $\frac{1}{n}({}^{2n}C_n)$

(d) 0

44. The number of zeros at the end of 100! is _______

(a) 20 (b) 22 (c) 24 (d)26

45. The no. of five digit number that can be formed by using 1, 2, 3 only ,such that exactly three digit of the formed numbers are same is _______

(a) 30 (b) 60 (c) 90 (d) 120

46. The no.of ordered pairs of integers (x,y) satisfying the equation $x^2+6x+y^2=4$ is _______

(a) 2 (b) 4 (c) 6 (d) 8

47. If ${}^nC_4$,${}^nC_5$ and ${}^nC_6$ are in A.P then the value of n can be _______

(a) 14 (b) 11 (c) 9 (d) 5

48. The straight lines l_1,l_2,l_3 are parallel and lie in the same plane.A total number of m points are taken on l_1, n points on l_2, k points on l_3. The maximum number of triangles formed with vertices at these points are _______

(a) ${}^{m+n+k}C_3$ (b) ${}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$

(c) ${}^mC_3 + {}^nC_3 + {}^kC_3$ (d) $m+n+k - {}^{m+n+k}C_3$

49. The no. of ways in which the letter of the word "ARRANGE" can be arranged such that both R do not come together is _______

(a) 360 (b) 900 (c) 1260 (d) 1620

50. A committee of 12 persons is to be formed from 9 women and 8 men in which at least 5 woman have to be included in a committee.Then the no.of committee in which the women are in majority and men are in majority are respectively

(a) 4784,1008 (b) 2702,3360

(c) 6062,2702 (d) 2702,1008

51. A is a set containing n elements.A subset P of A is chosen.The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen.The number of ways of choosing P and Q so that $P \cap Q = \varnothing$ is _______

(a) $2^{2n} - {}^{2n}C_n$ (b) 2^n (c) 2^n-1 (d) 3^n

52. 12 Persons are to be arranged to a round table, If two particular persons among them are not to be side by side, the total no,of arrangments is : _______

(a) 9(10!) (b) 2(10)! (c) 45(8!) (d) 10 !

53. Nandan gives dinner party to six guests.the no of ways in which they may be selected from ten friends if two of the friends will not attend the party together is: _______

(a) 112 (b) 140 (c) 164 (d) 146

54. The no, of straight lines that can be drawn out of 10 points of which 7 are collinear is _______

(a) 22 (b) 23 (c) 24 (d) 25

55. The no of ways of arranging the letters AAAAABBBCCCDEEF in a row when no two c's are together is: _______

(a) $\frac{15!}{5!3!3!2!} - 3!$ (b) $\frac{15!}{5!3!3!2!} - \frac{13!}{5!3!2!}$

(c) $\frac{12!}{5!3!2!} \times \frac{{}^{13}P_3}{3!}$ (d) $\frac{12!}{5!3!2!} \times {}^{13}P_3$

56. The no of ways in which 10 persons can go in two cars so that there may be 5 in each car, supposing that two particular persons will not go in the same car is:

(a) $\frac{1}{2}\left({}^{10}C_5\right)$ (b) $\frac{1}{2}\left({}^{8}C_5\right)$ (c) $2\left({}^{8}C_4\right)$ (d) ${}^{8}C_4$

57. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty are: _______
(a) 3^8 (b) 21 (c) 5 (d) $^8C_3$

58. There are 10 points in a plane out of these 6 are collinear. If N is the number of triangles formed by joining these points then:
(a) $N \le 100$ (b) $100 < N < 140$ (c) $140 < N < 190$ (d) $N > 190$

59. If $n = {}^mC_2$, then $^nC_2$ equal to._______
(a) $^{m+1}C_4$ (b) $^{m-1}C_4$ (c) $^{m+2}C_4$ (d) None of these

60. There are 3 copies each of 4 different books.The number of ways they can be arranged in a shelf is _______
(a) 369600 (b) 400400 (c) 420600 (d) 440720

61. The number of ways in which 5 identical balls can be kept in 10 identical boxes. If not more than one can go in to a box is _______
(a) $_{10}p_5$ (b) $^{10}C_5$ (c) 5 (d) 1

62. The vertices of a regular polygon of 12 sides are joined to form triangles.The no. of triangles which do not have their sides as the sides of the polygon is _______
(a) 96 (b) 108 (c) 112 (d) 220

63. 12 boys and 2 girls are to be seated in a row such that there are atleast 3 boys between the 2 girls.The no.of ways this can be done is m.12! where m =________
(a) $2.{}^{12}C_6$ (b) 20 (c) $^{11}P_2$ (d) $^{11}C_2$

64. The first 12 letters of the english alphabet are to be written at random so that there are exatly 4 letters between A and B. the number of ways this can be done is _______?
(a) 7.10! (b) 2.10! (c) 21.10! (d) 14.10!

65. How many different words canbe formed by the letters of the word MISSISSIPPI in which no two S are adjacent ?
(a) $8\ {}^6C_4\ {}^7C_4$ (b) $2 \times 7 \times {}^8C_4$ (c) $6 \times 8 \times {}^7C_4$ (d) $7\ {}^6C_4\ {}^8C_4$

66. From 6 different novels and 3 different dictionaries 4 novels and , 1 dictionary are to be selected and arranged in a row on the shelf so that the dictionary is always in the middle.Then the no, of such arrangements is _______
(a) Less then 500
(b) At least 500 but less then 750
(c) At least 750 but less then 1000
(d) At least 1000

67. At an election 3 wards of a town are convassed by 4,5 and 8 men respectively. If there are 20 volunteers then the no. of ways they can be alloted to different wards is ?
(a) $^{20}P_4.{}^{20}P_5.{}^{20}P_8$ (b) $^{20}C_4\ {}^{20}C_5\ {}^{20}C_8$
(c) $^{20}C_4\ {}^{16}C_5\ {}^{11}C_8$ (d) $\frac{1}{3!}\ {}^{20}C_4\ {}^{16}C_5\ {}^{11}C_8$

68. The no, of ways in which ten candidates $A_1, A_2 ... A_{10}$ can be ranked, if A_1 is always above A_2 is _______
(a) 2.8! (b) 9! (c) 10! (d) 5.9!

69. A round table conference is to be held among 20 delegates of 20 countries.The no, of ways they can be seated if two particular delegates are never sit together is.
(a) 17.18! (b) 18.19! (c) $\frac{20!}{2}$ (d) 19!.2

70. Ten different letters of an alphabet are given Words with 5 letters are formed from the given letters. The no, of words which have atleast one letter repeated is _______

(a) 69750 (b) 30240 (c) 9948 (d) 10680

71. 8 chairs are numbered 1 to 8 Two women and 3 men wish to occupy one chair each first the women choose the chairs marked, 1 to 4 then the men select from among the remaining chairs. The no, of seating arrangments is

(a) 1250 (b) 1360 (c) 1440 (d) 1560

72. There are 4 balls of different colours and 4 boxes of colours same as those of the balls. The no, of ways in which the balls, one in each box could be placed such that a ball does not go to a box of its own colour is _______

(a) 5 (b) 6 (c) 9 (d) 12

73. Let T_n denote the no of triangles which can be formed using the vertices of regular polygon of n sides if $T_{n+1} - T_n = 21$, then n = ______

(a) 4 (b) 5 (c) 6 (d) 7

74. The no of 9 digit numbers formed using the digit 223355888 such that odd digits occupy even places is _______

(a) 16 (b) 36 (c) 60 (d) 80

75. The range of the function $f\left(x\right) = {}^{7-x}P_{x-3}$ is _______

(a) {1,2,3} (b) {1,2,3,4}

(c) {1.2,3,4,5,} (d) {1,2,3,4,5,6}

76. Five digit numbers divisible by 3 are formed using the digits 0,1,2,3,4,5,6,7 without repetition. The no, of such numbers are _______

(a) 936 (b) 480 (c) 600 (d) 216

77. A student is to answer out of 13 questions in an examination such that he must choose atleast 4 from the first five questions. The no, of choice available to him is _______

(a) 140 (b) 196 (c) 180 (d) 346

78. The number of ways of distributing 8 identical balls in 3 distinct boxes so that no box is empty is _______

(a) 5 (b) ${}^8C_3$ (c) 38 (d) 21

79. The no, of positive integer solution of the equation $\left[\frac{x}{99}\right] = \left[\frac{x}{101}\right]$ is _______

(a) 2500 (b) 2499 (c) 1729 (d) 1440

80. The greatest integer n such that $(42)^n$ divides 2007! is _______

(a) 329 (b) 330 (c) 331 (d) 332

81. If the letters of the word SACHIN are arranged in all possible ways and these words are written in dictionary order then the word SACHIN appers at serial number _______

(a) 600 (b) 601 (c) 602 (d) 603

82. The product of n natural number $n \geq 2$ is _______

(a) not divvisible by n (b) divisible by n, but not by 2n

(c) divisible by 2n, but not by n! (d) divisible by n!

83. Nine hundred distinct n digit numbers are to be formed using only the 3 digits 2,5,7. The smallest value of n for which this is possible is _______

(a) 6 (b) 7 (c) 8 (d) 9

84. Let A be a set with n elements. The number of onto functions from A to A is _______

(a) n^n (b) n^n-n! (c) $\frac{n^n}{n!}$ (d) n!

85. The number of number that can be formed using all the digits from 1,2,3,4,3,2,1 so that odd digits occupy odd places is _______

(a) 12 (b) 18 (c) 24 (d) 36

86. 10 person are to be arranged around a round table. 3 persons wish to sit as a group numbar of ways the arrangement can be made is _______

(a) $9! \times 3!$ (b) $8! \times 3!$ (c) $7! \times {}^8P_3$ (d) $7! \times 3!$

87. How many number greater than 10 lac be formed from 2,3,0,3,4,2,3 _______

(a) 420 (b) 360 (c) 400 (d) 300

88. The average of the four digits numbers that can be formed by using each of the digits 3,5,7,and 9 exactly once in each number is _______

(a) 4444 (b) 5555 (c) 6666 (d) 7777

89. In an examination a question paper consists of 10 questions which is divided into two parts. i.e part i and part ii containing 5 and 7 questions respectively. A student is required to attempt 8 question in all selecting at least 3 from each part. In how many ways can a student select the questions _______

(a) ${}^5C_2.{}^7C_2+{}^5C_1$. ${}^5C_3+{}^5C_0.{}^7C_3$ (b) ${}^{12}C_5.{}^{12}C_7$

(c) ${}^5C_3.{}^7C_5$ (d) ${}^{12}C_8$

90. A library has a copies of one book, b copies of two books, C copies of each of three books and single copy of d books. The total no. of ways in which these books can be distributed is _______

(a) $\frac{(a+2b+3c+d)!}{a!b!c!}$ (b) $\frac{(a+2b+3c+d)!}{a!b^2!c^3!}$

(c) $\frac{(a+2b+3c+d)!}{a!(b!)^2(c!)^3}$ (d) $\frac{a+2b+3c+d}{a+b+c+d}$

91. Three boys and three girls are to be seated around a round table in a circle. Among them the boy X does not want any girl neighbour and the girl Y does not want any boy neighbour then the no. of arrangement is _______

(a) 2 (b) 4 (c) 2^3 (d) 3^3

92. 4 boys picked up 30 mangoes. In how many ways can they divide them, if all mangoes be identical _______

(a) ${}^{33}C_4$ (b) ${}^{33}C_2$ (c) 5456 (d) 6554.

93. The number of ways of dividing 15 men and 15 women into 15 couples each consisting of a man and a woman is _______

(a) 1240 (b) 1840 (c) 1820 (d) 2005

94. The number of times the digits 3 will be written when listing the integers from 1 to 1000 is _______

(a) 269 (b) 300 (c) 271 (d) 302

95. The number of ways of distributing 52 cards among four players so that three players have 17 cards each and the fourth player has just one card is _______

(a) $\frac{52!}{(17!)^3}$ (b) 52! (c) $\frac{52!}{(17!)}$ (d) $\frac{52!}{(17!)^2}$

96. In a circus there are 10 cages for accomodating 10 animals out of these 4 cages are so small that five out of ten animals can not enter into them.In how many ways will it be possible to accomodate 10 animals in these 10 cages?

(a) 66400 (b) 86400 (c) 96400 (d) 46900

97. The number of 4 digits number which do not contain 4 different digit is _______

(a) 2432 (b) 3616 (c) 4210 (d) 4464

98. A man has 7 relative, 4 of them ladies and 3 gentleman. his wife also have 7 relatives. 3 of them ladies and 4 gentlemen, They invite for a dinner partly 3 laddies and 3 gentlemen so that there are 3 of the men's relative and 3 of the wife's relative. The number of ways of invitation is _______

(a) 854 (b) 585 (c) 485 (d) 548

99. Find the number of chords that can be drawn through 16 points on a circle.

(a) 102 (b) 120 (c) 12 (d) ${}^{16}p_2$

100. The number of arrangements of two letter of the words "BANANA" in which two of N's do not apper adjacently is _______

(a) 40 (b) 60 (c) 80 (d) 100

101. The number of the factors of 20! is _______

(a) 4140 (b) 41040 (c) 4204 (d) 81650

102. If $\frac{1}{{}^4C_n} = \frac{1}{{}^5C_n} + \frac{1}{{}^6C_n}$, then value of n is _______

(a) 3 (b) 4 (c) 0 (d) none of this

103. The product of first n odd natural numbers equal.

(a) ${}^{2n}C_n \times {}^nP_n$ (b) $(\frac{1}{2})^n . {}^{2n}C_n \times {}^nP_n$

(c) $(\frac{1}{4})^n . {}^{2n}C_n \times {}^{2n}P_n$ (d) None of these

104. The number of ways in which a committee of 3 women and 4 men be chosen from 8 women and 7 men is formed if mr.A refuses to serve on the commitee if mr. B is a member of the committee is _______

(a) 420 (b) 840 (c) 1540 (d) none of these

105. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ then Value of $\sum_{r=0}^{n} \frac{n-2r}{{}^nC_r}$ is. _______

(a) $\frac{n}{2}a_n$ (b) $\frac{1}{4}a_n$ (c) $n.a_n$ (d) none of these.

106. The reminder when no.1!+2!+3!+4!+......+100! is divided by 240 is _______

(a) 153 (b) 33 (c) 73 (d) 187

107. There are three piles of identical yellow, black,and green balls and each pile contains at least 20 balls. The number of ways of selcting 20 balls if the number of black balls to be selected is thrice the number of yellow balls is _______

(a) 6 (b) 7 (c) 8 (d) 9

108. The number of integer a,b,c,d, such that a+b+c+d = 20 and a,b,c,d $\geq$ 0 is _______

(a) ${}^{24}C_3$ (b) ${}^{25}C_3$ (c) ${}^{26}C_3$ (d) ${}^{27}C_3$

109. The least positive integer k for which $k(n^2)(n^2-1^2)(n^2-2^2)(n^2-3^2)......[n^2-(n-1)^2]=r!$ for some positive integers r is _______

(a) 2002 (b) 2004 (c) 1 (d) 2_______

Hint

1. $^{n-1}C_5 + {}^{n-1}C_6 < {}^{n}C_7$

$\therefore\ {}^{n}C_6 < {}^{n}C_7$

$\therefore\ \frac{n!}{6!\,(n-6)!} < \frac{n!}{7!\,(n-7)!}$

$\therefore\ \frac{1}{n-6} < \frac{1}{7}$

$\therefore\ 7 < n-6$

$\therefore\ n > 13 \qquad \therefore\ n = 14$

2. Total arrangements = n !

Two particular books are together = (n-1) ! 2

Two particular books are not together $= n! - 2\,(n-1)!$

$= (n-2)\,(n-1)! = 480 = 4 \times 120$

$\therefore\ (n-2)\,(n-1)! = (6-2)\,(6-1)!$

$\therefore\ n = 6$

3.

Ten Thousand	Thousand	Hundrerd	Tens	Unit
600	600	600	600	600

$\therefore$ Sum : 66, 66, 600

4. M-2 times

T-2 times

a-2 times , H, E, I, c, S

$\rightarrow$ The words with two letters are alike $= {}^{3}C_2 \times \frac{4!}{2!.\,2!} = 18$

$\rightarrow$ words with two letters alike and 2 distinct $= {}^{3}C_2 \times {}^{7}C_2 \times \frac{4!}{2!} = 756$

$\rightarrow$ words with all letters distinct $= {}^{8}C_4 \times 4! = 1680$

$\therefore$ Total words $= 2454$

5. $\sum_{r=0}^{n} \frac{r}{{}^nC_r} = \frac{o}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{2}{{}^nC_2} + \ldots\ldots\ldots\ldots + \frac{n}{{}^nC_n}$

and $\sum_{r=0}^{n} \frac{r}{{}^nC_r} = \frac{n}{{}^nC_0} + \frac{n-1}{{}^nC_1} + \frac{n-2}{{}^nC_2} + \ldots\ldots\ldots + \frac{o}{{}^nC_n}$

$\therefore\ 2\sum_{r=0}^{n} \frac{r}{{}^nC_r} = \frac{n}{{}^nC_0} + \frac{n}{{}^nC_1} + \ldots\ldots\ldots\ldots + \frac{n}{{}^nC_n}$

$= n\left[\frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \ldots\ldots\ldots + \frac{1}{{}^nC_n}\right]$

$= n\sum_{r=0}^{n} \frac{1}{{}^nC_r}$

$= n\ a_n$

$\therefore \sum_{r=0}^{n} \frac{r}{{}^nC_r} = \frac{1}{2} n a_n$

6. ${}^nC_r = {}^nC_{r-1} \Rightarrow r + r - 1 = n \Rightarrow r = \frac{n+1}{2}$

now ${}^nP_r = {}^nP_{r+1}$

$\therefore\ \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$ $\therefore\ n - r = 1$

$\therefore\ n - \left(\frac{n+1}{2}\right) = 1$

$\therefore 2n - n - 1 = 2$

$\therefore n = 3$

7.

O	E	O	E	O	E	O	E	O

O = odd

E = Even

3, 3, 5, 5 are odd digits . 2, 2, 8, 8, 8 are even digits

$\therefore$ Total nine digit numbers are $\frac{{}^4C_4 \times 4!}{2!\ 2!} \times \frac{{}^5C_4 \times 5!}{3!\ 2!} = 60$

8. Here S-3 times

c-2 times U, E ones

$\therefore$ Total Words $= \dfrac{7!}{3!\,2!}$

$\rightarrow$ Words with 3S together $= \dfrac{5!}{2!}$

$\rightarrow$ words with all S not together $= \dfrac{7!}{3!\,2!} - \dfrac{5!}{2!} = 360$

9. coefficient of $t_5 = {}^nC_4$

coefficient of $t_6 = {}^nC_5$

coefficient of $t_7 = {}^nC_6$

$2 \times {}^nC_5 = {}^nC_4 + {}^nC_6$

by solving , We get $n = 7$ or 14.

10. 4 digit numbers $= 3 \times 5 \times 4 \times 3 = 180$

5 digit numbers $= 5 \times 5 \times 4 \times 3 \times 2 = 600$

6 digit numbers $= 5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

$\therefore$ Total numbers $= 1380$

11. Number of students gave i or more than i wrong answers $= a_i$

Number of students gave $i + 1$ wrong answers $= a_{i+1}$

Number of students gave i wrong answers

$= a_i - a_{i+1}$

Total number of answers.

$= 1(a_1 - a_2) + 2(a_2 - a_3) + 3(a_3 - a_4) + ... + ... + (k-1)(a_{k-1} - a_k) + ka_k$

$= a_1 + a_2 + ... a_k$

12. number of ways of placed six X's in 8 squares $= {}^8C_6 = {}^8C_2 = 28$

Here two squares of first and third row are empty

$\therefore$ required arrangements = 28 - 2 = 26

13. The number of five digit numbers except zero $= 5! = 120$

The number of five digit numbers except three $= 5! - 4!$

$= 120 - 24 \qquad = 96$

Total required numbers $= 120 + 96 = 216$

14. 4 lines are mutually intersects in ${}^{4}C_{2} = 6$ points.

4 circles are mutually intersects in ${}^{4}P_{2} = 12$ points.

also each line intersect each circle in two points.

$\therefore$ each line intersect 4 circles in 8 points.

$\therefore$ Number of points of intersection of 4 lines $4 \times 8 = 32$

$\therefore$ Total intersecting lines = $6 + 12 + 32 = 50$

15. Triangles constructed using 12 points $= {}^{12}C_{3} = 220$

Triangles constructed using 3 points on $\overleftrightarrow{AB} = {}^{3}C_{3} = 1$

Triangles constructed using 4 points on $\overleftrightarrow{BC} = {}^{4}C_{3} = 4$

Triangles constructed using 5 points on $\overleftrightarrow{AC} = {}^{5}C_{3} = 10$

$\therefore$ Total triangles $= 220 - 1 - 4 - 10 = 205$

16. The order of a, b, c is not changing

Number of arrangement by keeping order of a,b,c as given = $\frac{7!}{3!} = 7 \times 6 \times 5 \times 4 = 840$

17. ${}^{189}C_{35} + {}^{189}C_{x} = {}^{190}C_{x}$

$\therefore x = 36$

18. Here N Occurs 2 times

I Occurs 2 times

Words are there in dictionary list before the first word begins with E are such words which begins with a

$= \frac{10!}{2!\,2!} = 907,\ 200$

19. Here the arrangement of beads are identicle in both clockwise and anticlockwise..

$\therefore$ Total arrangement $= \frac{10!}{2\,(6!) \times 5!}$

20. there are two possiblities for each subjects.

$\therefore$ Total posiblities $2 \times 2 \times 2 \times 2 = 16$

$\therefore$ Number of ways to be fail $= 16 - 1 = 15$

21. Number of required words

$=$ Total words - number of words with . no letter being repeated \

$= 10^{5} - {}^{10}P_{5} = 69,\ 760$

22. $\sum_{r=0}^{n} r\ {}^nC_r\ P^r\ q^{n-r}$

$= {}^nC_1\ pq^{n-1} + {}^nC_2\, p^2\, q^{n-2} + ...$

$= npq^{n-1} + n\,(n-1)\,p^2\ q^{n-2} + ...$

$= np\,\left(q^{n-1} + (n-1)\,pq^{n-2} + ...\right)$

$= np\,(q+p)^{n-1} = np$

23.

X	Y	Z	r	Number of codes
3	2	0	5	$\frac{10\,!}{3!\,2!\,5!} = 2520$
3	2	1	4	12600
3	2	2	3	25200
3	2	3	2	25200
3	2	4	1	12600
3	2	5	0	2520
Total codes			80640	

24. Number of ways of selecting verticle sides

$= 1 + 3 + 5 + ... + (2n-1) = n^2$

Number of ways of selecting horizontal sides

$= m^2$

$\therefore$ Number of rectangles $= m^2.\ n^2$.

25. Integers formed using 1, 1, 1, 1, 1, 2, 3 $= \frac{7!}{5!} = 42$

Integers formed using 1, 1, 1, 1, 2, 2, 2 $= \frac{7!}{4!\,3!} = 35$

$\therefore$ Total integers $= 42 + 35 = 77$

26. using formulae ${}^{52}C_4$

27. $770 = 2 \times 5 \times 7 \times 11$

2 can be put in 4 ways

5 can be put in 4 ways

7 can be put in 4 ways

11 can be put in 4 ways

$\therefore$ Number of ways of solution $= N = 4^4 = 256$

28. $a = 182bc$

$\therefore {}^{x+2}P_{x+2} = 182 \times {}^{x}P_{11} \times {}^{x-11}P_{x-11}$

$\therefore (x+2)! = 182x!$

$\therefore x = 12, -15$

29. Sit of three persons are fixed.

$\therefore 12! \times 2$

30. Number of ways of arragement of a thing $= n$

Number of ways of arragement of a thing twice $= n \cdot n = n^2$

Similarly Total arrangement

$$n + n^2 + \dots\dots\dots + n^r = \frac{n(n^r - 1)}{n - 1}$$

31. Number of ways of section of drivers $= {}^2C_1 = 2$

Number of 2 seats from remaining 5 seats $= {}^5C_2 = 10$

$\therefore$ Total number of selection $= 2 \times 10 = 20$

32. ans. (D)

33. ${}^nc_2 = 153 \quad \therefore n = 18$

34. $4m = 2^a + 3^b + 5^c$

$= 2^a + (4-1)^b + (4+1)^c$

$= 4k + 2^a + (-1)^b + (1)^6$

$\therefore a = 1, \quad b = \text{even}, \quad c = \text{any}$

$a \neq 1, \quad b = \text{odd}, \quad c = \text{any}$

$\therefore$ Numbers $= 1 \times 2 \times 5 + 4 \times 3 \times 5 = 70$

35. Number of atleast r wrong answers $= 2^{n-r}$

number of students gave wrong answers of r questions $= 2^{n-r} - 2^{n-(r+1)}$

number of students who gave all wrong answers $= 2^o = 1$

Number of total wrong answers.

$= 1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + \dots\dots\dots\dots + (n-1)(2^1 - 2^o) + n(2^o)$

$= 2^{n-1} + 2^{n-2} + \dots\dots\dots + 2^0 = 2^n - 1$

$2^n - 1 = 4095$

$\therefore 2^n = 4096 = 2^{12}$

$\therefore n = 12$

36. Number of total functions $= 2^4 = 16$

Number of constant functions $= f_1(x) = a \quad \forall x \in E$

$f_2(x) = b \quad \forall x \in E$

which are not onto.

$\therefore$ Number of such functions $= 16 - 2 = 14$

37. 4 1 Number of group ${}^4C_4 \times {}^gC_1 = g$

3 2 Number of group ${}^4C_3 \times {}^gC_2 = 2g(g-1)$

Number of total dolls $g(1) + 2[2g(g-1)]$

$85 = 4g^2 - 3g$

by solving equation $g = 5$

38. coefficient of x^{11} in $(x + x^2 + \ldots\ldots\ldots x^6)^3$

coefficient of x^8 in $(1 - x^6)^3 (1 - x)^{-3}$

coefficient of x^8 in $(1 - 3x^6)(1 + {}^3C_1 x + {}^4C_2 x^2 + \ldots\ldots\ldots)$

$= {}^{10}C_8 - 3({}^4C_2) = 27$

39. Number of parallelogram

$= {}^pC_2\, {}^qC_2 + {}^qC_2\, {}^rC_2 + {}^rC_2\, {}^pC_2$

40. ${}^nC_2 - n = 90$ then $n = 15$

41. $N = ({}^{2n}C_2)({}^{2n-2}C_2)\ldots\ldots\ldots\ldots({}^2C_2)$

$$= \frac{(2n)!}{2^n}$$

$\therefore 2^n N = (2n)!$

42. ans. (a)

43. use $C_r = C_{2n+1-r}, \forall r$

44. $100! = 2^{97} \times 3^{48} \times 5^{24} \times \ldots$

45. $({}^3C_1) \cdot \frac{5!}{3!} = 60$

46. $x^2 + 6x + y^2 = 4$

$x^2 + 6x + 9 + y^2 = 13$

$\therefore (x + 3)^2 + y^2 = 13$

$\Rightarrow x + 3 = \pm 2, \quad y = \pm 3$

or $x + 3 = \pm 3, \quad y = \pm 2$

$\therefore$ Total 8 pairs can be obtained

47. from the given $2\left({}^nC_5\right) = {}^nC_4 + {}^nC_6$

$\therefore \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5} = 2$

$\therefore n = 14$

48. ${}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$

49. a occurs twice,

R occurs twice,

N, G and E occurs once,

$\therefore$ Total arrangement $= \frac{7!}{2!\,2!} = 1260$

Number of words with R come together $= \frac{6!}{2\,!} = 360$

Number of words with R do not come together $= 1260 - 360 = 900$

50. Number of committee with atleast 5 women

$= {}^9C_5 . {}^8C_7 + {}^9C_6 . {}^8C_6 + {}^9C_7 . {}^8C_5 + {}^9C_8 . {}^8C_4 + {}^9C_9 . {}^8C_3$

$= 1008 + 2352 + 2016 + 630 + 56 = 6062$

(i) Number of committee in which the women are in majority

$= 2016 + 630 + 56 = 2702$

(ii) Number of committee in which the men are in majority = 1008

51. Let $A = \{a_1, a_2, a_3, \ldots\ldots\ldots a_n\}$

For $a_i \in P$ (i) $a_i \in P$ & $a_i \in Q$

(ii) $a_i \notin P$ & $a_i \in Q$

(iii) $a_i \in P$ & $a_i \notin Q$

(iv) $a_i \notin P$ & $a_i \notin Q$

For (i), (ii) and (iii) $a_i \notin (P \cap Q)$

$\therefore$ The number of ways of choosing P and Q such that $P \cap Q = \varphi$ are 3^n

52. The number of ways of 12 persons are to be arrangment around round table $= 11!$

The arrangement in which two particular persons are side by side $= 10!\,(2!)$

$\therefore$ Required arrangements $= 11! - 10!\,(2!) = 10!\,(11-2)$

$= 9\,(10!)$

53. Numbers of ways of invitation

$= {}^{10}C_6 - {}^{8}C_4$

$= 210 - 70 = 140$

54. The number of required lines

$= {}^{10}C_2 - {}^{7}C_2 + 1 = 45 - 21 + 1 = 25$

55. Total 15 letters, here c is 3 times.

arrangements of letters except c in 12 places $= \dfrac{12!}{5!\,3!\,2!}$

2c are together $= \dfrac{{}^{13}P_3}{3!}$

$\therefore$ Total arrangements $= \dfrac{12!}{5!\,3!\,2!} \times \dfrac{{}^{13}P_3}{3!}$

56. arrangements of 8 parsons so that there are 4 persons in each car $= {}^{8}C_4$

arrangements of two particular persons $= 2$

$\therefore$ Total ways $= 2 \times {}^{8}C_4$

57. ${}^{8-1}C_{3-1} = {}^{7}C_2 = 21$

58. $N = {}^{10}C_3 - {}^{6}C_3 = 120 - 20 = 100$

59. $n = {}^{m}C_2 = \dfrac{m\,(m-1)}{2}$

$\therefore\ {}^{n}C_2 = \dfrac{n\,(n-1)}{2} = \dfrac{1}{2}\,\dfrac{m\,(m-1)}{2}\left[\dfrac{m\,(m-1)}{2} - 1\right]$

$= \dfrac{1}{8}\,m\,(m-1)\,(m^2 - m - 2)$

$$= 3\left[\frac{1}{24}(m+1)(m)(m-1)(m-2)\right]$$

$$= 3\left({}^{m+1}C_4\right)$$

60. Total arrangements $= \dfrac{12!}{3! \times 3! \times 3! \times 3!} = 369600$

61. Number of ways to keep 5 balls in 5 boxes = 1

62. Total no of triangles $= {}^{12}C_3 = 220$

Triangles with two sides of polygon = 12

Triangles with one side of polygon $= 8 \times 12 = 96$

$\therefore$ required triangles $= 220 - 12 - 96 = 112$

63. arrangment in two girls are together $= 2 \times 13 = 26$

$\rightarrow$ one boy sit between two girls $= 2 \times 12 = 24$

$\rightarrow$ two boy sit between two girls $= 2 \times 11 = 22$

$\rightarrow$ Total arrangements $= 14!$

$\therefore$ required arrangements $= 14! - (26 + 24 + 22)12! = 110.12! = {}^{11}P_2\ 12!$

$\therefore m = {}^{11}P_2$

64. Number of ways arranging a and b are 1, 6 ; 2, 7 ; 3, 8 ; ....; 7, 12

arrangement of remaining letters in 10 places are $= 10!$

$\therefore$ Total arrangements $= 2 \times 7 \times 10! = 14 \times 10!$

65. $\rightarrow$ arrangement of seven letters MIIIIPP $= \dfrac{7!}{4!\,2!}$

$\rightarrow$ arrangement of four S between 8 place s of 7 letters $= {}^{8}C_4$

$\therefore$ Total required arrangement $= \dfrac{7!}{4!\,2!}\ {}^{8}C_4 = 7\ {}^{6}C_4\ {}^{8}C_4$

66. N = Novel and D = Dictionary, NNDNN

selection of D = 3

arrangement of NNNN $= {}^{6}P_4 = 360$

$\therefore$ Total arrangements $= 3 \times 360 = 1080$

67. Type of allotments of 4 volunteers in first ward $= {}^{20}C_4$

Type of allotments of 5 volunteers in second ward out of 16 persons $= {}^{16}C_5$

Type of allotments of 8 volunteers in third ward out of 11 persons $= {}^{11}C_8$

$\therefore$ Total allotments $= {}^{20}C_4\ {}^{16}C_5\ {}^{11}C_8$

68. Number of ways to rank 10 candidates = 10!

In this half ranks a_1 is always above a_2 and in remolning half ranks a_2 is always above a_1

$\therefore$ required number of ranks $= \frac{10!}{2} = 5 \times 9!$

69. number of arrangements of 18 members except two particular members $= 17!$

number of arrangements of two particular members in between 18 places $= {}^{18}P_2 = 18 \times 17$

$\therefore$ Total arrangements $= 17! \times 18 \times 17 = 17 \times 18!$

70. Number of words with 5 letters $= 10^5$

Number of words with 5 letters without repeatations $= {}^{10}P_5$

$\therefore$ Required number of words $= 10^5 - {}^{10}P_5 = 69760$

71. Sitting arrangements of 2 women in the chairs marked 1 to 4 numbers $= {}^4P_2 = 12$

Sitting arrangements of 3 men in the remaining chairs $= {}^6P_3 = 120$

$\therefore$ Total arrangements $= 12 \times 120 = 1440$

72. Total arrangement $= 4!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$

$= 12 - 4 + 1 = 9$

73. $T_n = {}^nC_3$

$\therefore T_{n+1} - T_n = 21$

$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21 \Rightarrow {}^nC_1 = 21 \Rightarrow n = 7$

74. arranging odd number 3355 at even palces $= \frac{4!}{2!\,2!} = 6$

arranging even numbers 2,2,8,8,8 at odd places $= \frac{5!}{2!\,3!} = 10$

$\therefore$ Total numbers $= 6 \times 10 = 60$

75. to define ${}^{7-x}P_{x-3}$ we have

$x-3 \geq 0$ and $7-x \geq x-3$

$\therefore$ $x \geq 3$ and $x \leq 5$

$\therefore$ Domain $= \{2, 3, 4\}$

$\therefore f(3) = {}^4P_0 = 1, \quad f(x) = {}^3P_1 = 3, \quad f(5) = {}^2P_2 = 2$

$\therefore$ Range $= \{1, 2, 3\}$

76. Selection of five digits

$\{1, 3, 4, 6, 7\}, \{0, 1, 2, 3, 6\}, \{0, 1, 3, 4, 7\}, \{0, 1, 4, 6, 7\}, \{0, 2, 3, 6, 7\}, \{0, 2, 3, 4, 6\}$

$\therefore$ Total number $= 5! + 5(5! - 4!) = 120 + 5 \times 96 = 600$

77. ${}^5C_4\ {}^8C_6 + {}^5C_5\ {}^8C_5$

$= 140 + 56 = 196$

78. Required arrangements $= {}^{8-1}C_{3-1} = {}^7C_2 = 21$

79. If $\left[\dfrac{x}{99}\right] = \left[\dfrac{x}{101}\right] = n$

$n = 0 \Rightarrow x = 0, 1, 2, \ldots\ldots\ldots.98$

$n = 1 \Rightarrow x = 101, 102, 103, \ldots\ldots\ldots.197$

$n = 2 \Rightarrow x = 202, 203, 204, \ldots\ldots\ldots.296$

$\therefore$ Total solutions

$= [\ 99 + 97 + 95 + \ldots\ldots\ldots\ldots + 3 + 1\ ] - 1 = 2500 - 1 = 2499$

80. $42^n = 2^n . 3^n . 7^n$

$\therefore\ n = \left[\dfrac{2007}{7}\right] + \left[\dfrac{2007}{7^2}\right] + \left[\dfrac{2007}{7^3}\right] + \ldots\ldots\ldots.$

$= 286 + 40 + 5 = 331$

81. Number of words starting with a, c, H, I, N $= 5 \times 5! = 600$

$\therefore$ Serial number of S a c H I N is 601

82. $\dfrac{(m+1)(m+2)\ldots\ldots(m+n)}{n!} = \dfrac{m+n}{n}$ integer.

$\therefore$ $(m+1)(m+2)\ldots\ldots\ldots\ldots(m+n)$ is divisible by n!

83. out of n digits the digit must be 2, 5 or 7.

$\therefore$ number of n digits $= 3^n$

$\therefore 3^n > 900 \Rightarrow n = 7, 8, 9$

$\therefore$ The smallest value of n is 7

84. Here n(a) = n

Domain = range = a

$\therefore$ number of onto functions = n !

85. arrangement of odd digits 1, 1, 3, 3 in 4 odd places and even digit 2, 2, 4 in even places

$$= \frac{4!}{2!\,2!} \cdot \frac{3!}{2!\,1!} = 18$$

86. arrangement of 3 Particular persons +7 other persons on circular table $= 7\,!$

arrangement of 3 Particular persons $= 3\,!$

$\therefore$ Total arrangements $= 7\,! \times 3\,!$

87. For the number greater than 10 lac we need 7 digit.

here 2 is 2 times

3 is 3 times

0 and 4 are ones

$$\therefore \text{ Total numbers } = \frac{7\,!}{2!\,3!} - \frac{6\,!}{2!\,3!} = 420 - 60 = 360$$

= 360

88. $$\text{average} = \frac{\text{sum of numbers}}{\text{Total of numbers}}$$

$$= \frac{3!\left(3+5+7+9\right)\times 1111}{4!}$$

= 6666

89.

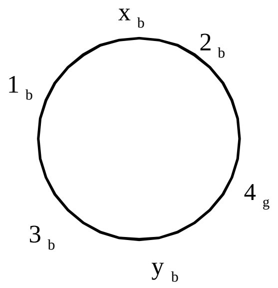
Selections :

		Part(i)	Part(ii)	Types of selections
		I	II	
No. of que.	(i)	3	5	${}^5C_3 \cdot {}^7C_5 = {}^5C_2 \,.\, {}^7C_2$
	(ii)	4	4	${}^5C_4 \cdot {}^7C_4 = {}^5C_1 \cdot {}^7C_3$
	(iii)	5	3	${}^5C_5 \cdot {}^7C_3 = {}^5C_0 \cdot {}^7C_4$
			Total	${}^5C_2 \cdot {}^7C_2 + {}^5C_1 \cdot {}^7C_3 + {}^5C_0 \cdot {}^7C_4$

90. Total no. of books = $a + 2b + 3c + d$

No. of distributions = $\dfrac{(a + 2b + 3c + d)!}{a!\,(b!)^2\,(c!)^3}$

91.



If we arrange the boys 1_b, 2_b and x_b and three girls 3_g, 4_g & y_g according to the given figure the x_b or y_g will not be neighbour.

$\therefore$ Total arrangements = ${}^2P_2 \times {}^2P_2 = 2! \times 2! = 2 \times 2 = 4$

92. Each child have any number of mangoes

$\therefore$ Total ways $= {}^{30+4-1}C_{4-1} = {}^{33}C_3 = 5456$

93. ans. (a)

Type of selection of first couple $= 15 \times 15 = 15^2$

similarly Type of selection of second couple $= 14 \times 14 = 14^2$

$\therefore$ Total number of couples $= 15^2 + 14^2 + \ldots\ldots\ldots\ldots + 2^2 + 1^2$

$$= \sum_{i=1}^{15} i^2 = \frac{15 \times 16 \times 31}{6} = 1240$$

94. Total numbers between 1 to 999 will be of the type x y z , where $x,y,z \in \{0,1,2,3,4,5,6,7,8,9\}$

3 Occurs ones in a numbers $= {}^3C_1 (9 \times 9) = 3\,(9^2)$

3 Occurs twice in a numbers $= {}^3C_2\ 9$

3 Occurs thries in a numbers is the number 333

$\therefore$ Total Possible numbers

$= 1 \times (3 \times 9^2) + 2 \times (3 \times 9) + 3 \times 1 = 300$

95. ways of distribution

$= {}^{52}C_{17} \cdot {}^{35}C_{17} \cdot {}^{18}C_{17} \cdot {}^{1}C_{1}$

$$= \frac{52!}{17!\,35!} \cdot \frac{35!}{17!\,18!} \cdot \frac{18!}{17!\,1!} \cdot 1 = \frac{52!}{(17!)^3}$$

96. arrangement of little animals that can not accomodate in small cage $= {}^6P_5$

arrangement of remaining 5 animals $= {}^5P_5 = 5!$

$\therefore$ Total accomodations $= {}^6P_5 \cdot {}^5P_5 = 720 \times 120 = 86400$

97. Total Numbers

$= 9 \times 10 \times 10 \times 10 - 9 \times 9 \times 8 \times 7$

$= 9000 - 4536 = 4464$

98.

	Man	Wife	Selection
	(3G, 4L)	(3L, 4G)	
(1)	3 L	3G	${}^4C_3\, {}^4C_3 = 16$
(2)	2L, 1G	1L, 1G	${}^4C_2\, {}^3C_1 \cdot {}^3C_1\, {}^4C_2 = 324$
(3)	1L, 2G	2L, 1G	${}^4C_1 {}^3C_2\, {}^3C_2\, {}^4C_1 = 144$
		3G, 3L	${}^3C_3 \cdot {}^3C_3 = 1$

$\therefore$ Total Invitations = 485

99. ans. (b)

Number of chords $= {}^{16}C_2 = 120$

(all points are on the circle so no points are collinear)

100. Here
a - 3 times
N - 2 times
b - ones

$\therefore$ Total arrangements $= \dfrac{6!}{3!\,2!} = 60$

2N are adjacent $= \dfrac{5!}{3!} = 20$

$\therefore$ required arrangements $= 60 - 20 = 40$

101. $20! = 2^{18} \times 3^8 \times 5^4 \times 7^2 \times 11 \times 13 \times 17 \times 19 = 41040$

102. $\dfrac{1}{{}^4C_n} = \dfrac{1}{{}^5C_n} + \dfrac{1}{{}^6C_n}$

$\therefore \dfrac{{}^5C_n}{{}^4C_n} = 1 + \dfrac{{}^5C_n}{{}^6C_n}$

$\therefore\ \dfrac{5!\,n!\,(4-n)!}{n!\,(5-n)!\times 4!} = 1 + \dfrac{5!\,n!\,(6-n)!}{n!\,(5-n)!\times 6!}$

$\therefore\ \dfrac{5}{5-n} = 1 + \dfrac{6-n}{6}$

$\therefore\ 30 = 6\,(5-n) + (5-n)\,(6-n)$

$\therefore\ 30 = 30 - 6n + 30 - 5n - 6n + n^2$

$\therefore\ n^2 - 17n + 30 = 0$

$\therefore\ (n-15)\,(n-2) = 0$

$\therefore\ n = 2$

because $n > 6$ is not possible

103. $1\times 3\times 5\times ....\times (2n-1) = \dfrac{(2n)!}{2^n\,(n!)} = \left(\dfrac{1}{2}\right)^n\ {}^{2n}C_n\cdot {}^{n}P_n$

104. women can be selected in ${}^8c_3$ ways, men can be selected in $= {}^7C_4 - {}^5C_2$ ways

105. $\displaystyle\sum_{r=0}^{n}\frac{n-2r}{{}^nC_r} = \sum_{r=0}^{n}\frac{n-r}{{}^nC_r} - \sum_{r=0}^{n}\frac{r}{{}^nC_r}$

$\displaystyle = \sum_{r=0}^{n}\frac{n-r}{{}^nC_{n-r}} - \sum_{r=0}^{n}\frac{r}{{}^nC_r}$

$= 0$

106. If $r \ge 6$ then $r\,!$ is divisible by 240

given number is divisible by 240

$\therefore$ remainder $= 1\,! + 2\,! + \ldots + 5\,! = 153$

107. $x =$ No. of yellow balls

$2x =$ No. of black balls

$y =$ No. of green balls

here $x + 2x + y = 20$

$\therefore\ 3x + y = 20$

$\therefore\ y = 20 - 3x$

Now $0 \le y \le 20$ $\quad\therefore\ 0 \le 20 - 3x \le 20$

$\therefore\ 0 \le 3x \le 20$

$\therefore\ 0 \le x \le 6$

$\therefore$ ways of selecting yellow ball = 7

108. $a = x-1,\quad b = y-1,\quad c = z-1,\quad d = w-1$

Here, x, y, z , w $\geq$ 0 and

$x-1 + y-1 + z-1 + w - 1 = 20$

$\therefore\ x + y + z + w = 24$

$\therefore$ Non zero integer solutions of the equation are = ${}^{24+4-1}C_{4-1} = {}^{27}C_3$

109. $K\left(n^2\right)\left(n^2-1^2\right)\left(n^2-2^2\right)\left(n^2-3^2\right)........\left[n^2-(n-1)^2\right] = r!$

$K\left(n^2\right)(n-1)(n+1)(n-2)(n+2)(n-3)(n+3).........(n+n-1)\cdot(n-n+1) = r!$

$\therefore\ K\cdot n\cdot 1\cdot 2...........(n-1)n\ (n+1)(n+2).........(2n-1) = r!$

$\therefore\ kn(2n-1)! = r!$

$\therefore\ K = 2$

LHS = $(2n)! = r!$

answer

1	a	41	a	81	b
2	b	42	a	82	d
3	d	43	a, d	83	b
4	c	44	c	84	d
5	c	45	b	85	b
6	a	46	d	86	d
7	c	47	a	87	b
8	c	48	b	88	c
9	c	49	b	89	a
10	d	50	d	90	c
11	a	51	d	91	b
12	c	52	a	92	c
13	a	53	b	93	a
14	c	54	d	94	b
15	c	55	c	95	a
16	d	56	c	96	b
17	c	57	b	97	d
18	a	58	a	98	c
19	c	59	d	99	b
20	a	60	a	100	a
21	a	61	d	101	b
22	a	62	c	102	d
23	c	63	c	103	b
24	b	64	d	104	d
25	c	65	d	105	d
26	c	66	d	106	a
27	d	67	c	107	b
28	b	68	d	108	d
29	a	69	a	109	d
30	c	70	a		
31	b	71	c		
32	d	72	c		
33	c	73	d		
34	b	74	c		
35	b	75	a		
36	a	76	c		
37	d	77	b		
38	b	78	d		
39	d	79	b		
40	d	80	c		

●●●

Unit-5

Principle of Mathematical Induction

Important Points

If statement P(n) of natural variable $n \in N$ is given, the Principle of Mathematical Induction is useful to verify the validity of the given statement, $\forall\ n \in N$.

The Principle of Mathematical Induction :-

Let P(n) be a statement involving natural number n.

The statement P(n) is true $\forall\ n \in N$, if

(1) P(1) is true,

(2) P(k), $k \in N$ is true $\Rightarrow$ P(k+1), $k \in N$ is true, then P(n), $\forall\ n \in N$ is true.

Note:- 1. Principle of Mathematical Induction verifies the validity of statements involving natural number variable only.

2. Formula involving natural number variable cannot be derived, but only its validity can be verified.

Use of Principle of Mathematical Induction in some special types of variable:-

(1) Variable type 1:

The statement P(n), $n \in N$ is given. If for positive integer k_0, $P(k_0)$ is true and for $k \geq k_0$, $k \in N$ P(k) is true $\Rightarrow$ P(k+1) is true, then P(n) is true for all $n \geq k_0$, $k \in N$.

(2) Variable type 2:

The statement P(n), $n \in N$ is given.

If (1) P(1) and P(2) are true and

(2) for positive integer k, P(k) and P(k+1) are true $\Rightarrow$ P(k+2) is true, then $\forall n \in N$, P(n) is true.

Question Bank

(1) For all $n \in N-\{1\}$, $7^{2n} - 48n - 1$ is divisible by ....

(a) 25 (b) 26 (c) 1234 **(d) 2304**

(2) $\forall$ $n \in N$, $P(n): 2.7^n + 3.5^n - 5$ is divisible by ....

(a) 64 (b) 676 (c) 17 **(d) 24**

(3) For all $n \geq 2$, $n^2(n^4 - 1)$ is divisible by ....

(a) 60 (b) 50 (c) 40 (d) 70

(4) For $n \in N$ $10^{n-2} > 81n$, if .....

(a) $n > 5$ **(b) $n \geq 5$** (c) $n < 5$ (d) $n > 6$

(5) For each $n \in N$, the correct statement is ....

(a) $2^n < n$ (b) $n^2 > 2^n$ **(c) $n^4 < 10^n$** (d) $2^{3n} > 7n + 1$

(6) If $a_n = 2^{2^n} + 1$, then for $n > 1$, $n \in N$, last digit of a_n is ....

(a) 3 (b) 5 (c) 8 **(d) 7**

(7) If $P(n): 4^n/(n+1) < (2n)!/(n!)^2$, then P(n) is true for ....

(a) $n \geq 1$ (b) $n > 0$ (c) $n < 0$ **(d) $n \geq 2$,** $n \in N$

(8) By principle of mathematical induction,

$\forall n \in N$, $\cos\theta\cos2\theta\cos4\theta \ldots \cos[(2^{n-1})\theta] =$........

(a) $\sin2^n\theta / 2^n \sin\theta$ (b) $\cos2^n\theta / 2^n\sin\theta$

(c) $\sin2^n\theta / 2^{n-1} \sin\theta$ (d) None of these

(9) By principle of mathematical induction, $\forall n \in N$,

$1/(1.2.3) + 1/(2.3.4) + \ldots + 1/\{n(n+1)(n+2)\} = \ldots\ldots$

(a) $n(n+1)/4(n+2)(n+3)$ **(b) $n(n+3)/4(n+1)(n+2)$**

(c) $n(n+2)/4(n+1)(n+3)$ (d) None of these

(10) By principle of mathematical induction, $\forall n \in N$, $5^{2n+1} + 3^{n+2} . 2^{n-1}$ is divisible by ....

(a) 19 (b) 18 (c) 17 (d) 14

(11) The product of three consecutive natural numbers is divisible by ....

(a) 6 (b) 5 (c) 7 (d) 4

(12) For all $n \in N$, $a^n - b^n$ is always divisible by.......... (a and b are distinct rational numbers)

(a) $2a - b$ (b) $a + b$ **(c) $a - b$** (d) $a - 2b$

(13) If $x^{2n-1} + y^{2n-1}$ is divisible by $x+y$, then n is ....

(a) Positive integer (b) Only for an even positive integer

(c) an odd positive integer (d) $\forall n, n \geq 2$

(14) The inequality $n! > 2^{n-1}$ is true for ....

(a) $n > 2$, $n \in N$ (b) $n < 2$ (c) $n \in N$ (d) None of these

(15) The smallest positive integer n for which $n! < \{(n+1)/2\}^n$ holds, is...

(a) 1 **(b) 2** (c) 3 (d) 4

(16) The greatest positive integer, which divides $(n+2)(n+3)(n+4)(n+5)(n+6)$ for all $\forall n \in N$, is .....

(a) 120 (b) 4 (c) 240 (d) 24

(17) $x(x^{n-1} - n\alpha^{n-1}) + \alpha^n(n-1)$ is divisible by $(x - \alpha)^2$ for ....

(a) n>1 (b) n>2 **(c) For all n∈N** (d) None of these

(18) For each n∈N, $3^{2n} - 1$ is divisible by ....

(a) 8 (b) 16 (c) 32 (d) None of these

(19) For each n∈N, $2^{3n} - 7n - 1$ is divisible by ....

(a) 64 (b) 36 **(c) 49** (d) 25

(20) For each n∈N, $10^{2n-1} + 1$ is divisible by ....

(a) 11 (b) 13 (c) 9 (d) None of these

(21) For each n∈N, $2.4^{2n+1} + 3^{3n+1}$ is divisible by ....

(a) 2 (b) 9 (c) 3 **(d) 11**

(22) Let P(n):$n^2 + n + 1$ is an odd integer.If it is assumed that

P(k) is true ⇒ P(k+1) is true. Therefore, P(n) is true .....

(a) for n>1 (b) for all n∈N

(c) for n>2 **(d) None of these**

(23) If P(n):$3^n < n!$, n∈N, then P(n) is true ....

(a) for n ≥ 6 **(b) for n ≥ 7**, n∈N

(c) for n ≥ 3 (d) for all n

(24) If P(n):1+3+5+.......+(2n–1) = n^2 is ....

(a) True for n>1 **(b) true for all n∈N**

(c) true for no n (d) None of these

(25) If $\forall n \in N$, P(n) is a statement such that, if P(k) is true $\Rightarrow$P(k+1) is true for $k \in N$, then P(n) is true ....

(a) for all n>1 (b) for all $n \in N$

(c) for all n>2 **(d) nothing can be said**

(26) Let $P(n) = 1+3+5+ \ldots + (2^n - 1) = 3 + n^2$. Then, which of the following is true ?

(a) P(1) is true **(b) P(k) is true $\Rightarrow$ P(k+1) is true**

(c) P(k) is true, P(k+1) is not true (d) both (a) and (b) are true

(27) If matrix $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \in N$. (use principle of Mathematical Induction)

(a) $A^n = n.A - (n-1)\, I$ (b) $A^n = 2^{n-1}.A + (n-1)\, I$

(c) $A^n = n.A + (n-1)\, I$ (d) $A^n = 2^{n-1}.A - (n-1)\, I$

(28) $S_n = 2.7^n + 3.5^n - 5$, $n \in N$ is divisible by the multiple of ....

(a) 5 (b) 7 **(c) 24** (d) None of these

(29) $10^n + 3(4^{n+2}) + 5$ is divisible by ($n \in N$) ....

(a) 7 (b) 5 **(c) 9** (d) 17

(30) $\forall n \in N$, $(3+5^{1/2})^n + (3-5^{1/2})^n$ is .....

(a) Even natural number (b) Odd natural number

(c) Any natural number (d) Rational number

(31) The remainder, when 5^{99} is divided by 13, is ....

(a) 6 **(b) 8** (c) 9 (d) 10

(32) For all positive integral values of n, $3^{3n} - 2n + 1$ is divisible by ....

(a) 2 (b) 4 (c) 8 (d) 12

(33) If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by ....

(a) 113 (b) 123 **(c) 133** (d) None of these

(34) For $n \in N$, P(n): $2^n (n-1)! < n^n$ is true, if ....

(a) $n < 2$ **(b) $n > 2$** (c) $n \geq 2$ (d) Never

Answers

(1) (d) 2304 (2) (d) 24 (3) (a) 60 (4) (b) $n \geq 5$

(5) (c) $n^4 < 10^n$ (6) (d) 7 (7) (d) $n \geq 2$

(8) (a) $\sin 2^n\theta / 2^n \sin\theta$ (9) (b) $n(n+3) / 4(n+1)(n+2)$

(10) (a) 19 (11) (a) 6 (12) (c) $a - b$ (13) (a) Positive integer

(14) (a) $n > 2$ (15) (b) 2 (16) (a) 120 (17) (c) all $n \in N$

(18) (a) 8 (19) (c) 49 (20) (a) 11 (21) (d) 11

(22) (d) None of these (23) (b) for $n \geq 7$

(24) (b) true for all $n \in N$ (25) (d) nothing can be said

(26) (b) P(k) is true $\Rightarrow$ P(k+1) is true (27) (a) $A^n = n.A-(n-1)I$

(28) (c) 24

(29) (c) 9 (30) (a) Even natural number (31) (b) 8

(32) (a) 2 (33) (c) 133 (34) (b) $n > 2$

Hints

(1)

P(1): 0 = 0×2304 P(2): 2304=1×2304 ∴ P(1) and P(2) are true --(1)

Let P(k):$7^{2k} - 48k - 1 = m \times 2304$, m∈N and

P(k+1) : $7^{2k+2} - 48(k+1) - 1 = m'\ (2304)$, m'∈N be true . ------ (2)

Now, P(k+2): $7^{2k+4} - 48(k+2) - 1 = 49 \times 7^{2k+2} - 48(k+1) - 49$

$= 49 \times 7^{2k+2} - 48(k+1) - 49 = 49(7^{2k+2} - 1) - 48(k+1)$

$= 49(2304m' + 48k + 48) - 48k - 48$ (…. (2))

$= 49 \times 2304m' + 49 \times 48k + 49 \times 48 - 48k - 48$

$= 49 \times 2304m' + 48 \times 48k + 48 \times 48 = 2304(49m' + k + 1)$

$= 2304 \times m''$, where m''=49m'+k+1 is positive integer.

∴ Ans. (d) 2304

(2)

∀n∈N માટે P(n): $2.7^n + 3.5^n - 5$

P(1): 24 , P(2):98+75−5=168=7×24

Ans. (a) 24

(3)

For every positive integers n ≥ 2 , P(n): $n^2(n^4 - 1)$

P(2): 4 × 15 = 60, P(3): 9 × 80=60 × 12

∴ From option Ans. (a) 60

(4) For n∈N , P(n):$10^{n-2} > 81n$

P(1) : 0.1 > 81 isn't true , P(2): 1> 162 isn't true , P(3):10 > 243 isn't true, as the same way P(4) isn't true , but P(5): 1000 > 405 is true and P(6): 10000 > 486 is true ∴ Ans. (b) n ≥ 5

(5)

Here for n = 1, $2^n < n$ isn't true,

$n^2 > 2^n$ isn't true,

$n^4 < 10^n$ is true,

$n^{3n} > 7n + 1$ isn't true,

Ans. (c) $n^4 < 10^n$

(6)

For $n = 2$, $a_2 = 2^{2^2} + 1 = 17 = 10 + 7$

Let $a_k = 2^{2^k} + 1 = 10m + 7$ be true where $k > 1, m \in N$ ... (1)

Now, $a_{k+1} = 2^{2^{k+1}} + 1 = \left(2^{2^k}\right)^2 + 1 = (10m + 6)^2 + 1$ (by (1))

$= 10(10m^2 + 12m + 3) + 7$

$\therefore$ Digit of one's place of a_n is 7.

$\therefore$ Ans. (c) 7

(7)

P(n): $4^n/(n+1) < (2n)!/(n!)^2$, $n \in N$

P(1) isn't true and $n < 0$ isn't possible.

$\therefore$ (a), (b), (c) options are not possible .

$\therefore$ Ans. **(d) $n \geq 2$,** $n \in N$

(8) For n=1, by P(n):$\cos\theta \cos 2\theta \cos 4\theta \ldots \cos[(2^{n-1})\theta]$ $\therefore$ P(1): $\cos\theta$

in option (a) n = 1 we get $\cos\theta$. $\therefore$ Ans. **(a) $\sin 2^n\theta / 2^n \sin\theta$**

(9)

For n = 1,

$1/(1.2.3) = 1/6$

Now, for n = 1, value of only option **(b) n(n+3) /4(n+1)(n+2) is** 1/6

∴ Ans. **(b) n(n+3) /4(n+1)(n+2)**

(10)

For n = 1, $P(1): 5^{2+1} + 3^{1+2} . 2^{1-1}$

$= 125 + 27 = 152 = 19 \times 8$

Let $P(k) = 5^{2k+1} + 3^{k+2} . 2^{k-1} = 19m,\ m \in N$ ---- (1)

$P(k+1) = 5^{2k+3} + 3^{k+3} . 2^{k} = 5^2 . 5^{2k+1} + 3.3^{k+2} . 2.2^{k-1}$

$= 25(19m - 3^{k+2} . 2^{k-1}) + 6.\ 3^{k+2} . 2^{k-1}$ (by (1))

$= 25.19m - 19.\ 3^{k+2} . 2^{k-1}$

$= 19(25m - 3^{k+2} . 2^{k-1})$

$= 19m'$

∴ Ans. (a) 19

(11)

Product of three consecutive natural numbers P(n) : n(n+1)(n+2)

P(1)= 6 which is divisible by 6.

P(2) = 24 which is divisible by 6.

∴ Ans. (a) 6

(12)

For every $n \in N$, $P(n) : a^n - b^n$

$P(1) = a - b$ and $P(2) = a^2 - b^2 = (a - b)(a+b)$

$\therefore$ Ans. (c) $a - b$

(13)

$P(n)$: $x^{2n-1} + y^{2n-1} = \lambda(x + y)$ where λ is a polynomial.

$P(1)$: $x + y$ is divisible by $x+y$

$P(2)$: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$\therefore$ Ans. **(a) Positive integer**

(14)

$P(n)$: $n! > 2^{n-1}$

Now $P(1)$ and $P(2)$ are not true, but $P(3)$ is true.

Let $P(k)$:$k! > 2^{k-1}$, $k>2$ be true.

$P(k+1)$:$(k+1)! > 2^k$

L.H.S of $P(k+1) = (k+1)! = k!(k+1)$

$> 2^{k-1}(k+1) = 2^k.(k+1)/2$

$> 2^k$

$\therefore$ Ans. (a) $n > 2$

(15) For smallest positive integer n, $P(n)$:$n! < \{(n+1)/2\}^n$,

$P(1)$:$1 < 1$ isn't true, $P(2)$:$2 < 9/4$ is true. $P(3)$:$6<8$ is true $P(4)$ is true. $\therefore$ **Ans. (b) 2**

(16)

For $\forall n \in N$, for which greatest positive integer, does $(n+2)(n+3)(n+4)(n+5)(n+6)$ divide ?

$P(n): (n+2)(n+3)(n+4)(n+5)(n+6), \quad n \in N$

$P(1) = 3.4.5.6.7 = 120.21$ $\quad$ $P(2) = 4.5.6.7.8 = 120.56$

$P(3) = 5.6.7.8.9 = 120.126$ $\quad$ $P(4) = 6.7.8.9.10 = 120.252$

$P(5) = 7.8.9.10 = 120.42$ $\quad$ $P(6) = 8.9.10.11.12 = 120.99.13$

$\therefore$ Ans. **(a) 120**

(17)

$$P(n) : x(x^{n-1} - n\alpha^{n-1}) + \alpha^{n}(n-1) = g(x).(x-\alpha)^2$$

$$P(1) = 0$$

$$P(k) : x(x^{k-1} - k\alpha^{k-1}) + \alpha^{k}(k-1) = g(x).(x-\alpha)^2$$

$$P(k+1) : x(x^{k} - (k+1)\alpha^{k}) + \alpha^{k+1}(k) = g'(x).(x-\alpha)^2$$

$$\text{L.H.S.} = x[kx\alpha^{k-1} - (k-1)\alpha^{k} + g(x).(x-\alpha)^2 - (k+1)\alpha^{k}] + \alpha^{k+1}k$$

$$= kx^2\alpha^{k-1} - 2kx\alpha^{k} + g(x).x.(x-\alpha)^2 + k\alpha^{k+1}$$

$$= g(x).x.(x-\alpha)^2 + (x^2 - 2x\alpha + \alpha^2)k\alpha^{k-1}$$

$$= (x-\alpha)^2[g(x).x + k\alpha^{k-1}]$$

$$= g'(x).(x-\alpha)^2$$

$$= \text{R.H.S.}$$

$\therefore$ Ans. **(c) all $n \in N$**

(18) For each $n \in N$, $P(n) : 3^{2n} - 1$

$P(1) = 8$, $\quad P(2) = 80 = 10.8$ $\quad \therefore$ Ans. **(a) 8**

(19)

For each $n \in N$, $P(n) : 2^{3n} - 7n - 1$

$P(1) = 0$ $P(2) = 49$ $P(3) = 512 - 21 - 1 = 490 = 49.10$

$\therefore$ Ans. **(c) 49**

(20)

For each $n \in N$, $P(n) : 10^{2n-1} + 1$

$P(1) = 11,$

$P(2) = 1001 = 11.91$

$\therefore$ Ans. **(a) 11**

(21)

$\forall n \in N$, $P(n) : 2.4^{2n+1} + 3^{3n+1}$

$P(1) = 209 = 11.19$

$P(2) = 11.385$

$\therefore$ Ans. **(d) 11**

(22) $P(n) : n^2 + n + 1 = n(n+1) + 1$

$P(1) : 3$ which is true.

$P(n) : n^2 + n + 1 = n(n+1) + 1$which is always odd number

$\therefore$ Ans. **(b) $\forall n \in N$**

(23) $P(n):3^n < n!$, $n \in N$

$P(1) : 3^1 < 1$ is not true. $P(3): 3^3 < 3!$ is not true.

$P(6) : 3^6 < 6!$ is not true.

$P(7) : 3^7 < 7!$ is true. $\therefore$ Ans. **(b) $n \geq 7$**

(24)

$P(1): 1 = 1$

$P(k): 1+3+5+.......+(2k-1) = k^2.$

$P(k+1): 1+3+5+.......+(2k-1) + (2k+1) = (k+1)^2.$

$L.H.S. = 1+3+5+.......+(2k-1) + (2k+1)$

$= k^2 + 2k + 1 = (k+1)^2 = R.H.S.$

$\therefore$ Ans. **(b) true for all n∈N**

(25)

P(1) validity cannot be checked because statement P(n) is given

$\therefore$ Ans. **(d) nothing can be said**

(26)

$P(1) : 1 = 4$ is not true.

Let $P(k) : 1+3+5+ \dots + (2k - 1) = 3 + k^2$ be true.

$P(k+1) = 1+3+5+ \dots + (2k - 1) + (2k+1)$

$= 3 + k^2 + 2k + 1 = (k+1)^2 + 3 = R.H.S.$

$\therefore$ Ans.**(b) P(k) is true ⇒ P(k+1) is true.**

(27)

$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$P(1): A = A - (1-1) I = A$ $\therefore$ P(1) is true.

P(k) is true ⇒ **P(k+1) is true, k** ∈N

$\therefore$ Ans. (a) $A^n = n. A-(n-1) I$

(32)

$\forall n \in N,\ P(n): 3^{3n} - 2n + 1$

$P(1): 26 = 2 \times 13$

$P(2): 726 = 2 \times 343$

$P(3): 19683 - 6 + 1 = 19678 = 2 \times 9839$

$\therefore$ Ans.**(a) 2**

(33)

$\forall n \in N,\ P(n) = 11^{n+2} + 12^{2n+1}$

$P(1): 11^{1+2} + 12^{2+1} = 133 \times 23,$

$P(2): 11^{2+2} + 12^{4+1} = 14641 + 248832 = 263473 = 133 \times 1981$

$\therefore$ Ans. **(c) 133**

(34)

For $n \in N,\ P(n) = 2^{n}(n-1)! < n^{n}$

$P(1): 2 < 1$ is not true.

$P(2): 4 < 4$ is not true.

$P(3): 16 < 27$ is true.

Same as P(4) is true.

$\therefore$ Ans. **(b) n > 2**

Unit - 6

Binomial Theorem

Important Points

- Cr = nCr = $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

- For x, y $\in$ R and n $\in$ N

  $(x+y)^n = {}_nC_o\, x^n y^o + {}_nC_1 x^{n-1} y + n\,c_2\, x^{n-2}\, y^2 + \text{---} + n\,c_n\, x^o\, y^n = \sum_{r=0}^{n} nc_r\, x^{n-r}\, y^r$

  e.g $\sum_{r=0}^{10} 10c_r\, 2^{10-r} (-5)^4 = (2-5)^{10} = 3^{10}$

  (r+1) th term in expansion of $(x+y)^n$ is

  $Tr+1 = nc_r\, x^{n-r}\, y^r$

- For x,y $\in$ R and n $\in$N

  $(x-y)^n = n\,C_o\, x^n y^o - nc_1\, x^{n-1}\, y^1 + nc_2\, x^{n-2}\, y^2 + \text{---} + (-1)^n\, nc_n\, x^o\, y^n$

  $nCr = \sum_{r=0}^{n} (-1)^r\, nc_r\, x^{n-r}\, y^{r,}$

  (r+1) th term in expansion of $(x-y)^n$ is $Tr+1 = (-1)^n\, nC_r\, x^{n-r}\, y^r$

- Number of terms in exprression of $(x+y)^n$ or $(x-y)^n$ is n+1.

  for each term, the sum of power of x and y is n

- **<u>MIDDLE TERM(S)</u>**

If n is even then middle term is $\left(\frac{n}{2}+1\right)$ th term It is given by $nC_{\frac{n}{2}}\ x^{\frac{n}{2}}$

If n is odd then middle term is $\left(\frac{n+1}{2}\right)$ th term and $\left(\frac{n+3}{2}\right)$ th term. These are given by

$nC_{\frac{(n-1)}{2}}\, x^{\frac{(n-1)}{2}}\, y^{\frac{(n+1)}{2}}$ and $nC_{\frac{(n+1)}{2}}\, x^{\frac{(n+1)}{2}}\, y^{\frac{(n-1)}{2}}$

- **The Greatest Co-efficient**
- If n is even, the greatest co-efficient in the expansion of $(x+y)^n$ is $nC_{\frac{n}{2}}$
- If n is odd, there are two greatest co-efficient in the expansion of $(x+y)^n$ These are $nC_{\frac{(n-1)}{2}}$ and $nC_{\frac{(n+1)}{2}}$
- **The Greatest Term**

  If $x>o, y>o, n\in N$ then to find the greatest term in the expansion of $(x+y)^n$ find $K=\frac{(n+1)y}{x+y}$.

  if k is an integer then the expansion of $(x+y)^n$ has two greatest terms is, Kth and (k+1) th terms.

  if k is not an integer then the expansion of $(x+y)^n$has just one greatest term and it is given by $T_{\propto}+1$.where $\propto=[K]$.

- $(x+y)^n + (x-y)^n = 2\ \{nC_0\, x^n\, y^o + n\,C_2\, x^{n-2}\, y^2 + nC_4\, x^{n-4}\, y^4$ +---+ $\left(\left[\frac{r}{2}\right]+1\right)$ th term

$(x+y)^n - (x-y)^n = 2\ \{nC_1\ x^{n-1}\ y + n\ C_3\ x^{n-3}\ y^3 + --- + \left(\left[\frac{n+1}{2}\right]\right)$ th term}

e.g $\left(1+\sqrt{2}x\right)^9 + \left(1-\sqrt{2}x\right)^9$ has $\left[\frac{9}{2}\right]+1=4+1=5$ terms.

- $(1+x)^n = nC_o\ x^o + nC_1\ x^1 + nC_2\ x^2 + --- + n\ C_n\ x^n\ ;\ n \in N$

- Co-efficient of x^r in expasion of $(1+x)^n$ is = $\binom{n}{r} = n\ C_r$

- Co-efficient of (r+1)th term in expansion of $(1+x)^n$ is = $\binom{n}{r} = n\ C_r$

- $(a_1+a_2+---+ {}^{a}m)^n = \sum \frac{n\ !}{n!\ n_2!...n_m\ !} a_1^{n_1} a_2^{n_2} ......a_m^{n_m}$

where $n_1, n_2, n_3, ...n_m \in N$ and

$n_1 + n_2 + n_3 + ...+n_m = n$

Numbers of terms in expansion = $(n + m - 1)^C(m-1)$

e.g. Number of terms in expansion of $(x+y+z)^n = \frac{(n+1)\ (n+2)}{2}$ sum of coefficients in $(x+y+z)^n = 3^n$

- **<u>Some properties of the Binomial coefficients</u>**

(1) $n\ Co + n\ C_1 + n\ C_2 + ...+n\ C_n = 2^n$

(2) $n\ Co + n\ C_2 + n\ C_4 + ...+ = 2^{n-1}$

(3) $n\ C_1 + n\ C_3 + n\ C_5 + ...+n\ C_n = 2^n-1$

(4) If $a_o, a_1, a_2, ...\ a_n$ are in A.P then

$a_o\ n\ c_o + a_1\ n\ c_1 + a_2\ n\ c_2 + ...+ a_n\ {}_n c_n = (a_o + a_n)\ 2^{n-1}$

(5) $n\ C_o - n\ C_1 + n\ C_2 - ....+ (-1)^n\ n\ C_n = o$

(6) If three consecutive binomial co-efficients $n\,C_{r-1},\ n\,C_{r},\ n\,C_{r+1}$

are in A.P then $r = \frac{1}{2}\left(n+\sqrt{n+2}\right)$

(7) Co-efficient of x^n in $(1+x)^{2n}=2$ {co efficient of x^n in $(1+x)^{2n-1}$}

(8) co-efficient of $\frac{1}{x}$ in expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n = \frac{(2n)!}{(n-1)i(n+1)!}$

- To find remainder when x^n is divideel by y, try to express x or some power of x as $ky \pm 1$ and then apply binomial theorem.

Question Bank

1. If coefficients of x^7 and x^8 are equal in expansion of $\left(2+\frac{x}{3}\right)^n$ then n = ________

(a) 55 (b) 56 (c) 54 (d) 58

2. The constant term in expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9, x \neq o$ is

(a) $\frac{5}{18}$ (b) $\frac{7}{18}$ (c) $\frac{5}{17}$ (d) $\frac{7}{17}$

3. Coefficients of middle terms in expansion of $\left(2-\frac{x^3}{3}\right)^7$ are...

(a) $-\frac{560}{27}, -\frac{280}{81}$ (b) $\frac{560}{27}, -\frac{280}{81}$

(c) $-\frac{560}{27}, \frac{280}{81}$ (d) $\frac{560}{27}, \frac{280}{81}$

4. Middle term in expansion of $\left(\frac{2}{x}-3xy\right)^{12}$ is______

(a) 14370048 y^6 (b) 14370024 y^6

(c) 43110144 y^6 (d) 43110124 y^6

5 If middle term is Kx^m in expansion of $\left(x+\frac{1}{x}\right)^{12}$ then m =______

(a) -2 (b) -1 (c) 0 (d) 1

6. Co efficient of middle term in expansion of $\left(x-\frac{x^3}{5}\right)^8$ =_____

(a) $\frac{14}{625}$ (b) $\frac{70}{625}$ (c) $\frac{14}{125}$ (d) $\frac{70}{125}$

7. Index number of middle term in expansion of $\left(1+a+\frac{a^2}{4}\right)^n$ is ______

(a) $\frac{n}{2}+1$ (b) $\frac{n+1}{2}$ (c) n+1 (d) $\frac{n+3}{2}$

8. In the expansion of $(x+y)^{13}$ the co efficients of 3 rd term and ____th terms are equal.

(a) 12 (b) 11 (c) 8 (d) 13

9. In the expansion of $(x-y)^{10}$, (co efficient of $x^7 y^3$) - (co-efficient of $x^3 y^7$) =_____

(a) $10 C_7$ (b) $2 .10 C_7$ (c) $10 C_7 + 10 C_1$ (d) 0

10. In the expansion of $\left(a^{\frac{2}{5}}+b^{\frac{1}{3}}\right)^{35}$ a $\neq$ b, the number of terms in which the power of a and b are integers are ______

(a) 1 (b) 2 (c) 3 (d) 4

11. 6 th term in the expansion of $\left(\frac{1}{x^{\frac{8}{3}}}+x^{2\log_{10}x}\right)^8$ is 5600 then x =_____

(a) 2 (b) $\sqrt{5}$ (c) $\sqrt{10}$ (d) 10

12. If p and Q are coefficients of x^n in expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ then_____

(a) P = Q (b) P = 2 Q (c) 2P = Q (d) P + Q = 0

13 The constant term in expansion of $\left(\dfrac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\dfrac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$ is ______

(a) 210 (b) 105 (c) 70 (d) 35

14 If $w \neq 1$ is quberoot of 1 then $\sum_{r=0}^{100} {}^{100}c_r\,(2+w^2)^{100-r}\,w^r$ =____

(a) -1 (b) 0 (c) 1 (d) 2

15. The Co-efficient of x^3 in $(1-x+x^2)^5$ is

(a) -30 (b) -20 (c) -10 (d) 30

16. It coefficients of middle terms in expansion of $(1+\lambda x)^8$ and $(1-\lambda x)^6$ are equal then λ =__

(a) $\dfrac{2}{7}$ (b) $\dfrac{-2}{7}$ (c) $\dfrac{-3}{7}$ (d) None of these

17. $(1-x)^m (1+x)^n = 1+a_1x + a_2x^2+......$; $m,n \in N$ and $a_1=a_2=10$ then (m,n) =______

(a) (45, 35) (b) (35, 20) (c) (35, 45) (d) (20, 45)

18. The expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$ is a plynomial of degree

(a) 5 (b) 6 (c) 7 (d) 8

19. The Co-efficient of x^r in expansion of $S =(x+3)^{n-1} + (x+3)^{n-2} (x+2) + (x+3)^{n-3} (x+2)^2 +...+ (x+2)^{n-1}$ is.................... .

(a) $nCr\,(3^r - 2^r)$ (b) $nCr\,(3^{n-r}-2^{n-r})$ (c) $3^{n-r}- 2^{n-r}$ (d) $3^{n-r} + 2^{n-r}$

20. Number of terms in expansion of $\left(\sqrt{x}+\sqrt{y}\right)^{10}+\left(\sqrt{x}-\sqrt{y}\right)^{10}$ is.................... .

(a) 5 (b) 6 (c) 7 (d) 8

21. The co-efficient of middle term in expansion of $(2x - 3y)^4$ is.................... .

(a) -96 (b) 216 (c) -216 (d) 96

22. Constant term in expansion of $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4$ is.................... .

(a) 5 (b) -5 (c) 4 (d) -4

23. $\left(\sqrt{2}+1\right)^5+\left(\sqrt{2}-1\right)^5$ =________

(a) 58 (b) $58\sqrt{2}$ (c) -58 (d) $-58\sqrt{2}$

24. If sum of Even terms are denoted by E and sum of odd terms are denoted by O in expansion of $(x + a)^n$ then $O^2 - E^2$ =_______

(a) $x^2 - a^2$ (b) $(x^2 - a^2)^n$ (c) $x^{2n} - a^{2n}$ (d) None of these

25. $\sum_{r=o}^{n} {}_nC_r \ 4^r$ = _____

(a) 4^n (b) 5^n (c) 4^{-n} (d) 5^{-n}

26. $(10.1)^5$ =_______

(a) 105101.501 (b) 105101.0501

(c) 105101.00501 (d) 105101.05001

27. If a,b $\in$ N, a $\neq$ b then for r $\in$ N, $a^n - b^n$ is divisible by ________

(a) a-b (b) b-a (c) both(a)and(b) (d) None of these

28. $(1.1)^{10000}$ ______1000

(a) > (b) < (c) = (d) None of these

29. 10th term in expansion of $\left(2x^2+\frac{1}{x^2}\right)^{25}$ is...........

(a) $\frac{1760}{x^2}$ (b) $\frac{1760}{x^3}$ (c) $\frac{880}{x^2}$ (d) $\frac{880}{x^3}$

30. The 11th term from last, in expansion of $\left(2x+\frac{1}{x^2}\right)^{25}$ is

(a) $25\,C_{15}\frac{2^{10}}{x^{20}}$ (b) $-25\,C_{15}\frac{2^{10}}{x^{20}}$

(c) $-25\,C_{14}\frac{2^{11}}{x^{11}}$ (d) $25\,C_{14}\frac{2^{11}}{x^{11}}$

31. 16th term and 17th term are equal in expansion of $(2+x)^{40}$ then x=______

(a) $\frac{17}{24}$ (b) $\frac{17}{12}$ (c) $\frac{34}{13}$ (d) $\frac{34}{15}$

32. 3rd term in expansion of $\left(\frac{1}{x}+x^{\log_{10}x}\right)^5$ is 1000 then x=______

(a) 10 (b) 100 (c) 1000 (d) None

33. $\left\{x^{\sqrt{\frac{1}{\log x+1}}}+x^{\frac{1}{12}}\right\}^6$ has 200 is as 4th term and x>1 then x = ______

(a) 10 (b) 100 (c) 1000 (d) None

34. $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{-\frac{1}{8}\log_3(5^{x-1}+1)}\right\}^{10}$ has 180 is as a 9th term then x =______

(a) 1 (b) 2 (c) 3 (d) 0

35. If the sum of co-efficient of first three terms in expansion of $\left(a-\frac{3}{a^2}\right)^m$, $m \in N$, $a \neq o$ is 559 then m = _____

(a) 10 (b) 11 (c) 12 (d) 13

36. Co-efficient of x^5 in expansion of $(1+2x)^6 (1 - x)^7$ is......................... .

(a) 150 (b) 171 (c) 192 (d) 161

37. Constant term in expansion of $\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$ is........................... .

(a) 190 (b) 200 (c) 210 (d) 220

38. If the ratio of co-efficients of three consecutive terms in expansion of $(1+x)^n$ is 1 : 7 : 42 then n = ______

(a) 35 (b) 45 (c) 55 (d) 65

39. If n is even natural number then middle term in expansion of $\left(x+\frac{1}{x}\right)^n$ is............... .

(a) $\frac{n!}{\left(\frac{n}{2}\right)!}$ (b) $\frac{n!}{\left(\frac{n}{2}!\right)^2}$ (c) $\frac{2(n!)}{\left(\frac{n}{2}!\right)^2}$ (d) $\frac{n!}{2\left(\frac{n}{2}!\right)^2}$

40. When 5^{20} is divided by 48 then remainder is______

(a) 46 (b) 47 (c) 48 (d) 49

41. The remainder when 2^{3n} -7n+4 is divided by 49 is ....

(a) 0 (b) 1 (c) 4 (d) 5

42. $nC_o + 2 \cdot nC_1 + 3 \cdot nC_2 + \text{---} + (n+1) \cdot nC_n =$ ________

(a) $(n+1)\ 2^{n-1}$ (b) $(n+2)\ 2^{n-1}$ (c) $(n+1)\ 2^{n}$ (d) $(n+1)\ 2^{n-1}$

43. The least positive remainder when 17^{30} is divided by 5 is

(a) 2 (b) 4 (c) 3 (d) 1

44. $nC_1 - nC_2 + nC_3 - \dots\dots + (-1)^n\ nC_n =$ _____

(a) 0 (b) -1 (c) n (d) 1

45. $13c_1 + 13c_2 + \dots\dots\dots + 13c_6 =$ ______

(a) $2^{13}-1$ (b) 2^{13} (c) $2^{12}-1$ (d) $2^{14}-1$

46. $10C_1 + 10C_3 + 10C_5 + \dots\dots\dots\dots + 10C_9 =$ _____

(a) 512 (b) 1024 (c) 2048 (d) 1023

47. If the sum of co-efficieuts of expansion $(m^2x^2 + 2mx + 1)^{31}$ is zero then m = _____

(a) 1 (b) -1 (c) 2 (d) -2

48. If $(1-x+x^2)^n = a_0 + a_1x + a_2x^2 + \text{-------} + a_{2n}\ x^{2n}$ then $a_0 + a_2 + a_4 + \text{-----} + a_{2n} =$ ______

(a) $\frac{3^n-1}{2}$ (b) $\frac{1-3^n}{2}$ (c) $\frac{3^n+1}{2}$ (d) $\frac{3^{n+1}}{2}$

49. Sum of Co-efficient of last 15 terms in expansion of $(1+x)^{29}$ is

(a) 2^{15} (b) 2^{30} (c) 2^{29} (d) 2^{28}

50. Remainder when 2^{2000} is divided by 17 is................. .

(a) 1 (b) 2 (c) 8 (d) 12

51. Co-efficient of x^{53} in $\sum_{r=o}^{100} 100\ C_r\ (x-5)^{100-r}\ 4^r$ is _______

(a) $100\ c_{53}$ (b) $100\ c_{48}$ (c) $-100\ c_{53}$ (d) $100\ c_{51}$

52. The interval in which x(>o) must lie so that the greatest term in the expansion of $(1+x)^{2n}$ has the greatest coefficient is

(a) $\left(\frac{n-1}{n}, \frac{n}{n-1}\right)$ (b) $\left(\frac{n}{n+1}, \frac{n+1}{n}\right)$

(c) $\left(\frac{n}{n+2}, \frac{n+2}{n}\right)$ (d) None

53. $n\,C_1 + 2.\,n\,C_2 + 3.\,n\,C_3$ +-----+ $n.\,n\,C_n$ =_____

(a) $n.\,2^{n-1}$ (b) $(n-1)\,2^{n-1}$ (c) $(n+1)\,2^{n-1}$ (d) $(n-1)\,2^n$

54. Let $x > -1$ then statement $(1+x)^n > 1+nx$ is true for

(a) $\forall\, n \in N$ (b) $\forall\, n > 1$

(c) $\forall\, n > 1$ and $x \neq o$ (d) $\forall\, n \in R$

55. $(1+x)^n = {}_nC_0 + {}_nC_1\, x + {}_nC_2\, x^2$ +----- $nC_n x^n$. and

$\frac{nC1}{nC0} + 2\frac{nC2}{nC1} + 3\frac{nC3}{nC2} + ... + \frac{nCn}{nCn-1} = \frac{1}{k} n(n+1)$ then K = ______

(a) 3 (b) 6 (c) 2 (d) 12

56. The coefficient of $\frac{1}{x}$ in expansion of $(1+x)^n \left(1+\frac{1}{x}\right)^n$ is _______

(a) $2n\,C_n$ (b) $2n\,C_{(n-1)}$ (c) $\frac{1}{2}$ (d) $2n\,C_0$

57. Co-efficient of x^4 in expansion of $(1+x+x^2+x^3)^{11}$ is _______

(a) 330 (b) 990 (c) 1040 (d) 900

58. The sum of coefficient of middle terms in expasision of $(1+x)^{2n-1}$ is_______

(a) $(2n-1)\,C_n$ (b) $(2n-1)\,C_{(n-1)}$ (c) $2n\,C_n$ (d) $2n\,C_{(n+1)}$

59. Remainder when $8^{2n} - 62^{2n+1}$ is divided by 9 is ________

(a) 0 (b) 2 (c) 7 (d) 8

60. The number of rational terms in expansion of $\left(1+\sqrt{2}+\sqrt[3]{5}\right)^{6}$ are

(a) 22 (b) 12 (c) 11 (d) 7

61. The greatest term in expansion of $(3+2x)^{50}$ is _____ ; where $x=\frac{1}{5}$

(a) $50\,C_7\,3^{43}\left(\frac{2}{5}\right)^{7}$ (b) $50\,C_6\,3^{44}\left(\frac{2}{5}\right)^{6}$

(c) $50\,C_{43}\,3^{7}\left(\frac{2}{5}\right)^{43}$ (d) $50\,C_{44}\,3^{6}\left(\frac{2}{5}\right)^{44}$

62. It A and B are coefficients of x^r and x^{n-r} respectively in expansion of $(1+x)^n$ then =_____

(a) A+B = n (b) A=B (c) $A+B = 2^n$ (d) $A-B = 2^n$

63. Coefficients of (2r+4)th term and (r-2)th term are equal in expansion of $(1+x)^{18}$ then r =_____

(a) 4 (b) 5 (c) 6 (d) 7

64. $(1+px)^n = 1+24x + 252x^2 + $ -----then................ .

(a) p = 3, n = 8 (b) p = 2, n = 6 (c) p = 3, n = 6 (d) p = 3, n = 5

65. Sun of coefficients in expansion of $(1+x - 3x^2)^{4331}$ is ____

(a) 1 (b) -1 (c) 0 (d) 2^{4330}

66. R = $\left(3+\sqrt{5}\right)^{2n}$ and f=R-[R], Where [ ] is an integer part function then R(1-f) =_____

(a) 2^{2n} (b) 4^{2n} (c) 8^{2n} (d) 4^{2n}

67. R = $\left(\sqrt{2}+1\right)^{2n+1}$, $n \in N$ and f= R - [R], Where [ ] is an integer part function then Rf=____

(a) 2^{2n+1} (b) 2^{2n-1} (c) $2^{2n}-1$ (d) 1

68. $\left(5^{\frac{1}{2}}+7^{\frac{1}{8}}\right)^{1024}$ has number of rational terms = ________

(a) 0 (b) 129 (c) 229 (d) 178

69. Number of rational terms in expansion of $\left(4^{\frac{1}{5}}+7^{\frac{1}{10}}\right)^{45}$ =_______

(a) 40 (b) 5 (c) 41 (d) 8

70. If sum of co-efficients in expansion of $\left(2x+\frac{1}{x}\right)^{n}$ is zero then find constant term

(a) 1120 (b) 512 (c) 1020 (d) 1050

71. Co-efficient of x^{-3} in expansion of $\left(x-\frac{a}{x}\right)^{11}$ is................... .

(a) $-792\ a^5$ (b) $-923\ a^7$ (c) $-792\ a^6$ (d) $-330\ a^7$

72. If coefficient of 2^{nd}, 3^{rd} and 4^{th} terms are in A.P for $(1+x)^n$ then n =____

(a) 28 (b) 14 (c) 7 (d) $\frac{7}{2}$

73. Number of terms in expansion of $(x_1+x_2+\text{-----}+x_r)^n$ is _______

(a) $(n + 1)\ C_4$ (b) $(n+r-1)\ C_{(r-1)}$ (c) $(n-r+1)\ C_{(r-1)}$ (d) $(n+r-1)C_r$

74. The greatest term in expansion of $(3+5x)^{15}$ is ____ where $x=\frac{1}{5}$

(a) $15\ C_3\ (3^{13})$ (b) $15\ C_4\ (3^{12})$ (c) $15\ C_4\ (3^{10})$ (d) $15\ C_4\ (3^{11})$

75. The greatest term in expansion of $(1+x)^{10}$ is ___ ; where $x = \frac{2}{3}$

(a) $210\left(\frac{3}{2}\right)^6$ (b) $210\left(\frac{2}{3}\right)^6$ (c) $210\left(\frac{2}{3}\right)^4$ (d) $210\left(\frac{3}{2}\right)^4$

76. $\binom{n-1}{1} + \binom{n-1}{2} + ... + \binom{n-1}{n-1}$ = ______ ; n>1

(a) 2^n-1 (b) 2^{n-2} (c) $2^{n-1}-1$ (d) 2^{n-1}

77. $\binom{n}{0} + 3\binom{n}{1} + 5\binom{n}{2} + ... + (2n+1)\binom{n}{n}$ = _____ ; $n \in N$

(a) $(n+2)2^n$ (b) $(n+1)2^n$ (c) $n2^n$ (d) $(n+1)2^{n+1}$

78. $\left(\frac{a^{1/3}}{b^{1/6}} + \frac{b^{1/2}}{a^{1/6}}\right)^{21}$ has same power of a and b for (r+1) th term then r = ______

(a) 8 (b) 9 (c) 10 (d) 11

79. coefficients of 5th, 6th and 7th terms are in A.P. for expansion of $(1+x)^n$ then n = ____

(a) 7 or 12 (b) -7 or 14 (c) 7 or 14 (d) -7 or 12

80. $\left[\left(\sqrt{2}+1\right)^8\right]$ =.................; where [ ] is integer part function.

(a) 1151 (b) 1152 (c) 1153 (d) 1154

81. $\left[\left(\sqrt{3}+1\right)^6\right]$ =.......................; where [ ] is integer part function..

(a) 415 (b) 416 (c) 417 (d) 418

82. $\dfrac{19^3+6^3+3(19)(6)(25)}{3^6+6(243)(2)+(15)(81)(4)+(20)(27)(8)+(15)(9)(16)+(6)(3)(32)+2^6}$ =____

(a) 1 (b) 5 (c) 2 (d) 6

83. If rth term in the expansion of $\left(ax^p+\dfrac{b}{x^q}\right)^n$ is constant, then r =_____

(a) $\dfrac{qn}{p+q}+1$ (b) $\dfrac{pn}{p-q}+1$ (c) $\dfrac{pn}{p+q}+1$ (d) $\dfrac{pn}{p-q}-1$

84. If rth term in expansion of $\left(2x^3+\dfrac{5}{x^2}\right)^{10}$ is constant then r =______

(a) 6 (b) 7 (c) 4 (d) 5

85. If rth term in the expansion of $\left(x+\dfrac{1}{2x}\right)^{12}$ is constant then r =_____

(a) 5 (b) 6 (c) 7 (d) 8

86. If 4th term in the expansion of $\left(px+\dfrac{1}{x}\right)^n$ is constant then n =_____

(a) 3 (b) 4 (c) 5 (d) 6

87. If middle term in the expansion of $\left(2-\dfrac{x^3}{3}\right)^7$ is $\dfrac{a}{27}x^9$ then a = ______

(a) 560 (b) -560 (c) $\dfrac{280}{3}$ (d) $-\dfrac{280}{3}$

88. If rth term in expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$, $x \neq o$ is constant then r = _____

(a) 6 (b) *7* (c) *8* (d) 9

89. The constant term in expansion of $\left(\frac{a}{x^q}+bx^p\right)^n$ is____ th term

(a) $\frac{qn}{p+q}+1$ (b) $\frac{pn}{p+q}+1$ (c) $\frac{pn}{p+q}+1$ (d) $\frac{pn}{p+q}-1$

90. _______th term is costant term in expansion of $\left(\frac{3}{x^2}+\frac{\sqrt{x}}{3}\right)^{10}$, $x \neq o$

(a) 4 (b) 7 (c) 8 (d) 9

91. If the sum of Co-efficient is 4096 in expansion of $(x+y)^n$ then the greatest Co-efficient is ______

(a) 792 (b) 924 (c) 1594 (d) 2990

92. Co-efficient of x^r is denoted by a_r in expansion of $(x+1)^{p+q}$ then

(a) $a_p = a_q$ (b) $a_p = -a_q$ (c) $a_p\ a_q = +1$ (d) None

93. If co-efficients of (r+2)th term and 3r th term are equal in expansion of $(1+x)^{2n}$, $n,r \in N$, r>1, n>2 then n =______

(a) 3r (b) 3r +1 (c) 2r (d) 2r +1

94. Numbers of rational terms in expansion $\left(3^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^{256}$ are ____

(a) 33 (b) 34 (c) 35 (d) 32

95. s(k) : 1+3+5+-----+ (2k-1) = 3+k^2 then which statemeut is true ?

(a) s(k) $\Rightarrow$ s (k+1) (c) s(k) $\Rightarrow$ s (k+1)

(c) s (1) is true (d) Result is proved by Principle of Mathematical induction

96. The co-efficients of the middle terms in the binomial expansions in powers of x of $(1+\propto x)^4$ and $(1-\propto x)^6$ is the same if $\propto$ equals

(a) $-\frac{3}{10}$ (b) $\frac{10}{3}$ (c) $\frac{-5}{3}$ (d) $\frac{3}{5}$

97. The Co-efficient of x^n in the expansion of $(1+x)(1-x)^n$ is

(a) $(-1)^{n-1}(n-1)^2$ (b) $(-1)^n(1-n)$

(c) n-1 (d) $(-1)^{n-1}n$

98. If the co-efficients of rth, (r+1) th and (r+2) th terms in the biromial expansion of $(1+y)^m$ are in A.P. then m and r satisfy the equation

(a) $m^2 - m(4r+1)+4r^2-2 = 0$ (b) $m^2 - (4r-1)m+4r^2+2 = 0$

(c) $m^2 - (4r-1)m+4r^2 - 2= 0$ (d) $m^2 - (4r+1)m+4r^2+2 = 0$

99. If the Co-efficient of x^7 in $\left(ax^2+\frac{a}{bx}\right)^{11}$ equals the co-efficients of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$ then

(a) $\frac{a}{b}=1$ (b) ab=1 (c) a-b=1 (d) a+b=1

100. If x is so small that terms with x^3 and higher powers of x may be neglected then $\frac{(1+x)^{\frac{3}{2}}-\left(1+\frac{x}{2}\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as............

(a) $\frac{-3}{8}x^2$ (b) $\frac{1}{2}x-\frac{3}{8}x^2$ (c) $1-\frac{3}{8}x^2$ (d) $3x+\frac{3}{8}x^2$

101. In the binomial expansion of $(a-b)^n$, $n \geq o$, the sum of 5^{th} and 6^{th} terms is zero then $\frac{a}{b}$ = ___________

(a) $\frac{5}{n-4}$ (b) $\frac{6}{n-5}$ (c) $\frac{n-5}{6}$ (d) $\frac{n-4}{5}$

Hints

1. $T_{r+1} = \binom{n}{r} a^{n-r} b^r$; $0 \le r \le n$

$= \binom{n}{r} 2^{n-r} \left(\frac{x}{3}\right)^r = \binom{n}{r} \frac{2^{n-r}}{3^r} x^r$

Coefficieut of x^7 = coefficieut of x^8

$\therefore \binom{n}{7} \frac{2^{n-7}}{37} = \binom{n}{8} \frac{2^{n-8}}{38}$

$\therefore \binom{n}{7} 2.3 = \binom{n}{8} \Rightarrow \frac{n-7}{8} = 6 \Rightarrow n = 55$

2. $T_{r+1} = \binom{n}{r} a^{n-r} b^r$; $0 \le r \le n$

$= \binom{9}{r} \left(\frac{3x^2}{2}\right)^{9-r} \left(-\frac{1}{3x}\right)^r$

$= \binom{9}{r} \frac{3^{9-2r}}{2^{9-r}} (-1)^r x^{18-3r}$ ________(1)

For constant term. 18 - 3r = 0 => r = 6

From (1) $T_{6+1} = \binom{9}{6} \frac{3^{9-12}}{2^{9-6}} (-1)^6 = \binom{9}{6} \frac{3^{-3}}{2^3} (+1) = \frac{7}{18}$

3. Because of n=7, $\frac{7+1}{2} = 4$ and $\frac{7+3}{2} = 5$ => T_4 and T_5 are middle terms

Now $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{7}{r} (2)^{7-r} \left(-\frac{x^3}{x}\right)^r = 7\,c_r\, \frac{2^{7-r}}{3^r} (-1)^r\, x^{3r}$

$\therefore\ T_4 = 7C_3\, \frac{2^4}{3^3}\, (-1)^3\, x^9 = -\frac{560}{27} x^9$

and $T_5 = 7C_4\, \frac{2^3}{3^4}\, (-1)^4\, x^{12} = \frac{280}{81} x^{12}$

4. Because of n=12 is even, $\frac{n-2}{2} = 7 \Rightarrow T_7$ is middle term

$\therefore\ T4 = 7c_3 \binom{12}{6} a^{12-6}\, b^6 = 12\, C_6 \left(\frac{2}{x}\right)^6 (-3xy)^6$

7. $\left(1 + a + \frac{a^2}{4}\right)^n = \left\{\left(1 + \frac{a}{2}\right)^2\right\}^n = \left(1 + \frac{a}{2}\right)^{2n}$

here 2n is even, $\frac{2n+2}{2}$ = (n+1) th term is middle term.

8. Here n = 13,

$\therefore$ Total terms = 13 + 1 = 14

$\therefore$ coefficieut of 3 rd term = coefficieut of third term from last

= 14-2

=12

10. $T_{r+1} = n\,C_r\, a^{n-r}\, b^r$; $0 \le r \le n$

$= 35\,C_r \left(a^{\frac{2}{5}}\right)^{n-r} \left(b^{\frac{1}{3}}\right)^{r}$

$= 35\,C_r\; a^{14-\frac{2r}{5}} \cdot b^{\frac{r}{3}}$

$\therefore$ r will be multiple of 5 and 3 both.

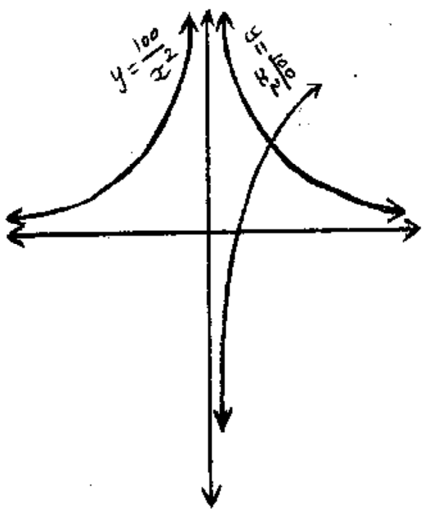
$\therefore$ r will be multiple of 15

$\therefore$ r = 15 K ; $0 \le r \le 35$

$\therefore$ r = 0, 15, 30

$\therefore$ 1st, 16th , and 31th has integer power of a and b

11. $T_6 = t_{5+1} = 8\,C_5 \left(\frac{1}{x^{\frac{8}{3}}}\right)^{8-5} (x^2 \log_{10} x)^5$



$5600 = \frac{8i}{5i\;3i} \left(\frac{1}{x^8}\right) x^2 \log_{10} x$

$5600 = 56\, x^2 (\log_{10} x)^5$

$\frac{100}{x^2} = (\log_{10} x)^5$ ________(1)

$\frac{100}{x^2} = y$; $y = (\log_{10} x)^5$

From graph, The point of intersection is (10,1)

$\therefore$ x=10

OR C heck equation o By putting Answers.

12. p = $2nC_n$; Q = $(2n - 1)\,C_n$

$$\therefore \frac{p}{Q} = \frac{2n\,C_n}{(2n-1)\,C_n} = \frac{(2n)!}{(n!)\,(n!)} \times \frac{n!\,(n-1)!}{(2n-1)!} = 2 \Rightarrow P = 2Q$$

13. $$\left\{ \frac{\left(x^{\frac{1}{3}} + 1^{\frac{1}{3}}\right)\left(x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1\right)}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}{\sqrt{x}\left(\sqrt{x}-1\right)} \right\}^{10}$$

$$= \left\{ x^{\frac{1}{3}} + 1 - \left(1 + x^{\frac{-1}{2}}\right) \right\}^{10}$$

$$= \left\{ x^{\frac{1}{3}} - x^{\frac{-1}{2}} \right\}^{10} \Rightarrow T_{r+1} = n\,C_r\,(a)^{n-r}\;b^r\,(-1)^r$$

$$= 10\,C_r \left(x^{\frac{1}{3}}\right)^{10-r} \left(x^{\frac{-1}{2}}\right)^{r} (-1)^r$$

$$= 10\,C_r\; x^{\frac{10-r}{3}}\; x^{\frac{-r}{2}}\,(-1)^r$$

$$= 10\,C_r\; x^{\frac{20-5r}{6}}\,(-1)^r$$

Now $\frac{20-5r}{6} = 0 \Rightarrow$ r = 4

14 $S = (2+\omega^2+\omega)^{100}$

$= (1+1+\omega+\omega^2)^{100}$

$= (1+0)^{100} = 1$

15. $(1-x+x^2)^5 = (1+x(x-1))^5$

$= 5C_o + 5C_1\, x(x-1) + 5C_2\, x^2 (x-1)^2 + 5C_3\, x^3 (x-1)^3 + ...$ coefficients of x^3

$= -2 .5C_2 - 5C_3$

$= -20-10$

$= -30$

17. $(1-x)^m (1+x)^n = (1-mx+m\,C_2x^2 + ...)(1+nx+nC_2\, x^2 + ...)$

$= 1 + (n-m)x + (nC_2+mC_2-mn)x^2 + ...$

$= 1 + a_1\, x + a_1\, x^2 + ...$

$\therefore$ n - m = 10 $\Rightarrow$ n = m + 10

and $nC_2 + mC_2 - mn = 10 \Rightarrow \frac{1}{2}n(n-1) + \frac{1}{2}m(m-1)-mn = 10$

$\Rightarrow n(n-1) + m(m-1) -2mn = 20$

$\Rightarrow (m+10)(m+9) + m(m-1) -2m(m+10) = 20$

$\Rightarrow m^2 +19m + 90 + m^2 - m-2m^2 - 20m = 20$

$\Rightarrow -2m=-70 \Rightarrow m= 35$

$\therefore$ n = 45

18 $(a+b)^5 + (a-b)^5 = 2[a^5+5C_2\, a^3 b^2 + 5C_4\, ab^4]$

$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5 = 2[x^5 +10x^3(x^3-1) + 5x(x^3-1)^2]$

is poloynomial of degree 7

19. S = $\dfrac{(x+3)^{n-1}\left[1-\left(\dfrac{x+2}{x+3}\right)^n\right]}{1-\left(\dfrac{x+2}{x+3}\right)}$ $\left[\because \text{For G.P } S_n = \dfrac{a(1-r^n)}{1-r}\right]$

= $(3+x)^n - (2+x)^n$

$\therefore$ coefficient of $x^r = n\,C_r\,(3^{n-r} - 2^{n-r})$

22. $\left(1+\dfrac{x}{2}-\dfrac{2}{x}\right)^4 = \left\{1+\left(\dfrac{x}{2}-\dfrac{2}{x}\right)\right\}^4$

$= 4\,C_o + 4\,C_1\left(\dfrac{x}{2}-\dfrac{2}{x}\right) + 4\,C_2\left(\dfrac{x}{2}-\dfrac{2}{x}\right)^2 + 4\,C_3\left(\dfrac{x}{2}-\dfrac{2}{x}\right)^3 + 4\,C_4\left(\dfrac{x}{2}-\dfrac{2}{x}\right)^4$

$= -5 - 4x + \dfrac{x^2}{2} + \dfrac{x^3}{2} + \dfrac{x^4}{16} + \dfrac{16}{x} + \dfrac{8}{x^2} - \dfrac{32}{x^3} + \dfrac{16}{x^4}$

24. $O = n\,C_o\,x^n a^o + n\,C_2\,x^{n-2} a^2$ +-------

$E = n\,C_1\,x^{n-1} a + n\,C_3\,x^{n-3} a^3$ +-------

$\therefore$ $O + E = (x+a)^n$

$o - E = (x-a)^n$

So that $(O + E)(O - E) = (x+a)^n (x-a)^n$

$\therefore$ $O^2 - E^2 = (x^2 - a^2)^n$

25. $(1+x)^n = \sum_{r=o}^{n} nC_r x^r$ By Putting x=4

$(1+4)^n = \sum_{r=o}^{n} nC_r 4^n$

26. $(10.1)^5 = (10+0.1)^5$ then apply binomial theorem

32. $T_3 = T_{2+1} = 5\,C_2 \left(\frac{1}{x}\right)^{5-2} \left(x^{\log 10^x}\right)^2 = 1000$

$\Rightarrow 10\ x^{-3}\ x^{2\log}\ 10^x = 1000$

$\Rightarrow x^{2\log}10^{x-3} = 10^2$

$\Rightarrow (2\ \log_{10}x-3)\ roy_{10}x = 2$

$\Rightarrow (2y-3)y = 2\ [\because \log_{10}x=y]$

$\Rightarrow 2y^2 - 3y - 2 = 0$

$\Rightarrow y = 2 \Rightarrow \log_{10}\text{-}x = 2 \Rightarrow x=10^2 \Rightarrow x=100$

33. $T_4 = 200$

$T_{3+1} = 200$

$$6\,C_3 \left\{ x\sqrt{\frac{1}{\log_{10} x+1}} \right\}^{6-3} \left(x^{\frac{1}{12}} \right)^3 = 200$$

$$20 \left\{ x^{\frac{1}{\log_{10} x+1}} \right\}^{\frac{3}{2}} x^{\frac{1}{4}} = 200$$

$$x^{\frac{3}{2(\log_{10} x+1)}+\frac{1}{4}} = 10$$

$$\frac{3}{2(109_{10}x+1)} + \frac{1}{4} = 109_x 10 = \frac{1}{109_{10}x}$$

$$\frac{3}{2(y+1)} + \frac{1}{4} = \frac{1}{y}$$

$y^2 + 3y - 4 = 0 \Rightarrow y = 1,-4$

$\log_{10}x=1$ [-4 is not possible]

$\therefore$ $x=10^1$

$\therefore$ $\boxed{x=10}$

34. $T_9 = {}^{10}C_8 \left\{\sqrt{25^{x-1}+7}\right\}^{10-8} \left\{\left(5^{x-1}+1\right)^{\frac{-1}{8}}\right\}^8 = 180$

$\frac{25^{x-1}+7}{5^{x-1}+1} = 4 \Rightarrow \frac{y^2+7}{y+1} = 4$ [$\because$ $y = 5^{x-1}$]

$\Rightarrow$ $y^2-4y+3 = 0$

$\Rightarrow$ $y = 3, -1$

$\Rightarrow$ $x = \log_5 15$ or $x = 1$

35. $m C_o + (-3)\, m C_1 + (9)\, m C_2 = 559$; $m \in N$

$1 - 3m + \frac{9}{2} m(m-1) = 559$

$3m^2 - 5m - 372 = 0$

$(m-12)(3m+31) = 0$

$\therefore$ $m = 12$

38. $n C_{r-1} : n C_r : n C_{r+1} = 1 : 7 : 42$

$\frac{n C_{r-1}}{n C_r} = \frac{1}{7} \Rightarrow \frac{r}{n-r+1} = \frac{1}{7} \Rightarrow$ $n - 8r + 1 = 0$ ______(1)

and $\frac{n C_r}{n C_{r+1}} = \frac{7}{42} \Rightarrow \frac{r+1}{n-r} = \frac{1}{6} \Rightarrow$ $n-7r-6 = 0$ _____(2)

By solving equation (1) & (2) we get $n = 55$, $r=7$

40. $5^4 = 625 = 13\ (48) + 1$

$\therefore 5^4 = ky+1$; $k = 13$, $y = 48$

$\therefore (5^4)^5 = (ky+1)^5$

$5^{20} = 1 + 48m,\ m \in N$

42. See theory

44. See theory

47. $(m^2 x^2 + 2mx+1)^{31} = (mx + 1)^{62} = a_0 + a_1 x + a_2 x^2 + \text{-------}$

Put $x=1$

$(m+1)^{62} = a_0 + a_1 + a_2 + \text{-------} = 0$

$\therefore m+1 = 0 \Rightarrow m = -1$

48. By Putting $x = 1$, $1 = a_0 + a_1 + a_2 + \text{-------} + a_{2n}$ ______(1)

Putting $x = -1$, $3^n = a_0 - a_1 + a_2 - \ldots\ldots\ldots\ldots + a_{2n}$ _______(2)

By adding equation (1) & (2) ×

We get $3^n - 1 = 2\ [a_0 + a_2 + \text{-------} + a_{2n}]$

49. $S = 29\,C_{15} + 29\,C_{16} + \text{-----} + 29\,C_{29}$

$S = 29\,C_{14} + 29\,C_{13} + \text{-----} + 29\,C_o$

$\therefore 2S = 2^{29} \Rightarrow S = 2^{28}$

52. $2n\,C_{(n-1)}\ x^{n-1} < 2nC_n\, x^n$ and $2nC_{(n+1)}\ x^{n+2} < 2nC_n\ x^n$

$$\therefore \frac{2n\,C_{(n-1)}}{2n\,C_n} < x < \frac{2n\,C_n}{2n\,C_{(n+1)}}$$

53. See theorey

55. here r $\dfrac{n\,C_r}{n\,C_4 - 1} = n - r + 1$

$$\therefore \sum_{r=1}^{n} r\frac{nC_r}{nC_{r-1}} = \sum_{r=1}^{n} [(n+1) - r] = n(n+1) - \frac{n}{2}(n+1)$$

$$= \frac{n}{2}(n+1)$$

56. $(1+x)^n \left(1+\frac{1}{x}\right)^n = \frac{(1+x)^{2n}}{x^n}$

$\therefore$ Co-efficient of x^{-1} in expansion of $(1+x)^n \left(1+\frac{1}{x}\right)^n$ = Co-efficient of x^{n-1} in expansion of $(1+x)^{2n}$

$= 2n\, C_{(n-1)}$

57. Simplify $(1+x)^{11} (1+x^2)^{11}$

58. Here nth and (n+1) th terms are middle terms and sum of coefficients

$(2n-1)\, C_{(n-1)} + (2n-1)\, C_n = 2n\, C_n$

59. $8^{2n} - 62^{2n+1} = (9-1)^{2n} - (63-1)^{2n+1}$

60. General term of expansion = $\frac{6\,!}{r!\, s!\, (6-r-s)!}(1)^{6-r-s}\left(\sqrt{2}\right)^r\left(\sqrt[3]{5}\right)^s$

$$= \frac{6\,!}{r!\, s!\, (6-r-s)!} 2^{\frac{r}{2}}\ 5^{\frac{s}{3}}$$

Here $2^{\frac{r}{2}}$ and $5^{\frac{s}{3}}$ must be rational

$\therefore$ r is a multiple of 2 and s is a multiple of 3

$\therefore$ $0 \le r \le 6$ and $0 \le s \le 6$

i. e $0 \le r+s \le 6$

$\therefore$ If r = 0 then s = 0, 3, 6 $\Rightarrow$ 3 terms

r = 2 then s = 0, 3, $\Rightarrow$ 2 terms

r = 4 then s = 0 $\Rightarrow$ 1 terms

r = 6 then s = 0 $\Rightarrow$ 1 terms

$\therefore$ Total terms = 3+2+1+1 = 07

61. Here K= $\frac{(n+1)y}{x+y} = \frac{(50+1)2x}{3+2x} = \frac{(51)\left(\frac{2}{5}\right)}{3+\frac{2}{5}} = 6$

$\therefore$ The greatest terms are T_6 and T_7

63. $18\ C_{(2r+3)} = 18\ C_{(r-3)}$

$\therefore$ 2r+3 = r-3 or 2r + 3 + r - 3 = 18

$\therefore$ r = -6 or r = 6

66. R = $\left(3+\sqrt{5}\right)^{2n}$ Suppose F = $\left(3-\sqrt{5}\right)^{2n}$

Now $0 < 3 - \sqrt{5} < 1 \Rightarrow 0 < F < 1$

$\therefore$ R + F = $\left(3+\sqrt{5}\right)^{2n} + \left(3-\sqrt{5}\right)^{2n}$

= 2 {Any Integer}

Which is even number

$\therefore$ f + [R] + F is ever number

Now $0 \le f < 1$ and $0 < F < 1 \Rightarrow 0 < f + F < 2$

$\therefore$ f + F = 1

$\therefore$ F = 1 -f

Now R (1-f) = RF

$= \left(3+\sqrt{5}\right)^{2n}\left(3-\sqrt{5}\right)^{2n}$

$= (9-5)^{2n}$

$= 4^{2n}$

67. $F = \left(\sqrt{2}-1\right)^{2n+1}$

$\therefore$ R - F = 2m,

$\therefore$ [R] + f - F = 2m

f - F = 2m-[R] is an integer = 0

$\therefore$ f = F

68. $T_{r+1} = 1024\,C_r\; 5^{512-\frac{r}{2}}\; 7^{\frac{r}{8}}$

$= \{1024\,C_r\,5^{512-r}\}\; 5^{\frac{r}{2}}\; 7^{\frac{r}{8}}$

$\therefore$ r will be multiple of 2 and 8 i.e. multiple of 8 and $0 \le r \le 1024$

$\therefore$ r has 129 values.

79. $t_{r+1} = 21\,Cr\left(\dfrac{a^{\frac{1}{3}}}{b^{\frac{1}{6}}}\right)^{21-r}\left(\dfrac{b^{\frac{1}{2}}}{a^{\frac{1}{6}}}\right)^{r}$

$= 21\,C_r\; a^{7-\frac{r}{2}}\; b^{\frac{2r}{3}-\frac{7}{2}}$

$\therefore\; 7-\dfrac{r}{2} = \dfrac{2r}{3}-\dfrac{7}{2} \Rightarrow r=9$

81. Suppose R+f = $\left(\sqrt{2}+1\right)^8$ Where R is an integer and 0 $\leq$ f < 1

$\therefore$ R = $\left[\left(\sqrt{2}+1\right)^8\right]$

Also 0 < $\left(\sqrt{2}+1\right)$ < 1 $\Rightarrow$ 0 < F = $\left(\sqrt{2}-1\right)^8$

$$= 2\left[8C_o\left(\sqrt{2}\right)^8 + 8C_2\left(\sqrt{2}\right)^6 + 8C_4\left(\sqrt{2}\right)^4 + 8C_6\left(\sqrt{2}\right)^2 + 8C_8\left(\sqrt{2}\right)^0\right]$$

$$= 2[577] = 1154$$

Also 0 $\leq$ f < 1 and 0 < F < 1 $\Rightarrow$ 0 < f + F < 2 $\Rightarrow$ f + F =1

$\therefore$ R + 1 = 1154 $\Rightarrow$ R = 1153

83. $\dfrac{(19+6)^3}{(3+2)^6} = 1$

85. Index = $\dfrac{\text{n p}}{p+q} + 1 = \dfrac{10\times 3}{3+2} + 1$ = 6+1 = 7th term

99. From r = $\dfrac{1}{2}\left(n+\sqrt{n+2}\right)$, r = $\dfrac{1}{2}\left(m+\sqrt{m+2}\right)$

2r - m = $\sqrt{m+2}$

$4r^2 - 4rm + m^2 = m+2$

$m^2 - 4rm - m + 4r^2 - 2 = 0$

$m^2 - m(4r+1) + 4r^2 - 2 = 0$

Answers

1-A	2-B	3-C	4-C	5-A	6-B	7-C	8-A	9-D	10-C
11-D	12-B	13-A	14-C	15-A	16-B	17-C	18-C	19-B	20-A
21-B	22-B	23-B	24-B	25-B	26-C	27-C	28-A	29-B	30-B
31-B	32-B	33-A	34-A	35-C	36-B	37-C	38-C	39-B	40-C
41-D	42-B	43-B	44-D	45-C	46-A	47-B	48-C	49-D	50-A
51-C	52-B	53-A	54-C	55-C	56-B	57-B	58-C	59-B	60-D
61-B	62-B	63-C	64-A	65-B	66-B	67-D	68-B	69-C	70-A
71-D	72-C	73-B	74-D	75-C	76-C	77-B	78-B	79-C	80-C
81-B	82-A	83-C	84-B	85-C	86-D	87-B	88-B	89-A	90-D
91-B	92-A	93-C	94-A	95-B	96-A	97-A	98-A	99-B	100-A
101-D									

Unit - 7

Sequence and Series

Important Points

Sequence

Any function $f : N \to R$ is called real sequence.

Any function $f : N \to C$ is called complex sequence.

Any function $f : \{1,2,3, ... n\} \to X$ is a finite sequence of a set x $(x \neq \phi)$.

A sequence is usually written as $\{f(n)\}$ or $\{a_n\}$ or $\{T_n\}$ or $\{t_n\}$, $f(n)$ or a_n or T_n is called the nth term of the sequence.

for example $1, \frac{1}{2}, \frac{1}{3}, ...$ is a sequence of whose nth term is $\frac{1}{n}$. This sequence is usually written as $\left\{\frac{1}{n}\right\}$

Series

For any sequence a_1, a_2, a_3 .... the sequence $\{a_1+a_2+a_3+ ... +a_n\}$ is called a series $(a_i \in C, \forall i)$

A series is finite or infinite according as the number of terms added is finite or infinite

Progressions (Sequence)

Sequences whose terms follow certain patterns are called progressions

Arithmetic Progression (A.P.)

A sequence $a_1, a_2, a_3, ...$ is said to be an arithmetic progression iff $a_{n+1} - a_n$ = non-zero constant, for all n. Hear this constant is called the common difference of the A.P. and is usually denoted by ' d '

A general A.P. is a, a + d, a + 2d, ..., a + (n-1) d... and $\boxed{T_n = a + (n-1) d}$ is the general term of A.P. Hear a is the first term of A.P. and d is the common difference of A.P.

Note that $d = T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = ...$

* nth term from the end = l - (n -1) d   where l = last term

Sum of the first n terms of an A.P.

$S_n = a + (a+d) + (a + 2d) + ... + [a + (n - 1) d]$

$= \frac{n}{2} [2a + (n - 1) d]$

$= \frac{n}{2} (a+l)$ where $\ell = T_n$ = last term

n = number of terms

a = first term

* Sum of nth term from the end = $[2l + (n - 1) d]$

* If the all terms of an A.P. are increased, decreased, multiplied and divided by the same non - zero constant, then they remain in A.P.

* In an A.P. sum of terms equidistant from the beginning and end is constant

i.e. $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2}$ = ...

* Three consecutive numbers in A.P. can be taken as a-d, a, a+d

* Four consecutive numbers in A.P. can be taken as a-3d, a-d, a+d, a+3d

* 7Five consecutive numbers in A.P. can be taken as a-2d, a-d, a, a+d, a+2d

* Six consecutive numbers in A.P. can be taken as a-5d, a-3d, a-d, a+d, a+3d, a+5d.

Arithmetic Means (A.M.)

If a, A, b are in A.P. then A is called by arithmetic mean.

Hear $\boxed{A = \frac{a+b}{2}}$

n Arithmetic Mean between a and b

$A_1, A_2, A_3, ... A_n$ are said to be n A.M.s between two numbers a and b iff a, $A_1, A_2, ..., A_n$, b are in A.P. Hence $A_1 = a+d$, $A_2 = a + 2d$ ... ,$A_n = a + nd$

where $d = \frac{b-a}{n+1}$

* $A_1 + A_2 + ... + A_n = nA$ where $A = \frac{a+b}{2}$

Harmonic Progression

Non - zero numbers $a_1, a_2, a_3, ..., a_n ...$ are said to be a harmonic progression (H.P.) iff

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots, \frac{1}{a_n} \cdots$, are in A.P.

Harmonic mean (H)

If a, H, b are in H.P., then H is called the harmonic mean (H.M.) berween a and b

Hear $\boxed{H = \frac{2ab}{a+b}}$

n Harmonic mean between a and b

If a, $H_1, H_2, ..., H_n$, b are in H.P., then $H_1, H_2, ... H_n$ are called n harmonic mean between a and b

Geometric Progression (G.P.)

A sequence $a_1, a_2, a_3, ..., a_n, ...$ of non zero numbers is said to be a geometric progression (G.P.)

iff $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = ... = \frac{a_{n+1}}{a_n}$ = a constant for all $n \in N$

This constant is called the common ratio of the G.P. and it is denoted by 'r'

A general G.P. is a, ar, ar^2, ... ,ar^{n-1},...

nth term of G.P. is $\boxed{T_n = ar^{n-1}}$

Sum of a G.P.

S_n = sum of first n terms of the G.P.

$= a + ar + ar^2 + ... + ar^{n-1}$

$= a\frac{(r^n - 1)}{r-1}$ if r > 1

$$= a\frac{\left(1-r^n\right)}{1-r} \quad \text{if } r < 1$$

$= na$ if $r = 1$

$S = a + ar + ar^2 + \ldots$ up to infinity

$$= \frac{a}{1-r} \quad \text{where } -1 < r < 1$$

Geometric Mean (G.M.)

If three positive real numbers a, G, b are in G. P. then G is called the geometric mean between a and b

Hear $\boxed{G = \sqrt{ab}}$ $\therefore G^2 = ab$

n Geometric Means

Positive real numbers $G_1, G_2, G_3, \ldots, G_n$ are said to be n G.M.s between two positive numbers a and b iff a, $G_1, G_2, \ldots, G_n$, b are in G.P.

If r is the common ration of this G.P., then

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} \text{ and } G_1 = ar,\ G_2 = ar^2, \ldots, G_n = ar^n$$

$$\text{Hear } G_1 . G_2 . G_3 . \ldots . G_n = \left(\sqrt{ab}\right)^n = (ab)^{\frac{n}{2}} = G^n$$

* If each term of a G.P. is multiplied or divided by a non - zero number, the resulting progres sion is also a G.P.

* Three numbers in G.P. can be taken as $\frac{a}{r}$, a, ar

* Four numbers in G.P. can be taken as $\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3

* Five numbers in G.P. can be taken as $\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2

Arithmetico Geometric Series (A.G.P.)

If P_1, P_2, P_3, ...be an A.P. and a_1, a_2, a_3, ... be a G.P. then $p_1 q_1, p_2 q_2, p_3 q_3$, ... is said to be an arithmetico geometric progression. A general A.G.P. is a, (a+d) r, $(a+2d)r^2$, $(a+3d)r^3$,...

Sum of an A.G.P.

$$S_n = \Sigma\left\{\left(a+(n-1)d\right)r^{n-1}\right\} = \frac{a}{1-r} + \frac{dr\left(1-r^{n-1}\right)}{(1-r)^2} - \frac{\{a+(n-1)d\}r^n}{1-r}, \ (r \neq 1)$$

$$S_\infty = \lim_{n\to\infty} S_n = \frac{a}{1-r} + \frac{dr}{(1-r)^2}, \ (-1 < r < 1)$$

T_n = n th term of A.G.P. = $\{a + (n-1)d\}r^{n-1}$

Series of natural numbers

$$\sum n = \sum_{r=1}^{n} r = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$

$$\sum n^2 = \sum_{r=1}^{n} r^2 = 1^2 + 2^2 + 3^2 + ...+ n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum n^3 = \sum_{r=1}^{n} r^3 = 1^3 + 2^3 + 3^3 + ...+ n^3 = \frac{n^2(n+1)^2}{4}$$

* If the formula of S_n is given we can obtain the formula for the corresponding sequence $\{a_n\}$ by $a_1 = s_1$ and $\forall$ $n \geq 2$, $a_n = s_n - s_{n-1}$

* A series is an A.P. iff its nth term is a linear expression in n.

* A sequence is both an A.P. and a G.P. iff it is a constant sequence.

* A series is an A.P. iff $S_n = \frac{n}{2}$ [2a + (n-1)d] is pure quadratic expression in n,

with no constant term.

* $A \geq G \geq H$

* In an A.P. of finitely many terms, sum of terms equidistant from the beginning and end is constant equal to the sum of the first and last terms.

* In a G.P. of finitely many terms, the product of terms equidistant from the beginning and end is constant equal to the sum of the first and last terms.

* An A.M. of n real numbers $a_1, a_2, a_3, ..., a_n$ is $A = \dfrac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$

* A G.M of n real numbers $a_1, a_2, a_3, ..., a_n$ ($a_i > 0$, i = 1, 2, ... , n) is $G = (a_1.a_2....a_n)^{\frac{1}{n}}$

* If $a_1, a_2, a_3, ...$ and $b_1, b_2, b_3, ...$ are in G.P.s then $a_1b_1, a_2b_2, a_3b_3, ...$ are in G.P. also.

* If $a_1, a_2, a_3, ... a_n$ are in G.P. $a_i > 0$, i = 1, 2, ...n then

$a_2 = \sqrt{a_1\ a_3}$, $a_3 = \sqrt{a_2\ a_4} = \sqrt{a_1\ a_5}$, $a_4 = \sqrt{a_3\ a_5} = \sqrt{a_2\ a_6} =$

$\sqrt{a_1\ a_7} = \ldots$ $a_{n-1} = \sqrt{a_{n-2}\ a_n}$ In sort $a_r = \sqrt{a_{r-k}\ a_{r+k}}$

where k = 0, 1, 2, ... n-r and $k \leq r-1$, r = 1 2, ... n-1

QUESTION BANK

1. If the 1st term and common ratio of a G.P. are 1 and 2 respectively then

   $s_1 + s_3 + s_5 + ... + s_{2n-1}$ = ___________

   (A) $\frac{1}{3}$ $(2^{2n}$-5n+4)   (B) $\frac{1}{3}$ $(2^{2n+1}$-5n)

   (C) $\frac{1}{3}$ $(2^{2n+1}$-3n-2)   (D) $\frac{1}{3}$ $(2^{2n+1}-5n^2)$

2. $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+...100 \text{ terms}$ = _______

   (A) $2^{100} + 99$ (B) $2^{-100} + 99$ (C) $2^{-101} + 100$ (D) $2^{-99} + 99$

3. If for the triangle whose perimeter is 37 cms and length of sides are in G.P. also the length of the smallest side is 9 cms then length of remaining two sides are ___ and __

   (A) 12, 16   (B) 14, 14   (C) 10, 18   (D) 15, 13

4. Find a, b and c between 2 and 18 such that a+b+c=25, 2,a,b are in A.P. and b,c, 18 are in G.P.

   (A) 5, 8, 12   (B) 4, 8, 13   (C) 3, 9, 13   (D) 5, 9, 11

5 Find out four numbers such that, first three numbers are in G.P., last three numbers are in A.P. having common difference 6, first and last numbers are same.

   (A) 8, 4, 2, 8   (B) -8, 4, -2, -8   (C) 8, -4, 2, 8   (D) -8, -4, -2, -8

6. If the A.M. of two numbers a and b is equal to $\sqrt{10}$ times their G.M. then $\frac{a-b}{a+b}$ =___

   (A) $\frac{\sqrt{10}}{3}$ (B) $3\sqrt{10}$ (C) $\frac{9}{10}$ (D) $\frac{3}{\sqrt{10}}$

7. If the harmonic mean and geometric mean of two numbers a and b are 4 and $3\sqrt{2}$ respectively then the interval [a, b] = _______

(A) [3, 6] (B) [2, 7] (C) [4, 5] (D) [1, 8]

8. A.M of the three numbers which are in G.P. is $\frac{14}{3}$ If adding 1 in first and second number and subtracting 1 from the third number, resulting numbers are in A.P. then the sum of the squares of original three numbers is ______

(A) 91 (B) 80 (C) 84 (D) 88

9. If the H.M. of a and c is b, G.M. of b and d is c and A.M. of c and e is d, then the G.M. of a and e is ______

(A) b (B) c (C) d (D) ae

10. If a, b, c are in A.P. and geometric means of ac and ab, ab and bc, ca nad cb are d, e, f respectively then d^2, e^2, f^2 are in _____

(A) A. P. (B) G. P. (C) H. P. (D) A. G. P.

11. If two arithmetic means A_1, A_2, two geometric means G_1, G_2 and two harmonic means H_1, H_2 are inserted between two numbers p and q then _____

(A) $\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2}$ (B) $\frac{G_1 + G_2}{H_1 + H_2} = \frac{A_1 A_2}{H_1 H_2}$

(C) $\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 - A_2}{H_1 - H_2}$ (D) $(A_1 + A_2)(H_1 + H_2) = G_1 G_2 H_1 H_2$

12. $\Sigma\left(\Sigma 2^{1-n}\right)$ = _______

(A) $2n - 2 + 2^{1-n}$ (B) $2n - 2 + 2^{n-1}$

(C) $2n - 2 + 2^{-n}$ (D) $2n - 2 + 2^{n+1}$

13 If $(666 \ldots n \text{ times})^2 + (8888 \ldots n \text{ times}) = (4444 \ldots K \text{ times})$ then K = ______

(A) n + 1 (B) n (C) 2n (D) n^2

14 If $(m+1)^{th}$, $(n+1)^{th}$ and $(r+1)^{th}$ terms of an A.P. are in G.P. and m, n, r are in H.P. then the common difference of the A.P. is ________

(A) $-\frac{a}{n}$ (B) $-\frac{n}{2a}$ (C) $\frac{2a}{n}$ (D) $\frac{-2a}{n}$

15. 2 + 6 + 12 + 20 + 30 + ... 100 terms = _______

(A) $\frac{1020300}{3}$ (B) $\frac{1030200}{3}$ (C) $\frac{1003200}{3}$ (D) $\frac{1023200}{3}$

16. If any terms of an A.P. is non - zero and d $\neq$ 0 then $\sum_{r=1}^{n-1} \frac{1}{a_r \, a_{r+1}}$ =_______

(A) $\frac{n}{a_1 \, a_n}$ (B) $\frac{n-1}{a_1 \, a_n}$ (C) $\frac{n+1}{a_1 \, a_n}$ (D) $\frac{2n}{a_1 \, a_n}$

17. If S_1, S_2, S_3, ... , S_n are the sums of infinite G.P.s. whose first terms ars 1, 2, 3, ..., n and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \frac{1}{n+1}$ respectively, then $\sum_{i=1}^{n} S_i$ = _______

(A) $\frac{n\,(n+3)}{2}$ (B) $\frac{n\,(n+4)}{2}$ (C) $\frac{n\,(n-3)}{2}$ (D) $\frac{n\,(n+1)}{2}$

18. 1 + 3 + 7 + 13 + ... 100 terms = ________

(A) $\frac{1010000}{2}$ (B) $\frac{1000200}{3}$ (C) $\frac{1015050}{3}$ (D) $\frac{1051050}{3}$

19. 1 + 5 + 14 + 30 + ... n terms = ________

(A) $\frac{(n+2)\,(n+3)}{12}$ (B) $\frac{n\,(n+1)\,(n+5)}{12}$

(C) $\frac{n\,(n+2)\,(n+3)}{12}$ (D) $\frac{n\,(n+1)^2\,(n+2)}{12}$

20. $4 + 18 + 48 + \ldots$ n terms = ________

(A) $\frac{n(n+1)(n+2)(3n+5)}{12}$ (B) $\frac{n(n+1)(n+2)(5n+3)}{12}$

(C) $\frac{n(n+1)(n+2)(7n+1)}{12}$ (D) $\frac{n(n+1)(n+2)(9n-1)}{12}$

21. $2 + 12 + 36 + 80 + \ldots$ n terms = _______

(A) $\frac{n(n+1)(n+2)(3n+5)}{24}$ (B) $\frac{n(n+1)(n+2)(3n+1)}{12}$

(C) $\frac{n(n+1)(n+3)(n+5)}{24}$ (D) $\frac{n(n+1)(n+2)(n+3)}{12}$

22. $\frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \frac{9}{400} + \ldots$ infinite terms = ________

(A) 0.8 (B) 0.9 (C) 1 (D) 0.99

23. $\frac{1^3}{1} + \frac{1^3+2^3}{2} + \frac{1^3+2^3+3^3}{3} + \ldots$ up to n terms = _______

(A) $\frac{n(n+1)(n+2)(5n+3)}{48}$ (B) $\frac{n(n+1)(n+3)(n+5)}{24}$

(C) $\frac{n(n+1)(n+2)(7n+1)}{48}$ (D) $\frac{n(n+1)(n+2)(3n+5)}{48}$

24. $\frac{1^3}{1} + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + \ldots$ 15 terms = ______

(A) 446 (B) 680 (C) 600 (D) 540

25. $\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+...100$ terms

(A) $\frac{25}{160}$ (B) $\frac{1}{6}$ (C) $\frac{25}{151}$ (D) $\frac{25}{152}$

26. 1 + 3 + 7 + 15 + ... 10 terms = ______

(A) 2012 (B) 2046 (C) 2038 (D) 2036

27. $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}+...+\tan^{-1}\frac{1}{9703}=$ ________

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\tan^{-1}(0.98)$

28. If n = ______ then $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is an A.M. of a and b

(A) 2(B) 3 (C) 1 (D) 0

29. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is G.M. of a and b then n = ______ (a, b $\in R^+$ $a\neq b$)

(A) 0(B) 1 (C) $-\frac{1}{2}$ (D) -2

30. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is H.M. of a and b then n = _______ (a, b $\in R^+$ $a\neq b$)

(A) 0(B) -1 (C) $-\frac{1}{2}$ (D) -2

31. A sequence log a, $\log\left(\frac{a^2}{b}\right)$, $\log\left(\frac{a^3}{b^2}\right)$ is ______ (where $a\neq b$)

(A) G. P. (B) A. P. (C) H. P. (D) A. G. P.

32. If a, b, c are in A. P. and geometric means of ac and ab, ab and bc, ca and cb are d, e, f respectively then e + f, f + d and d + e are in _______

(A) G. P. (B) A. P. (C) H. P. (D) A. G. P.

33. Find out three numbers which are in G. P. such that their summation is 13 and the sum of their squares is 91

(A) 3, 1, 9 (B) 1, 3, 9 (C) 1, 9, 3 (D) $\frac{13}{3}, \frac{13}{3}, \frac{13}{3}$

34. If $S_1, S_2, S_3, ..., S_n$ be the sum of n terms of n A.P. s whose first terms are 1, 2, 3, ..., n respectively and common differences are 1, 3, 5, ... (2n-1) respectively then

$$\sum_{r=1}^{n} S_r = \_\_\_\_\_\_\_$$

(A) $\frac{n^3(n+1)}{2}$ (B) $\frac{n^2(n^2-1)+2}{2}$

(C) $\frac{n(n^3+1)}{2}$ (D) $\frac{n^2(n^2+1)}{2}$

35. 0.4 + 0.44 + 0.444 + ... to 2n terms = _____

(A) $\frac{4}{81}$ (18n + 1 + 100^{-n}) (B) $\frac{4}{81}$ (18n - 1 + 100^{-n})

(C) $\frac{4}{81}$ (18n - 1 + 10^{-n}) (D) $\frac{4}{81}$ (18n - 1 + 100^{n})

36. The 11th, 13th and 15th terms of any G.P. are in ______

(A) G. P. (B) A. P. (C) H. P. (D) A. G. P.

37. If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A. P. then a, b, c are in _______

(A) G. P. (B) A. P. (C) H. P. (D) A. G. P.

38. If the sum of first 101 terms of an A. P. is 0 and If 1 be the first term of the A. P. then the sum of next 100 terms is ______

(A) -101 (B) 201 (C) -201 (D) -200

39. If A_1 and A_2 be the two A. M. s between two numbers 7 and $\frac{1}{7}$

then $(2A_1 - A_2)(2A_2 - A_1)$ = ______

(A) 49 (B) $\frac{48}{7}$ (C) $\frac{50}{7}$ (D) 1

40. For an A. P., $S_{100} = 3 S_{50}$ The value of $S_{150} : S_{50}$ =_______

(A) 8(B) 3 (C) 6 (D) 10

41. If 1, A_1, A_2, A_3, ... A_n, 31 are in A. P. and $A_7 : A_{n-1} = 5 : 9$ then n =______

(A) 28 (B) 14 (C) 15 (D) 13

42. In a G. P., the last term is 1024 and the common ratio is 2. Its 20 th term from the end is ______

(A) $\frac{1}{512}$ (B) $\frac{1}{1024}$ (C) $\frac{1}{256}$ (D) 512

43. The sum of the numbers $1 + 2.2 + 3.\ 2^2 + 4.\ 2^3 + ... + 50.\ 2^{49}$ is ______

(A) $1 + 49.\ 2^{49}$ (B) $1 + 49.\ 2^{50}$ (C) $1 + 50.\ 2^{49}$ (D) $1 + 50.\ 2^{50}$

44. First term of a G. P. of 2n terms is a, and the last term is l The product of all the terms of the G. P. is ______

(A) $(a\ell)^{\frac{n}{2}}$ (B) $(a\ell)^{n-1}$ (C) $(a\ell)^n$ (D) $(a\ell)^{2n}$

45. The series 1. 1! + 2. 2! + 3. 3! + ... + n. n! = _______

(A) (n + 1)! -n (B) (n + 1)! -1 (C) n! -1 + n (D) n! +1 -n

46. If an A. P., $T_{35} = -50$ and $d = -3$ then S_{35} =______

(A) 35 (B) 38 (C) 32 (D) 29

47. If p th term and q th term of an A. P. are $\frac{1}{qr}$ and $\frac{1}{pr}$ respectively, then r th term of the A. P. = ________

(A) $\frac{1}{pqr}$ (B) 1 (C) $\frac{1}{pq}$ (D) pq

48. If a set A = {3, 7, 11, ..., 407} and a set B = {2, 9, 16, ..., 709}

then $n(A \cap B)$ = _______

(A) 13 (B) 14 (C) 15 (D) 16

49. If $S_n = an + bn^2$, for an A. P. where a and b are constants, then common difference of A. P. will be _______

(A) 2b (B) a+b (C) 2a (D) a-b

50. If $\{a_n\}$ is an A. P. then $a_1^2 - a_2^2 + a_3^2 - a_4^2 + ... + a_{99}^2 - a_{100}^2 =$ ______

(A) $\frac{50}{99}\left(a_1^2 - a_{100}^2\right)$ (B) $\frac{100}{99}\left(a_{100}^2 - a_1^2\right)$

(C) $\frac{50}{51}\left(a_1^2 + a_{100}^2\right)$ (D) None of this

51. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then bc^2, ca^2, ab^2 are in ______

(A) A. P. (B) G. P. (C) H. P. (D) A. G. P.

52. If the first, second and last terms of an A. P. are a, b and 2a respectively, the sum of the series is ________

(A) $\frac{4a^2}{b-a}$ (B) $\frac{2a^2+2ab}{b-a}$ (C) $\frac{2ab+a^2}{b-a}$ (D) $\frac{2a^2-2ab}{a-b}$

53. Sum of products of first n natural numbers taken two at a time is _____

(A) $\dfrac{n(n^2-1)(3n+2)}{24}$ (B) $\dfrac{n(n+1)^2(3n+2)}{72}$

(C) $\dfrac{n^2(n+1)(3n+2)}{48}$ (D) $\dfrac{n(n+1)(n+2)(3n+2)}{96}$

54. $\dfrac{3}{1^2}+\dfrac{5}{1^2+2^2}+\dfrac{7}{1^2+2^2+3^2}$ + ... upto n terms = _____

(A) $\dfrac{6n^2}{n+1}$ (B) $\dfrac{6n}{n+1}$ (C) $\dfrac{6(2n-1)}{n+1}$ (D) $\dfrac{3(n^2+1)}{n+1}$

55. The nth term of the sequence $\dfrac{1}{1+\sqrt{x}}, \dfrac{1}{1-x}, \dfrac{1}{1-\sqrt{x}}$, ... is _______

(A) $\dfrac{1+\sqrt{x}(n^2-2)}{1-x}$ (B) $\dfrac{1+\sqrt{x}(n-1)}{1+\sqrt{x}}$

(C) $\dfrac{1+\sqrt{x}(n-2)}{1-x}$ (D) $\dfrac{3-\sqrt{x}(n+2)}{3(1-x)}$

(56) The sum of the series a - (a + d) + (a + 2d) - (a + 3d) + ... up to 50 terms is ______

(A) - 50d (B) 25d (C) a + 50d (D) - 25d

(57) The numbers of terms in the A. P. a,b,c,........, x is _______

(A) $\dfrac{x+b+a}{c-b}$ (B) $\dfrac{x+b-2a}{c-b}$ (C) $\dfrac{x+b+2a}{c-b}$ (D) $\dfrac{x-b+2a}{c-b}$

(58) If the sides of a right triangle are in A. P., then the sum of the sines of the two acute angles is ______

(A) $\dfrac{7}{5}$ (B) $\dfrac{8}{5}$ (C) $\dfrac{1}{5}$ (D) $\dfrac{6}{5}$

(59) If $\log_3 2$, $\log_3 (2^x - 5)$ and $\log_3 \left(2^x - \frac{7}{2}\right)$ are in A. P. then x = ______

(A) 2 (B) 3 (C) 4 (D) 2 or 3

(60) If an be the nth term of a G. P. of positive numbers and $\sum_{n=1}^{100} a_{2n} = \alpha$, $\sum_{n=1}^{100} a_{2n-1} = \beta$ such that $\alpha \neq \beta$, then the common ratio of the G. P. is ________

(A) $\frac{\alpha}{\beta}$ (B) $\frac{\beta}{\alpha}$ (C) $\sqrt{\frac{\alpha}{\beta}}$ (D) $\sqrt{\frac{\beta}{\alpha}}$

(61) If the numbers p, q, r are in A. P., then $2^{p^2}, 2^{pq}, 2^{pr}$ are in ________

(A) A. P. (B) G. P. (C) H. P. (D) A. G. P.

(62) If the angles A < B < C of a ΔABC are in A. P. then _______

(A) $c^2 = a^2 + b^2 - ab$ (B) $c^2 = a^2 + b^2$

(C) $b^2 = a^2 + c^2 - ac$ (D) $a^2 = b^2 + c^2 - bc$

(63) $\frac{1}{3} + \frac{2}{3^2} + \frac{1}{3^3} + \frac{2}{3^4} + \frac{1}{3^5} + \frac{2}{3^6}$ + ... up to ∞ = _______

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{7}{8}$ (D) $\frac{5}{8}$

(64) $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3}$ + ... up to ∞ = _______

(A) $\frac{16}{35}$ (B) $\frac{11}{8}$ (C) $\frac{35}{16}$ (D) $\frac{7}{16}$

(65) If sec (x-y), sec x and sec (x+y) are in A. P., then cos x sec $\left(\frac{y}{2}\right)$ = ...

($y \neq 2n\pi$, $n \in I$

(A) $\pm\sqrt{2}$ (B) $\pm\frac{1}{\sqrt{2}}$ (C) ± 2 (D) $\pm\frac{1}{2}$

(66) $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}$ + ... n terms = _______

(A) $n+2^{-n}-1$ (B) $2^{-n} - n + 1$ (C) $\frac{2^n - n + 1}{4}$ (D) $2^{-n}+n^2-1$

(67) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A. P. , then $\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)$ = _______

(A) $\frac{4b^2 - 3ac}{abc}$ (B) $\frac{4}{ac}-\frac{3}{b^2}$ (C) $\frac{4}{ac}-\frac{5}{b^2}$ (D) $\frac{4b^2 + 3ac}{ab^2c}$

(68) The sum of the series $\frac{3}{4}+\frac{5}{36}+\frac{7}{144}$, ... up to 11 terms is _______

(A) $\frac{120}{121}$ (B) $\frac{143}{144}$ (C) 1 (D) $\frac{144}{143}$

(69) If the sides of a $\triangle ABC$ are in A. P. and the greatest angle is double the smallest. The ratio of the sides of $\triangle ABC$ is ________

(A) 3 : 4 : 5 (B) 5 : 12 : 13 (C) 4 : 5 : 6 (D) 5 : 6 : 7

(70) 6 th term of the sequence $\frac{7}{3}, \frac{35}{6}, \frac{121}{12}, \frac{335}{24}$, ...is ________

(A) $\frac{2113}{96}$ (B) $\frac{2112}{96}$ (C) $\frac{865}{48}$ (D) $\frac{2111}{96}$

(71) If $x_1, x_2, x_3, \ldots x_n \in R - \{0\}$ such that $\left(\sum_{i=1}^{n-1} x_i^2\right)\left(\sum_{i=2}^{n} x_i^2\right) \le \sum_{i=1}^{n-1} (x_i\, x_{i+1})$, then $x_1, x_2, x_3, \ldots, x_n$ are in ________

(A) A. P. (B) G. P. (C) H. P. (D) none of these

(72) The greatest value of n for which $1+\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^n}<2$ is _____ $(n \in N)$

(A) 100 (B) 10 (C) 1000 (D) none of these

(73) The coefficient of x^8 in the product (x+1) (x+2) (x+3) ... (x+10) is ________

(A) 1024 (B) 1300 (C) 1320 (D) 1360

(74) The sum of 20 terms of the series 12 + 16 + 24 + 40 + ... is ________

(A) 8335 (B) 8348 (C) 8356 (D) 8363

(75) If $\frac{1}{b-c}, \frac{1}{2b-x}$ and $\frac{1}{b-a}$ are in A. P., then $a - \frac{x}{2}$, $b - \frac{x}{2}$, $c - \frac{x}{2}$ are in ____

(A) A. P. (B) G. P. (C) H. P. (D) A. G. P.

(76) If 1, $\log_y x$, $\log_z y$, $-15 \log_x z$ are in A. P. then the common difference of this A. P. is _________

(A) 1(B) 2 (C) -2 (D) 3

(77) If the function f satisfies the relation f (x+y) = f(x) f(y) for all x, y $\in N$, Further if f(1) = 3 and

$\sum_{r=1}^{n}$ f(a+r) $=\frac{81}{2}(3^n-1)$ then a =_______

(A) 4(B) 2 (C) 1 (D) 3

(78) If for an A. P. $\{a_n\}$, $a_1 + a_5 + a_{15} + a_{26} + a_{36} + a_{40} = 210$ then s_{40} =________

(A) 2100 (B) 700 (C) 1400 (D) none of these

(79) If a, 4, b are in A. P. and a, 2, b are in G. P. then $\frac{1}{a}$, 1, $\frac{1}{b}$ are in ______

(A) G. P. (B) A. P. (C) H. P. (D) A. G. P.

(80) For all x, y $\in R^+$ the value of $\frac{(1+x+x^2)(1+y+y^2)}{xy}$ = _____

(A) < 9 (B) ≤ 9 (C) > 9 (D) ≥ 9

(81) In a G. P. the first term is a, second term is b and the last term is c, then the sum of the series is _________

(A) $\frac{c^2 - ab}{c - a}$ (B) $\frac{b^2 - ac}{b - c}$ (C) $\frac{a^2 - bc}{a - b}$ (D) $\frac{a^2 - bc}{a + b}$

(82) If $a_1, a_2, ... a_{10}$ be in A. P. , $\frac{1}{h_1}, \frac{1}{h_2} ... \frac{1}{h_{10}}$ be in A. P. and $a_1 = h_1 = 2$, $a_{10} = h_{10} = 3$

then $a_4 h_7$ = _____

(A) $\frac{1}{6}$ (B) 6 (C) 3 (D) 2

(83) If $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A. P. then $\frac{H + a}{H - a} + \frac{H + b}{H - b}$ = ____

(A) 2 (B) 4 (C) 0 (D) 1

(84) In an A. P., $S_m : S_n = m^2 : n^2$ The ratio of p^2 th term to q^2 term is _____

(A) $\frac{2p^2 + 1}{2q^2 + 1}$ (B) $\frac{2p^2 - 1}{2q^2 - 1}$ (C) $\frac{2p - 1}{2q - 1}$ (D) $\frac{p^2 - 2}{q^2 - 2}$

(85) Sum of numbers in the nth row of the following arrangement is _____

1

2 3 4

5 6 7 8 9

10 11 12 13 14 15

....

(A) $n^3 + (n + 1)^3 - 8$ (B) $n^3 - (n + 1)^3 + 8$

(C) $n^3 + (n - 1)^3$ (D) $(2n - 1)^3$

(86) If A is the A. M. between a and b, then $\frac{A - 2b}{A - a} + \frac{A - 2a}{A - b}$ = ______

(A) -8 (B) 2 (C) 4 (D) -4

(87) If a, b, c are in G. P. , a, x, b are in A. P. and b, y, c are in A. P., then $\frac{a}{x} + \frac{c}{y}$ = _____

(A) 1(B) $\frac{1}{2}$ (C) 2 (D) 4

(88) Sum to infinity of the series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + ...$ is _____

(A) $\frac{5}{16}$ (B) $\frac{35}{16}$ (C) $\frac{16}{35}$ (D) $\frac{7}{16}$

(89) If the pth term of a G. P. is x and q th term is y, then the nth term is _____

(A) $\left(\frac{x^{n-p}}{y^{n-q}}\right)^{\frac{1}{p-q}}$ (B) $\left(\frac{x^{n+q}}{y^{n+p}}\right)^{\frac{1}{p-q}}$

(C) $\left(\frac{x^{n-q}}{y^{n-p}}\right)^{\frac{1}{p-q}}$ (D) $\left(\frac{x^{n-q}}{y^{n-p}}\right)^{\frac{1}{p+q}}$

(90) The nth term of an A. P. is p^2 and the sum of the first n terms is s^2 The first term is _

(A) $\frac{p^2n + 2s^2}{n}$ (B) $\frac{2s^2 + p^2n}{n^2}$ (C) $\frac{ps^2 - p^2s}{n}$ (D) $\frac{2s^2 - p^2n}{n}$

(91) If an A. P. a = 1, $S_n : (S_{2n} - S_n)$ = constant, $\forall$ n $\in N$ then the common difference d =_____

(A) 4(B) $\frac{1}{2}$ (C) 2 (D) 3

(92) If $\propto, \beta$ are the roots of ax^2-bx+c=o and γ, δ are the roots of px^2-qx+r=α and $\alpha, \beta, \gamma, \delta$ are in A. P. then the common difference = _____

(A) $\frac{aq - bp}{8ap}$ (B) $\frac{aq - bp}{4ap}$ (C) $\frac{bp - aq}{4ap}$ (D) $\frac{bp - aq}{8ap}$

(93) If α, β are the roots of ax^2-bx+c=o and γ, δ are the roots of px^2-qx+r=o and If $\alpha, \beta, \gamma, \delta$ are in G. P. then the common ratio is = _____

(A) $\left(\frac{ar}{cp}\right)^{\frac{1}{4}}$ (B) $\left(\frac{ar}{cp}\right)^{\frac{1}{8}}$ (C) $\left(\frac{ap}{cr}\right)^{\frac{1}{4}}$ (D) $\left(\frac{ar}{cp}\right)^{\frac{-1}{4}}$

(94) In a ΔABC angles A, B, C are in increasing A. P. and sin (B + 2C) = $\frac{-1}{2}$ then A = _____

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) $\frac{\pi}{6}$

(95) If sum of n terms of A. P. is given by $s_n = (a - 2)n^3 + (b - 1)n^2 + (c - 3)n + d$, where a, b, c are independent of n, then the common difference d = _____

(A) b - 1 (B) 2(b - 1) (C) 2(b + 1) (D) 2(b - 3)

(96) If three positive real numbers a, b, c are in A. P. and if abc = 64 then the minimum value of b is _______

(A) 6 (B) 5 (C) 4 (D) 3

(97) In a ΔABC, a, b, c are in A. P., then cot $\frac{A}{2}$ cot $\frac{C}{2}$ = ____

(A) 2 (B) -3 (C) 3 (D) -2

(98) If 2, b, c, 23 are in G. P. then $(b - c)^2 + (c - 2)^2 + (23 - b)^2$ =_____

(A) 625 (B) 525 (C) 441 (D) 442

Hints

1. Hear a = 1, r = 2, for G. P

$\therefore$ $s_1 + s_3 + s_5 + ... S_{2n-1} = 1 + 1\,\frac{(1-2^3)}{1-2} + \frac{1(1-2^5)}{1-2} + ...n$ terms

$= 1 + 1(2^3 - 1) + (2^5 - 1) + ...$ n term

$= (2 + 2^3 + 2^5 + ...n \text{ term}) - (1 + 1 + 1 + ...n \text{ terms})$

$= 2\frac{(2^{2n} - 1)}{2^2 - 1} - n \qquad \because \quad r = 2^2$

$= \frac{1}{3}(2^{2n+1} - 3n - 2)$

(2) Required sum $= \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...100$ terms

$= \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + ...100$ term

$= (1 + 1 + 1 + ...100 \text{ terms}) - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...100 \text{ terms}\right)$

$= 100 - \frac{1}{2}\frac{\left(1 - \left(\frac{1}{2}\right)^{100}\right)}{1 - \frac{1}{2}}$

$= 100 - 1 + \left(\frac{1}{2}\right)^{100} = 2^{-100} + 99$

(3) As the
length of sides of a triangle in G.P.
Let they are 9, 9r and $9r^2$ $(r > 1)$
Now perimeter = $9 + 9r + 9r^2 = 37$

$\therefore$ $9r^2 + 9r - 28 = 0$ $\therefore$ $(3r + 7)(3r - 4) = 0$

$\therefore$ $r = \frac{-7}{3}$ or $r = \frac{4}{3}$ but $r > 0$, $r = -\frac{7}{3}$ is rejected. $\therefore$ $r = \frac{4}{3}$

$\therefore$ Hence the sides of the triangle are 9, 12, 16
$\therefore$ The lengths of the other two sides are 12 and 16

(4) Using the given conditions we get $a + b + c = 25$......... (1)
$2a = 2 + b$ .... (2) and
$c^2 = 18b$ .... (3)

by (1) & (2) we get $\frac{2+b}{2} + b + c = 25$ $\Rightarrow$ $3b + 2c = 48$ ....(4)

by (3) & (4) we get $3\left(\frac{c^2}{18}\right) + 2c = 48$ $\Rightarrow$ $c^2 + 12c - 288 = 0$

$\Rightarrow$ $(c - 12)(c + 24) = 0$
$\Rightarrow$ $c = 12$ or $c = -24$ but c can not be – ve
$\therefore$ $c = 12$ $\therefore$ $144 = 18b$ $\therefore$ $b = 8$ and $a = 25 - 8 - 12 = 5$
$\therefore$ $a = 5$, $b = 8$, $c = 12$

(5) Let the four numbers are $a + 6$, $a - 6$, a, $a + 6$
$\therefore$ $(a - 6)^2 = (a + 6)a$ $\Rightarrow$ $a = 2$
$\therefore$ required four numbers are 8, –4, 2, 8

(6) since AM. = $\sqrt{10}$ (G.M.)

$\Rightarrow$ $\frac{a+b}{2} = \sqrt{10}\sqrt{ab}$

$\Rightarrow$ $(a+b)^2 = 40\,ab$ $\Rightarrow$ $a^2 + b^2 = 38\,ab$ $\Rightarrow$ $\frac{a^2b^2}{2ab} = 19$

$\Rightarrow$ $\frac{a^2 + b^2 - 2ab}{a^2 + b^2 + 2ab} = \frac{18}{20}$ $\Rightarrow$ $\left(\frac{a-b}{a+b}\right)^2 = \frac{9}{10}$ $\Rightarrow$ $\frac{a-b}{a+b} = \frac{3}{\sqrt{10}}$

(7) Hear H = 4 and G = $3\sqrt{2}$

$\Rightarrow \quad \frac{2ab}{a+b} = 4$ and $ab = G^2 = 18$

$\Rightarrow \quad 2(18) = 4\,(a + b)$

$\Rightarrow \quad a + b = 9$

$\Rightarrow \quad a = 3, b = 6$

$\Rightarrow \quad [a, b] = [3, 6]$

(8) Let the three numbers in G.P. be a, ar, ar^2

since A.M. $= \frac{14}{3}$

$\Rightarrow \quad \frac{a+ar+ar^2}{3} = \frac{14}{3}$

$\Rightarrow \quad a + ar + ar^2 = 14$ ....(1)

also a + 1, ar + 1, $ar^2 - 1$ in A.P.

$\Rightarrow \quad a + 1 + ar^2 - 1 = 2(ar + 1)$

$\Rightarrow \quad a + ar^2 = 2ar + 2$

$\Rightarrow \quad ar^2 - 2ar + a = 2$ ....(2)

Now (1) ÷ (2)

$\Rightarrow \quad \frac{1+r+r^2}{r^2-2r+1} = \frac{14}{2} = 7$

$\Rightarrow \quad 2r^2 - 5r + 2 = 0$

$\Rightarrow \quad r = 2$ or $r = \frac{1}{2}$

If r = 2 then (1) $\Rightarrow$ a = 2

Hence the required number are 2, 4, 8

$\therefore \quad a^2 + (ar)^2 + (ar^2)^2 = 4 + 16 + 64 = 84$

(9) Hear $b = \frac{2ac}{a+c}$, $c^2 = bd$ and $\frac{c+e}{2} = d$

$\therefore \quad c^2 = \left(\frac{2ac}{a+c}\right)\cdot\left(\frac{c+e}{2}\right)$

$\Rightarrow \quad c(a+c) = a(c+e)$

$\Rightarrow \quad ac + c^2 = ac + ae$

$\Rightarrow \quad c^2 = ae$

$\therefore$ G.M. of a and e = c

(10) Since d is G.M. between ac and ab $\therefore$ $d^2 = ac \,.\, ab = a^2bc$

similarly $e^2 = ab^2c$ and $f^2 = abc^2$

Now a, b, c in A.P

$\Rightarrow$ a^2bc, ab^2c, abc^2 in A.P. $\Rightarrow$ $d^2, e^2\ f^2$ are in A.P.

(11) Hear p, A_1, A_2, q are in A.P. $\Rightarrow A_1 + A_2 = p + q$

and p, G_1, G_2, q are in G.P. $\Rightarrow G_1\,G_2 = pq$

also p, H_1, H_2, q are in H.P. $\Rightarrow \frac{1}{H_1} - \frac{1}{p} = \frac{1}{q} - \frac{1}{H_2}$

$$\Rightarrow \frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{p} + \frac{1}{q}$$

$$\Rightarrow \frac{H_2 + H_1}{H_1H_2} = \frac{q+p}{pq} = \frac{A_1 + A_2}{G_1G_2}$$

$$\Rightarrow \frac{G_1G_2}{H_1H_2} = \frac{A_1 + A_2}{H_1 + H_2}$$

(12) $\Sigma\left\{\Sigma\, 2^{1-n}\right\} = \Sigma\left\{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots n \text{ terms}\right\}$

$$= \Sigma\, 2\left(1 - \frac{1}{2^n}\right)$$

$$= 2\left(\Sigma 1 - \Sigma\frac{1}{2^n}\right)$$

$$= 2n - 2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n}\right)$$

$$= 2n - \left(1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^{n-1}}\right)$$

$$= 2n - \frac{\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}}$$

$$= 2n - 2\left(1 - \frac{1}{2^n}\right) = 2n - 2 + 2^{1-n}$$

(13) $(6 + 60 + 600 + \ldots \text{ n terms})^2 + (8 + 80 + 800 + \ldots \text{ n term})$

$= \quad 36(1 + 10 + 100 + \ldots \text{ n terms})^2 + 8(1 + 10 + 100 + \ldots \text{ n terms})$

$$= \quad 36\left(\frac{10^n - 1}{10 - 1}\right)^2 + 8\left(\frac{10^n - 1}{10 - 1}\right)$$

$$= \quad 36\left(\frac{10^{2n} - 2\cdot 10^n + 1}{9\times 9}\right) + 8\left(\frac{10^n - 1}{9}\right)$$

$$= \quad \frac{4}{9}\left(10^{2n} - 2\cdot 10^n + 1 + 2\cdot 10^n - 2\right) = \frac{4}{9}\left(10^{2n} - 1\right)$$

$$= \quad 4\left(\frac{10^{2n} - 1}{10 - 1}\right) = 4(1 + 10 + 100 + \ldots \text{ 2n terms})$$

$= 4 + 40 + 400 + \ldots$ 2n terms

$= 444 \ldots$ 2n times $= 444 \ldots$ k times

$\therefore \quad k = 2n$

(14) Hear for an A.P. $\{T_n\}$, T_{m+1}, T_{n+1} and T_{r+1} are in G.P.

$\therefore \quad T_{n+1}^2 = T_{m+1} \cdot T_{r+1}$

$\Rightarrow \quad (a + nd)^2 = (a + md)(a + rd)$

$\Rightarrow \quad (2n - m - r)a = \left(mr - n^2\right)d \quad \ldots\ldots \quad (1) \quad \because d \neq 0$

also m, n, r are in H.P.

$$\Rightarrow \quad n = \frac{2mr}{m + r}$$

$$\Rightarrow \quad mr = \frac{n(m + r)}{2}$$

Now (1) $\Rightarrow$ $(2n - m - r)a = \left[\frac{n(m + r)}{2} - n^2\right]d = \frac{n}{2}(m + r - 2n)d$

$$\Rightarrow \quad a = -\frac{n}{2}d$$

$$\Rightarrow \quad d = \frac{-2a}{n}$$

(15) 2 + 6 + 12 + 20 + ... up to 100 terms

$= \left(1^2+1\right)+\left(2^2+2\right)+\left(3^2+3\right)+\ldots$ up to 100 terms

$= \left[\Sigma n^2+\Sigma n\right]_{n=100} = \left[\frac{n(n+1)(n+2)}{3}\right]_{n=100}$

$= \frac{100\times101\times102}{3} = \frac{1030200}{3}$

(16) $\sum_{r=1}^{n-1}\frac{1}{a_r\,a_{r+1}} = \frac{1}{d}\sum_{r=1}^{n-1}\frac{d}{a_r\,a_{r+1}} = \frac{1}{d}\sum_{r=1}^{n-1}\frac{a_{r+1}-a_r}{a_r\,a_{r+1}}$

$= \frac{1}{d}\sum_{r=1}^{n-1}\left(\frac{1}{a_r}-\frac{1}{a_{r+1}}\right)$

$= \frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\ldots\ldots+\frac{1}{a_{n-1}}-\frac{1}{a_n}\right)$

$= \frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_n}\right) = \frac{1}{d}\left(\frac{a_n-a_1}{a_1\,a_n}\right) = \frac{1}{d}\left(\frac{(n-1)d}{a_1\,a_n}\right) = \frac{n-1}{a_1a_n}$

(17) $\sum_{i=1}^{n} s_i = \sum_{i=1}^{n}\frac{a}{1-r} = \frac{1}{1-\frac{1}{2}}+\frac{2}{1-\frac{1}{3}}+\frac{3}{1-\frac{1}{4}}+\ldots+\frac{n}{1-\frac{1}{n+1}}$

= 2 + 3 + 4 + ... + (n + 1)

$= \frac{n}{2}\left[4+(n-1)1\right] = \frac{n(n+3)}{2}$

(18) $S_n = 1+3+7+13+\ldots+a_n$

$S_n =$ 1 + 3 + 7 + ... + a_{n-1} + a_n

– – – – – –

$0 = 1 + 2 + 4 + 6 + \ldots + (a_n - a_{n-1}) - a_n$

$\therefore \quad a_n = 1+\left(2+4+6+\ldots(n-1)\right) = 1+\frac{n-1}{2}\left(4+(n-2)2\right)$

$= 1+(n-1)(n)$

$= n^2-n+1$

$\therefore \quad S_n = \Sigma\left(n^2-n+1\right)$

$$= \frac{n\left(n^2+2\right)}{3}$$

$\therefore$ $1 + 3 + 7 + 13 + \ldots 100 \text{ terms} = 100\left(\frac{10000+2}{3}\right) = \frac{1000200}{3}$

(19) 1 + 5 + 14 + 30 + ... n terms

$$= \quad 1^2 + \left(2^2+1^2\right) + \left(1^2+2^2+3^2\right) + \ldots n \text{ terms}$$

$$= \quad \Sigma\left(\Sigma n^2\right) = \Sigma \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)^2(n+2)}{12}$$

(20) 4 + 18 + 48 + 100 + ... n terms

$$= \quad \Sigma\, n(n+1)^2 = \Sigma\left(n^3+2n^2+n\right) = \frac{n(n+1)(n+2)(3n+5)}{12}$$

(21) 2 + 12 + 36 + 80 + ... n terms

$$= \quad \Sigma\, n^2(n+1)$$

$$= \quad \Sigma\, n^3 + \Sigma\, n^2$$

$$= \quad \frac{n(n+1)(n+2)(3n+1)}{12}$$

(22) $\frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \frac{9}{400} + \ldots \infty$

$$= \quad \frac{3}{(1\times 2)^2} + \frac{5}{(2\times 3)^2} + \frac{7}{(3\times 4)^2} + \frac{9}{(4\times 5)^2} + \ldots\infty$$

$$= \quad \frac{2^2-1^2}{1^2\times 2^2} + \frac{3^2-2^2}{2^2\times 3^2} + \frac{4^2-3^2}{3^2\times 4^2} + \ldots\infty$$

$$= \quad \left(\frac{1}{1^2}-\frac{1}{2^2}\right) + \left(\frac{1}{2^2}-\frac{1}{3^2}\right) + \left(\frac{1}{3^2}-\frac{1}{4^2}\right) + \ldots\infty$$

(23) Required sum = $\Sigma\left(\frac{\Sigma n^3}{n}\right) = \Sigma\ \frac{n^2(n+1)^2}{4n}$

$$= \frac{1}{4}\Sigma\left(n^3+2n^2+n\right)$$

$$= \frac{n(n+1)(n+2)(3n+5)}{48}$$

(24) $a_n = \frac{1^3+2^3+3^3+...+n^3}{1+2+3+...+n} = \frac{\Sigma n^3}{\Sigma n} = \frac{(\Sigma n)^2}{\Sigma n} = \Sigma n = \frac{n(n+1)}{2}$

$$\Sigma a_n = \Sigma\frac{n(n+1)}{2} = \frac{1}{6}n(n+1)(n+2)$$

Required sum $= [\Sigma a_n]_{n=15} \quad = \left[\frac{1}{6}n(n+1)(n+2)\right]_{n=15}$

$$= \frac{1}{6}\times 15\times 16\times 17 = 5\times 8\times 17$$

$$= 680$$

(25) $a_n = \frac{1}{(3n-1)(3n+2)} \quad = \frac{1}{3}\times\frac{(3n+2)-(3n-1)}{(3n-1)(3n+2)}$

$$= \frac{1}{3}\left[\frac{1}{3n-1}-\frac{1}{3n+2}\right]$$

$$\Sigma a_n = \frac{1}{3}\Sigma\left(\frac{1}{3n-1}-\frac{1}{3n+2}\right)$$

$$= \frac{1}{3}\left[\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)+...+\left(\frac{1}{3n-1}-\frac{1}{3n+2}\right)\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}-\frac{1}{3n+2}\right] = \frac{n}{2(3n+2)}$$

$\therefore$ Required sum $= \left(\frac{n}{2(3n+2)}\right)_{n=100} = \frac{100}{2(302)} = \frac{25}{151}$

(26) $S_{10} = 1+3+7+15+\ldots 10 \quad \text{terms}$

$$= \left(2^1-1\right)+\left(2^2-1\right)+\left(2^3-1\right)+\ldots 10 \ \text{ terms}$$

$$= \left(2+2^2+2^3+\ldots 10 \ \text{ terms}\right)-10$$

$$= 2\frac{\left(2^{10}-1\right)}{2-1}-10 = 2^{11}-2-10 = 2048-12$$

$$= 2036$$

(27) $\tan^{-1}\frac{1}{3} = \tan^{-1}\frac{2-1}{1+2.1} = \tan^{-1}2-\tan^{-1}1$

$\tan^{-1}\frac{1}{7} = \tan^{-1}\frac{3-2}{1+3.2} = \tan^{-1}3-\tan^{-1}2$

$\tan^{-1}\frac{1}{9703} = \tan^{-1}\frac{99-98}{1+99.98} = \tan^{-1}99-\tan^{-1}98$

$\therefore$ Required sum $= \tan^{-1}2-\tan^{-1}1+\tan^{-1}3-\tan^{-1}2+\ldots+\tan^{-1}99-\tan^{-1}98$

$$= \tan^{-1}99-\tan^{-1}1$$

$$= \tan^{-1}\frac{99-1}{1+99.1} = \tan^{-1}\frac{98}{100} = \tan^{-1}(0.98)$$

(28) Hear A.M. of a and b = $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\Rightarrow \quad \frac{a+b}{2} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\Rightarrow \quad a^{n+1}+a\cdot b^n+ba^n+b^{n+1} = 2a^{n+1}+2b^{n+1}$

$\Rightarrow \quad ab^n+ba^n = a^{n+1}+b^{n+1}$

$\Rightarrow \quad ab^n+b^{n+1} = a^{n+1}-ba^n$

$\Rightarrow \quad b^n(a-b) = a^n(a-b)$

$\Rightarrow \quad b^n = a^n$

$\Rightarrow \quad n = 0$

(29) Hear G.M. of a and b = $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$$\Rightarrow \left(\frac{a^{n+1}+b^{n+1}}{a^n+b^n}\right)^2 = ab$$

$$\Rightarrow a^{2n+2}+b^{2n+2}+2(ab)^{n+1} = ab\left(a^{2n}+b^{2n}+2a^nb^n\right)$$

$$\Rightarrow a^{2n+2}+b^{2n+2}+2a^{n+1}b^{n+1} = a^{2n+1}.\,b + b^{2n+1}.\,a + 2a^{n+1}\,b^{n+1}$$

$$\Rightarrow a^{2n+1}(a-b) = b^{2n+1}(a-b)$$

$$\Rightarrow a^{2n+1} = b^{2n+1}$$

$$\Rightarrow 2n+1=0 \Rightarrow n=\frac{-1}{2} \quad (\because\ a \ne b)$$

(30) Hear H.M. of a and b = $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$$\therefore \quad \frac{2ab}{a+b} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$$

$$\therefore \quad 2a^{n+1}b + 2ab^{n+1} = a^{n+2} + ab^{n+1} + ba^{n+1} + b^{n+2}$$

$$\therefore \quad ab^{n+1} + a^{n+1}b = a^{n+2} + b^{n+2}$$

$$\therefore \quad b^{n+1}(a-b) = a^{n+1}(a-b)$$

$$\therefore \quad b^{n+1} = a^{n+1}$$

$$\therefore \quad n = -1$$

(31) Hear $T_n = \log\left(\dfrac{a^n}{b^{n-1}}\right)$, $T_{n+1} = \log\left(\dfrac{a^{n+1}}{b^n}\right)$

$$\therefore \quad T_{n+1} - T_n = \log\left(\frac{a^{n+1}}{b^n}\right) - \log\left(\frac{a^n}{b^{n-1}}\right)$$

$$= \log\left(\frac{a^{n+1}}{b^n} \times \frac{b^{n-1}}{a^n}\right) = \log\frac{a}{b} \ne 0$$

$\therefore$ $\{T_n\}$ is an A.P.

(32) Since G.M. of ac and ab = d $\therefore$ $d^2 = a^2bc$

similarly $c^2 = ab^2c$ and $f^2 = abc^2$

Now a, b, c are in A.P.

$\Rightarrow$ $a^2bc,\ ab^2c,\ abc^2$ are in A.P.

$\Rightarrow$ $d^2,\ e^2,\ f^2$ are in A.P.

$\Rightarrow$ $d^2 + de + ef + df,\quad e^2 + de + ef + df,\quad f^2 + de + ef + df$ are in A.P.

$\Rightarrow$ $(d + e)(d + f),\ (e + d)(e + f),\ (f + d)(f + e)$ are in A.P.

$\Rightarrow$ $\dfrac{1}{e+f}, \dfrac{1}{d+f}, \dfrac{1}{d+e}$ are in A.P.

$\Rightarrow$ $e + f,\ f + d,\ d + e$ are in H.P.

(33) Let required three number are $\dfrac{a}{r}, a, ar$

Now $\dfrac{a}{r} + a + ar = 13$ ....(1)

$\dfrac{a^2}{r^2} + a^2 + a^2r^2 = 91$ ....(2)

$\Rightarrow$ $\left(\dfrac{a}{r} + a + ar\right)^2 = 169$

$\Rightarrow$ $\left(\dfrac{a^2}{r^2} + a^2 + a^2r^2\right)^2 + 2\left(\dfrac{a^2}{r} + a^2 + a^2r\right) = 169$

$\Rightarrow$ $91 + 2a\left(\dfrac{a}{r} + a + ar\right) = 169$

$\Rightarrow$ $2a(13) = 78$ $\Rightarrow$ $26a = 78$ $\Rightarrow$ $a = 3$

From

(1) $\Rightarrow$ $\dfrac{3}{r} + 3 + 3r = 13$

$\Rightarrow$ $3r^2 + 3r - 13r + 3 = 0$

$\Rightarrow$ $3r^2 - 10r + 3 = 0$

$\Rightarrow$ $(r - 3)(3r - 1) = 0$

$\Rightarrow \quad r = 3 \qquad \text{or} \qquad r = \frac{1}{3}$

If a = 3, r = 3 required three numbers are 1, 3, 9

If a = 3, $r = \frac{1}{3}$ required three numbers are 9, 3, 1

$\therefore$ Answer is (B) 1, 3, 9

(34) Hear $S_1 = \frac{n}{2}[2+(n-1)], S_2 = \frac{n}{2}[4+(n-1)3]...S_n = \frac{n}{2}[2n+(n-1)(2n-1)]$

$$\therefore \quad \sum_{r=1}^{n} S_r = \frac{n}{2}[2+4+6+...+2n] + \frac{n(n-1)}{2}[1+3+5+...+(2n-1)]$$

$$= \frac{n}{2}\cdot 2\,\Sigma n + \frac{n(n-1)}{2}\ \frac{n(1+2n-1)}{2}$$

$$= n\frac{n(n+1)}{2} + \frac{n(n-1)}{2}n^2 = \frac{n^2}{2}\left[n+1+n^2-n\right] = \frac{n^2(n^2+1)}{2}$$

(35) 0.4 + 0.44 + 0.444 + ... up to 2n terms

$= 4\,[0.1 + 0.11 + 0.111 + \text{... up to 2n terms}]$

$= \frac{4}{9}\,[0.9 + 0.99 + 0.999 + \text{... up to 2n terms}]$

$= \frac{4}{9}\,[(1-0.1) + (1-0.01) + (1-0.001) + \text{... up to 2n terms}]$

$$= \frac{4}{9}\left[2n - 0.1\frac{\{1-(0.1)^{2n}\}}{1-0.1}\right]$$

$$= \frac{4}{9}\left[2n - \frac{1}{9}\left(1-\frac{1}{100^n}\right)\right]$$

$$= \frac{4}{81}\left[18n - 1 + 100^{-n}\right]$$

(36) For Geometric sequence,

$T_n = ar^{10}$, $T_{13} = ar^{12}$, $T_{15} = ar^{14}$

Hence, $T_{11} \cdot T_{15} = ar^{10} \cdot ar^{14}$

$$= \left(ar^{12}\right)^2$$

$$= \left(T_{13}\right)^2$$

$\therefore$ T_{11}, T_{13}, T_{15} are in Geometric progression.

(37) $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in arithmetic sequence

$\therefore$ $\frac{b+c-a}{a}+2, \quad \frac{c+a-b}{b}+2, \quad \frac{a+b-c}{c}+2$ are in A.P.

$\therefore$ $\frac{b+c+a}{a}, \frac{c+a+b}{b}, \frac{a+b+c}{c}$ are in A.P.

$\therefore$ $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\therefore$ a, b, c are in H.P.

(38) Hear a = 1, $S_{101} = 0$

$$\Rightarrow \quad \frac{101}{2}\left[2+100\,d\right] = 0$$

$$\Rightarrow \quad d = \frac{-1}{50}$$

$\therefore$ sum of next 100 terms = $S_{201} - S_{101} = S_{201} - 0 = S_{201}$

$$= \frac{201}{2}\left[2(1) + 200\left(-\frac{1}{50}\right)\right]$$

$$= \frac{201}{2}[2-4] = -201$$

(39) Hear $7, A_1, A_2, \frac{1}{7}$ are in A.P.

$$\Rightarrow \quad A_1 - 7 = A_2 - A_1 = \frac{1}{7} - A_2$$

$$\Rightarrow \quad 2A_1 - A_2 = 7 \text{ and } 2A_2 - A_1 = \frac{1}{7}$$

$$\Rightarrow \quad (2A_1 - A_2)(2A_2 - A_1) = 7 \cdot \frac{1}{7} = 1$$

(40) For an A.P. $S_{100} = 3 S_{50}$

$$\Rightarrow \quad \frac{100}{2}\left[2a + 99d\right] = 3 \cdot \frac{50}{2}\left[2a + 49d\right]$$

$$\Rightarrow \quad 2a = 51d \quad ....(1)$$

$$\therefore \quad \frac{S_{150}}{S_{50}} = \frac{\frac{150}{2}\left[51d + 149d\right]}{\frac{50}{2}\left[51d + 49d\right]} = \frac{3(200d)}{100d} = 6$$

(41) Hear $d = \frac{b-a}{n+1} = \frac{31-1}{n+1} = \frac{30}{n+1}$ ....(1)

Also, $\frac{A_7}{A_{n-1}} = \frac{5}{9}$

$$\Rightarrow \quad \frac{1+7d}{31-2d} = \frac{5}{9} \quad \Rightarrow \quad d = 2 \quad ....(2)$$

From (1) and (2) $\frac{30}{n+1} = 2$

$$\Rightarrow \quad n + 1 = 15 \Rightarrow \quad n = 14$$

(42) If the G.P. is a, ar, ar^2, ... $\frac{l}{r}$, l then $l, \frac{l}{r}, \frac{l}{r^2}$..... ar^2, ar, a is also G.P. with first term $\ell = 1024$ and common ratio of this G.P. is $\frac{1}{r} = \frac{1}{2}$

$$\therefore \quad \text{Required term} = l\left(\frac{1}{r}\right)^{20-1} = 1024 \left(\frac{1}{2}\right)^{19} = \frac{1}{512}$$

(43) Let $S = 1 + 2.2 + 3.2^2 + 4.2^3 + ... + 50.2^{49}$ .... (1)

$\therefore$ $2 + 2.2^2 + 3.2^3 + 4.2^4 + ... + 50.2^{50}$ .... (2)

(2) – (1) $\Rightarrow$ $S = -1 - (1.2 + 1.2^2 + 1.2^3 + ... \text{49 terms}) + 50.2^{50}$

$$= -1 - 2\frac{(2^{49} - 1)}{2 - 1} + 50.2^{50} = -1 - 2^{50} + 2 + 50.2^{50}$$

$= 1 + 49.2^{50}$

(44) If r be the common ratio, then the product of first 2n terms = $P = a(ar)(ar^2) ...... \frac{l}{r}, l$ ......(1)

and also $P = l\left(\frac{l}{r}\right)\left(\frac{l}{r^2}\right).......(ar)a$ .... (2)

(1) × (2) $\Rightarrow$ $P^2 = (a\ell)(a\ell)(a\ell)$ .... up to 2n factors $= (a\ell)^{2n}$

$\Rightarrow$ $P = (a\ell)^n$

(45) For taking n = 1, $1 \times 1! = 1$

(A) $(n + 1)! - n = 2! - 1 = 1$

(B) $(n + 1)! - n = 2! - 1 = 1$

(C) $n! - 1 + n = 1 - 1 + 1 = 1$

(D) $n! + 1 - n = 1 + 1 - 1 = 1$

All possibilities are true, for n = 1

(A) $(n + 1)! - n = 3! - 2 = 4 \neq 5$ $\because n = 2$

(B) $(n + 1)! - 1 = 3! - 1 = 5$

(C) $n! - 1 + n = 2 - 1 + 2 = 3$

(D) $n! + 1 - n = 2 + 1 - 2 = 1$

Answer (B)

Second Method :

Required sum $= \Sigma nn!$

$= \Sigma[(n+1) - 1]\, n!$

$= \Sigma(n+1)! - n!$

$= \Sigma(n+1)! - \Sigma n!$

$= [(n+1)! + n! + (n-1)! + ... + 3! + 2! + 1!] - [n! + (n-1)! + ... + 3! + 2! + 1!]$

$= (n+1)! - 1$

(46) Hear n = 35, d = –3 $\ell = -50$

$\therefore$ $\ell = a + (n - 1)\, d = a + 34\,(-3)$

$\therefore$ $-50 + 102 = a$

$\therefore$ a = 52

Now $S_{35} = \frac{35}{2}\left[2(52) + 34(-3)\right] = \frac{35}{2}[104 - 102] = 35$

(47) Hear $a + (p-1)d = \frac{1}{qr}$ and $a + (q-1)d = \frac{1}{pr}$

$\therefore$ $a = d = \frac{1}{pqr}$

$\therefore$ $a + (r-1)\, d = \frac{1}{pqr} + (r-1)\frac{1}{pqr} = \frac{1}{pq}$

(48) Hear $A = \{4k - 1 / 1 \le k \le 102,\ k \in N\}$ and

$B = \{7k - 5 / 1 \le k \le 102,\ k \in N\}$

If $x \in A \cap B$ then for $\exists\, n, m \in N,\ 4n - 1 = 7m - 5$

$\therefore$ $m = \frac{4n+4}{7}$ m, n = 1, 2, ... 102

$\therefore$ n = 6, 13, 21, ... 98

$\therefore$ $n\,(A \cap B) = 14$

(49) C.D. $= T_2 - T_1 = (T_1 + T_2) - 2T_1 = S_2 - 2S_1$

$= (2a + 4b) - 2(a + b)$

= 2b

(50) Required sum $= -d\,(a_1 + a_2) - d(a_3 + a_4) + ... - d(a_{99} + a_{100})$

$= -d\,(a_1 + a_2 + a_3 + ... + a_{100})$

$= -d\,\frac{100}{2}[a_1 + a_{100}] \times \frac{(a_1 - a_{100})}{(a_1 - a_{100})}$

$= -\frac{50d\left(a_1^{\,2} - a_{100}^{\,2}\right)}{a_1 - (a_1 + 99d)} = \frac{50}{99}\left(a_1^{\,2} - a_{100}^{\,2}\right)$

(51) If the roots are α, β then $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\Rightarrow \quad \alpha + \beta = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow \quad -\frac{b}{a} = \frac{\frac{b^2}{a^2} - 2\frac{c}{a}}{\frac{c^2}{a^2}} \qquad \Rightarrow \qquad ab^2 + bc^2 = 2a^2c$$

$\Rightarrow$ bc^2, ca^2, ab^2 are in A.P.

(52) Hear $d = b - a$ and $\ell = 3a = a + (n-1)\,d$

$$\Rightarrow \quad n = \frac{a+b}{b-a}$$

$$\therefore \quad S_n = \frac{n}{2}(a+\ell) = \frac{1}{2}\left(\frac{a+b}{b-a}\right)(a+3a)$$

$$= \frac{2a\,(a+b)}{b-a} = \frac{2a^2 + 2ab}{b-a}$$

(53) $S = 1\times2 + 1\times3 + 1\times4 + \ldots + 1\times n + 2\times3 + 2\times4 + \ldots + 2\times n + 3\times4 + \ldots + (n-1)n$

Also $(1 + 2 + 3 + \ldots + n)^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 + 2S$

$$\therefore \quad S = \frac{(\Sigma n)^2 - \Sigma n^2}{2} = \frac{n(n^2-1)(3n+2)}{24}$$

(54) $T_n = \frac{2n+1}{\Sigma n^2} = \frac{6}{n(n+1)}$

$$S_n = \Sigma T_n = 6\,\Sigma\frac{1}{n(n+1)} = 6\Sigma\left(\frac{1}{n} - \frac{1}{n+1}\right) = 6\left(1 - \frac{1}{n+1}\right) = \frac{6n}{n+1}$$

(55) The given sequence is

$$\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}, \ldots\ldots.$$

i.e. $\frac{1-\sqrt{x}}{1-x}, \frac{1}{1-x}, \frac{1+\sqrt{x}}{1-x}$...... which is an A.P. with C.D. $d = \frac{\sqrt{x}}{1-x}$ and $T_1 = a = \frac{1-\sqrt{x}}{1-x}$

Hence $T_n = a = (n-1)d = \frac{1-\sqrt{x}}{1-x} + (n-1)\frac{\sqrt{x}}{1-x}$

$$= \frac{1+\sqrt{x}\,(n-2)}{1-x}$$

(56) a – (a + d) + (a + 2d) – (a + 3d) + ... + up to 50 terms

= (–d) + (–d) + (–d) + ... up to 25 terms

= - 25d

(58) Let the sides be a – d, a, a + d, d > 0 then $(a + d)^2 = a^2 + (a – d)^2$

$\Rightarrow$ a = 4d

Hence the three sides are 3d, 4d, 5d

$\therefore \quad \sin A + \sin C = \frac{4d}{5d} + \frac{3d}{5d}$

$$= \frac{7}{5}$$

(59) Hear $2 \log_3 (2^x – 5) = \log_3 2 + \log_3 \left(2^x - \frac{7}{2}\right)$

$\Rightarrow \quad (2^x - 5)^2 = 2\left(2^x - \frac{7}{2}\right)$

$\Rightarrow \quad t^2 – 12t + 32 = 0 \qquad \because 2^x = t$

$\Rightarrow \quad t = 4, 8 \qquad \Rightarrow \quad x = 2, 3$ but x = 2 ia not possible

$\therefore \quad x = 3$

(60) Let the G.P. is a, ar, ar^2... then

$\alpha = a_2 + a_4 + a_6 + ... + a_{200} = ar + ar^3 + ar^5 + ... + ar^{199}$

$= ar\,(1 + r^2 + r^4 + ... + r^{198})$ .... (1)

and $\beta = a(1 + r^2 + r^4 + ... + r^{198})$ ... (2)

(1) ÷ (2) $\Rightarrow r = \frac{\alpha}{\beta}$

(61) As p, q, r are in A.P.

$\Rightarrow \quad 2q = p + r$

$\Rightarrow \quad \left(2^{p}\right)^{2q} = \left(2^{p}\right)^{p+r}$

$\Rightarrow \quad 2^{2pq} = 2^{p^2}, 2^{pr}$

$\Rightarrow \quad \left(2^{pq}\right)^{2} = 2^{p^2}. 2^{pr}$

$\therefore \quad 2^{p^2}, 2^{pa}, 2^{pr}$ are in G.P.

(62) As A, B, C are in A.P.

$\therefore \quad 2B = A + C = 180° - B$

$\therefore \quad B = 60°$

$\therefore \quad \cos B = \dfrac{c^2 + a^2 - b^2}{2ac}$

$\Rightarrow \quad \dfrac{1}{2} = \dfrac{c^2 + a^2 - b^2}{2ac} \qquad \Rightarrow \qquad b^2 = a^2 + c^2 - ac$

(63) $\dfrac{1}{3} + \dfrac{2}{3^2} + \dfrac{1}{3^3} + \dfrac{2}{3^4} + \cdots$ up to ∞

$= \quad \left(\dfrac{1}{3} + \dfrac{1}{3^3} + \text{...up to } \infty\right) + 2\left(\dfrac{1}{3^2} + \dfrac{1}{3^4} + \text{... up to } \infty\right)$

$= \quad \dfrac{\frac{1}{3}}{1-\frac{1}{9}} + \dfrac{2\left(\frac{1}{q}\right)}{1-\frac{1}{9}} = \dfrac{3}{8} + \dfrac{2}{8} = \dfrac{5}{8}$

(64) Let $S = 1 + \dfrac{4}{5} + \dfrac{7}{5^2} + \dfrac{10}{5^3} + \text{.......} \infty$ ....(1)

$\therefore \quad \dfrac{1}{5} S = \dfrac{1}{5} + \dfrac{4}{5^2} + \dfrac{7}{5^3} + \text{........} \infty$ ....(2)

(1) – (2) $\Rightarrow \quad \dfrac{4}{5} S = 1 + \left(\dfrac{3}{5} + \dfrac{3}{5^2} + \dfrac{3}{5^3} + \ldots\right) = 1 + \dfrac{\frac{3}{5}}{1-\frac{1}{5}} = \dfrac{7}{4}$

$$\Rightarrow \quad S = \frac{7}{4} \times \frac{5}{4} = \frac{35}{16}$$

(65) Since sec(x – y), secx, sec(x + y) are in A.P.

$$\Rightarrow \quad \frac{2}{\cos x} = \frac{\cos(x+y) + \cos(x-y)}{\cos(x-y)\cos(x+y)}$$

$$\Rightarrow \quad 2(\cos^2 x - \sin^2 y) = \cos x\,(2\cos x \cos y)$$

$$\Rightarrow \quad \cos^2 x\,(1 - \cos y) = \sin^2 y$$

$$\Rightarrow \quad \cos^2 x = 2\cos^2\left(\frac{y}{2}\right) \text{ as } \cos y \neq 1$$

$$\Rightarrow \quad \cos x \sec\left(\frac{y}{2}\right) = \pm\sqrt{2}$$

(67) As $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., $\quad \therefore \quad \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

Hance, $\left(\frac{1}{a} + \frac{1}{b} - \frac{1}{c}\right)\left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right) = \left\{\frac{1}{b} + \left(\frac{1}{a} - \frac{1}{c}\right)\right\}\left\{\frac{1}{b} - \left(\frac{1}{a} - \frac{1}{c}\right)\right\}$

$$= \left(\frac{1}{b}\right)^2 - \left(\frac{1}{a} - \frac{1}{c}\right)^2$$

$$= \frac{1}{b^2} - \left\{\left(\frac{1}{a} + \frac{1}{c}\right)^2 - \frac{4}{ac}\right\}$$

$$= \frac{1}{b^2} - \left(\frac{2}{b}\right)^2 + \frac{4}{ac} = \frac{4}{ac} - \frac{3}{b^2}$$

(68) $a_n = \frac{3 + (n-1)2}{\{n(n+1)\}^2} = \frac{2n+1}{n^2(n+1)^2} = \frac{(n+1)^2 - n^2}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$$\therefore \quad S_{11} = \sum_{i=1}^{11} a_i = \left(\frac{1}{1^2} - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \ldots + \left(\frac{1}{11^2} - \frac{1}{12^2}\right)$$

$$= 1 - \frac{1}{144} = \frac{143}{144}$$

(69) Let the side be a – d, a, a + d, a, d > 0 and let the smallest angle be A, then greatest angle is 2A and the third angle will be 180° – 3A

$$\therefore \quad \frac{a-d}{\sin A} = \frac{a}{\sin(180° - 3A)} = \frac{a+d}{\sin 2A}$$

$$\therefore \quad 3 - 4\sin^2 A = \frac{a}{a-d} \text{ and } 2\cos A = \frac{a+d}{a-d}$$

$\therefore \quad a = 5d, \quad a - d = 4d, \quad a + d = 6d$

$\therefore \quad a - d : a : a + d = 4 : 5 : 6$

(70) The given sequence is $2 + \frac{1}{3}, 6 - \frac{1}{6}, 10 + \frac{1}{12}, ...$

Hence the 6^{th} term is $22 - \frac{1}{96} = \frac{2111}{96}$

(71) By using the identily $\left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right) - \left(\sum_{i=1}^{n} a_i b_i^2\right) =$

$$(a_1b_2 - a_2b_1)^2 + (a_1b_3 - a_3b_1)^2 + ... + (a_{m-1}b_m - a_m b_{m-1})^2$$

$$\left(\sum_{i=1}^{n-1} x_i^2\right)\left(\sum_{i=2}^{n} x_i^2\right) - \left(\sum_{i=1}^{n-1} (x_i x_{i+1})\right)^2 \le 0$$

$$\Rightarrow \quad (x_1x_3 - x_2 \cdot x_2)^2 + (x_2x_4 - x_3 \cdot x_3)^2 + ... + (x_{n-2}x_n - x_{n-1}x_{n-1})^2 \le 0$$

$$\Rightarrow \quad x_1 x_3 = x_2^2, \quad x_2 x_4 = x_3^2 ... x_{n-2} x_n = x^2_{n-1}$$

$$\Rightarrow \quad \frac{x_2}{x_1} = \frac{x_3}{x_2}, \quad \frac{x_3}{x_2} = \frac{x_4}{x_3}, ... \frac{x_{n-1}}{x_{n-2}} = \frac{x_n}{x_{n-1}}$$

$\Rightarrow \quad x_1, x_2, x_3 ... x_{n-1}, x_n$ are in G.P.

(72) $1 + \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^n}$

$$= \frac{1\left(1 - \left(\frac{1}{2}\right)^{n+1}\right)}{1 - \frac{1}{2}}$$

$$= 2\left(1 - \frac{1}{2^{n+1}}\right)$$

$$= 2 - \frac{1}{2^n} < 2 \quad \forall\, n$$

so (D) is correct

(73) Coefficient of $x^8 = 1 \times 2 = 1 \times 3 + \ldots + 9 \times 10 = \sum_{r=1}^{q} r\,(r+1)$

$$= \frac{1}{2}\left[(1+2+3+\ldots+n)^2 - \left(1^2 + 2^2 + 3^2 + \ldots + n^2\right)\right]_{n=10}$$

$$= \left[\frac{n(n+1)(n-1)(3n+2)}{24}\right]_{n=10} = 1320$$

(75) Since $\frac{1}{b-c}, \frac{1}{2b-x}$ and $\frac{1}{b-a}$ are in A.P.

$\therefore \quad \frac{2}{2b-x} = \frac{1}{b-c} + \frac{1}{b-a}$

$\therefore \quad ac - \frac{ax}{2} - \frac{cx}{2} = b^2 - bx$

$\therefore \quad ac - a\frac{x}{2} - c\frac{x}{2} + \frac{x^2}{4} = b^2 - bx + \frac{x^2}{4}$

$\therefore \quad \left(a - \frac{x}{2}\right)\left(c - \frac{x}{2}\right) = \left(b - \frac{x}{2}\right)^2$

$\therefore \quad a - \frac{x}{2}, b - \frac{x}{2}, c - \frac{x}{2}$ are in G.P.

(76) Let d be the common difference of this A.P. then $\log_y x = 1 + d, \log_z y = 1 + 2d$ and

$\log_x z = \frac{1+3d}{-15}$

$\therefore \quad \frac{\log x}{\log y} \times \frac{\log y}{\log z} \times \frac{\log z}{\log x} = (1+d)(1+2d)\left(\frac{1+3d}{-15}\right)$

$\therefore \quad (1+d)(1+2d)(1+3d) = -15$

$\therefore \quad 6d^3 + 11d^2 + 6d + 16 = 0$

$\therefore \quad d = -2$

(77) Since $f(x+y) = f(x)\,f(y)\ \forall\ x, y \in N$

$\therefore \quad f(n) = f(1 + 1 + 1 + ... + 1) \quad = f(1) . f(1) . f(1) ...f(1)$ n factors

$= 3^n \quad \forall\ n \in N \qquad \because\ f(1) = 3$

Hence $\sum_{r=1}^{n} f(a+r) = \sum_{r=1}^{n} 3^{a+r} = 3^a \cdot \frac{3(3^n - 1)}{3-1} = \frac{81}{2}(3^n - 1)$

$\therefore \quad 3^{a+1} = 81 \qquad \therefore \quad a = 3$

(78) Hear $a_1 + a_5 + a_{15} + a_{26} + a_{36} + a_{40} = 210$

$\Rightarrow \quad (a_1 + a_{40}) + (a_5 + a_{36}) + (a_{15} + a_{26}) = 210$

$\Rightarrow \quad 3(a_1 + a_{40}) = 210$

$\Rightarrow \quad a_1 + a_{40} = 70$

$\Rightarrow \quad S_{40} = \frac{40}{2}(a_1 + a_{40}) = 20 \times 70 = 1400$

(79) Hear $\frac{a+b}{2} = 4$ and $\sqrt{ab} = 2$

$\therefore \quad a + b = 8$ and $ab = 4$

$\therefore \quad \frac{1}{b} + \frac{1}{a} = 2$

$\therefore \quad \frac{\frac{1}{a} + \frac{1}{b}}{2} = 1 \qquad \therefore \quad$ A.M. of $\frac{1}{a}$ and $\frac{1}{b}$ is 1

$\therefore \quad \frac{1}{a}, 1, \frac{1}{b}$ are in A.P.

(80) We know that $a + \frac{1}{a} \geq 2$ for all $a > 0$

Now $\frac{(1 + x + x^2)(1 + y + y^2)}{xy} = \left(x + \frac{1}{x} + 1\right)\left(y + \frac{1}{y} + 1\right) \geq (2+1)(2+1) \geq 9$

(81) Hear $r = \frac{b}{a}$, $\ell = c = ar^{n-1}$

$\Rightarrow \quad ar^n = cr$

$\therefore \quad S_n = \frac{a(1-r^n)}{1-r} = \frac{a - ar^n}{1-r} = \frac{a - cr}{1-r} = \frac{a - c\left(\frac{b}{a}\right)}{1-\frac{b}{a}} = \frac{a^2 - bc}{a-b}$

(82) Let $a_{n+1} - a_n = d$ and $\frac{1}{h_{n+1}} - \frac{1}{h_n} = D$, $h = 1, 2, ...9$

Now $a_{10} = 3 \quad \Rightarrow \quad a_1 + 9d = 3 \quad \Rightarrow \quad 9d = 3 - 2 = 1 \quad \Rightarrow \quad d = \frac{1}{9}$

$\therefore \quad a_4 = \frac{7}{3}$

Again $\frac{1}{h_{10}} = \frac{1}{h_1} + 9D$

$\Rightarrow \quad 9D = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$

$\Rightarrow \quad D = -\frac{1}{54}$

$\therefore \quad \frac{1}{h_7} = \frac{1}{h_1} + 6D = \frac{1}{2} + 6\left(-\frac{1}{54}\right) = \frac{7}{18} \quad \Rightarrow \quad h_7 = \frac{18}{7}$

Hence $a_4 h_7 = \frac{7}{3} \times \frac{18}{7} = 6$

(83) Hear $\frac{2}{H} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \quad \frac{2}{H} = \frac{b+a}{ab}$

$\Rightarrow \quad \frac{H}{2} = \frac{ab}{a+b}$

$$\Rightarrow \quad \frac{H}{a} = \frac{2b}{a+b}$$

$$\Rightarrow \quad \frac{H+a}{H-a} = \frac{a+3b}{b-a} \qquad \text{.........} \quad (1)$$

similarly $\quad \frac{H+b}{H-b} = \frac{3a+b}{a-b} = -\left(\frac{3a+b}{b-a}\right) \qquad \text{.........}(2)$

(1) + (2) $\Rightarrow \frac{H+a}{H-a} + \frac{H+b}{H-b} = \frac{2b-2a}{b-a} = 2$

(84) Hear $\frac{S_m}{S_n} = \frac{m^2}{n^2} \qquad \Rightarrow \qquad \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]} = \frac{m^2}{n^2}$

$$\Rightarrow \qquad \frac{a+\left(\frac{n-1}{2}\right)d}{a+\left(\frac{n-1}{2}\right)d} = \frac{m}{n}$$

Let $\frac{m-1}{2} = p^2 - 1$ and $\frac{n-1}{2} = q^2 - 1$

$\therefore \quad m = 2p^2 - 1 \quad$ and $\quad n = 2q^2 - 1$

$$\therefore \quad \frac{a+\left(p^2-1\right)d}{a+\left(q^2-1\right)d} = \frac{2p^2-1}{2q^2-1}$$

$$\Rightarrow \quad \frac{p^2\text{th term}}{q^2\text{ th term}} = \frac{2p^2-1}{2q^2-1}$$

(85) Hear n th row is an A.P. of $2n - 1$ terms with common difference. = 1 and last term $= n^2 = l$

Hence, the required sum $= \frac{n}{2}\left(2\ell - (n-1)d\right)$

$= \frac{2n-1}{2}\left(2n^2 - (2n-1-1)1\right)$

$= (2n - 1)(n^2 - n + 1) = n^3 + (n - 1)^3$

(86) Hear $\frac{A-2b}{A-a} = \frac{\frac{a+b}{2} - 2b}{\frac{a+b}{2} - a} = \frac{a-3b}{b-a}$ and $\frac{A-2a}{A-b} = \frac{b-3a}{a-b}$

Hence $\frac{A-2b}{A-a} + \frac{A-2a}{A-b} = \frac{a-3b}{b-a} + \frac{b-3a}{a-b} = -4$

(88) $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

$$= \frac{1}{1-\frac{1}{5}} + \frac{3\left(\frac{1}{5}\right)}{\left[1-\left(\frac{1}{5}\right)\right]^2}$$

$$= \frac{5}{4} + \frac{15}{16} = \frac{35}{16}$$

(89) Hear $ar^{p-1} = x,\ ar^{q-1} = y$

$\Rightarrow \quad \frac{x}{y} = r^{p-q} \qquad \Rightarrow \qquad r = \left(\frac{x}{y}\right)^{\frac{1}{p-q}}$

Now $t_n = ar^{n-1} = ar^{p-1} \cdot r^{n-p} = x \cdot \left(\frac{x}{y}\right)^{\frac{n-p}{p-q}} = x \cdot \left(\frac{x^{n-p}}{y^{n-p}}\right)^{\frac{1}{p-q}}$

$$\Rightarrow \quad \left(\frac{x^{p-q}\cdot x^{n-p}}{y^{n-p}}\right)^{\frac{1}{p-q}} = \left(\frac{x^{n-q}}{y^{n-p}}\right)^{\frac{1}{p-q}}$$

(90) Given that $t_n = p^2$ and $S_n = s^2 \Rightarrow \quad \frac{n}{2}\left(a+p^2\right) = s^2$

$$\Rightarrow \quad a + p^2 = \frac{2s^2}{n}$$

$$\Rightarrow \quad a = \frac{2s^2}{n} - p^2 = \frac{2s^2 - p^2 n}{n}$$

(91) Hear $\frac{S_1}{S_2 - S_1} = \frac{S_2}{S_4 - S_2}$

$$\Rightarrow \quad S_1 S_4 = S_2{}^2$$

$$\Rightarrow \quad a\left[\frac{4}{2}\left(2a+3d\right)\right] = \left(a+a+d\right)^2$$

$\Rightarrow \quad 2ad = d^2 \quad \Rightarrow \quad d = 2a = 2(1) = 2 \quad \therefore \quad d = 2$

(92) Let $\alpha = a - 3d$, $\beta = a - d$, $\gamma = a + d$, $\delta = a + 3d$

$\therefore \quad \alpha + \beta = 2a - 4d = \frac{b}{a}$ .... (1)

$\gamma + \delta = 2a + 4d = \frac{q}{p}$ .... (2)

$(2) - (1) \Rightarrow 8d = \frac{q}{p} - \frac{b}{a} = \frac{aq - bp}{ap}$

$\Rightarrow$ Common Difference $= 2d = \frac{aq - bp}{4ap}$

(93) Let α, β, γ and δ are $\frac{a}{r_1{}^3}, \frac{a}{r_1}, ar_1, ar_1{}^3$ respectively

Now $\alpha\beta = \frac{a^2}{r_1^4} = \frac{c}{a}$ ....(1) and

$\gamma\delta = a^2 r_1^4 = \frac{r}{p}$ .... (2)

$(2) \div (1) \Rightarrow r_1^8 = \frac{r}{p} \times \frac{a}{c}$ $\therefore$ common ratio $= r_1^2 = \left(\frac{cr}{cp}\right)^{\frac{1}{4}}$

(94) Since A, B, C are in A.P. $A + C = 2B$

$\Rightarrow \quad \pi - B = 2B$

$\Rightarrow \quad B = \frac{\pi}{3}$

Now $\sin(B + 2C) = -\frac{1}{2} = \sin\frac{7\pi}{6}$

$\Rightarrow \quad B + 2C = \frac{7\pi}{6}$

$\Rightarrow \quad 2C = \frac{7\pi}{6} - \frac{\pi}{3} = \frac{5\pi}{6}$

$\Rightarrow \quad C = \frac{5\pi}{12}$

$\therefore \quad A = \pi - (B + C) = \pi - \left(\frac{\pi}{3} + \frac{5\pi}{12}\right)$

$= \pi - \frac{9\pi}{23} = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$

(95) as $S_n = \frac{n}{2}[2a + (n-1)d]$ is pure quadratic with no constant term

$\therefore \quad a = 2, \quad d = 0$

$\therefore \quad S_n = (b - 1)n^2 + (c - 3)n$

Now $\quad t_n = S_n - S_{n-1} = (b-1)n^2 + (c - 3)n - (b - 1)(n - 1)^2 - (c - 3)(n - 1)$

$= (b - 1)(2n - 1) + (c - 3)$

$\therefore \quad d = t_n - t_{n-1} \quad = (b - 1)(2n - 1) + (c - 3) - [(b - 1)(2n - 3) + c - 3]$

$= (b - 1)(2n - 1 - 2n + 3)$

$= (b - 1)2$

$= 2(b - 1)$

(96) since $A \geq G$.

$\Rightarrow \quad \dfrac{a+b+c}{3} \geq (abc)^{\frac{1}{3}}$

$\Rightarrow \quad \dfrac{3b}{3} \geq (64)^{\frac{1}{3}} \qquad \because \quad a + c = 2b$

$\Rightarrow \quad b \geq 4$

$\therefore$ minimum value of b is 4

(97) Hear $2b = a + c$

$\Rightarrow \quad 4R \sin B = 2R (\sin A + \sin C)$

$\Rightarrow \quad 4\sin\dfrac{B}{2}\cos\dfrac{B}{2} = 2\sin\dfrac{A+C}{2}\cos\dfrac{A-C}{2}$

$\Rightarrow 2\cos\left(\dfrac{A+C}{2}\right) = \cos\left(\dfrac{A-C}{2}\right) \Rightarrow \dfrac{\cos\left(\frac{A+C}{2}\right)}{\cos\left(\frac{A-C}{2}\right)} = \dfrac{1}{2}$

$\Rightarrow \dfrac{\cos\left(\frac{A+C}{2}\right) + \cos\left(\frac{A-C}{2}\right)}{\cos\left(\frac{A+C}{2}\right) - \cos\left(\frac{A-C}{2}\right)} = \dfrac{3}{-1} \Rightarrow \cot\dfrac{A}{2}\cot\dfrac{C}{2} = 3$

(98) Let r be the common ratio of the G.P. 2, b, c, 23

$\therefore\ b = 2r, \quad c = 2r^2, \qquad 23 = 2r^3$

$(b + c)^2 + (c - 2)^2 + (23 - b)^2 = (2r - 2r^2)^2 + (2r^2 - 2)^2 + (23 - 2r)^2$

$= 4(r - r^2)^2 + 4(r^2 - 1)^2 + (23 - 2r)^2$

$= 4[r^2 + r^4 - 2r^3 + r^4 + 1 - 2r^2] + 529 - 92r + 4r^2$

$= 8r^4 - 8r^3 - 4r^2 + 4 + 529 - 92r + 4r^2$

$= 8r^4 - 4r^3 - 92r + 533$

$= 8r\left(\dfrac{23}{2}\right) - 8\left(\dfrac{23}{2}\right) - 92r + 533$

$= 92r - 92 - 92r + 533 = 441$

Answers

1-C	2-B	3-A	4-A	5-C	6-D	7-A	8-C	9-C	10-A
11-A	12-A	13-C	14-D	15-B	16-B	17-A	18-B	19-D	20-A
21-B	22-C	23-D	24-B	25-C	26-D	27-D	28-D	29-C	30-B
31-B	32-C	33-B	34-D	35-B	36-A	37-C	38-C	39-D	40-C
41-B	42-A	43-B	44-C	45-B	46-A	47-C	48-B	49-A	50-A
51-A	52-B	53-A	54-B	55-C	56-D	57-B	58-A	59-B	60-A
61-B	62-C	63-D	64-C	65-A	66-A	67-B	68-B	69-C	70-D
71-B	72-D	73-C	74-B	75-B	76-C	77-D	78-C	79-B	80-D
81-C	82-B	83-A	84-B	85-C	86-D	87-C	88-B	89-C	90-D
91-C	92-B	93-A	94-B	95-B	96-C	97-C	98-C		

Unit-8

Limit and Continuity

Important Points

Real Value Function:

If function f : A $\rightarrow$ B (Where A $\subset$ R and B $\subset$ R)

is defined then f is a real function of a real value.

Some useful real functions of real value.

(1) Constant function :

$f : A \rightarrow B,\ f(x) = c,\ \forall x \in A$ (where $A \subset R$, $B \subset R$, $A \neq \phi$, $B \neq \phi$) is called a constant function.

Here c is fixed element of c.

N.B,d $R_f = \{c\}$

(2) Identity function:

$I_A : A \rightarrow A,\ I_A(x) = x,\ \forall x \in A$ is called identity function,

N.B. : Indentity function in one-one and onto.

(3) Modulus function:

$f : R \rightarrow R^+ U \{0\}$

$$f(x) = |x| = \left.\begin{array}{l} x,\ x \geq 0 \\ -x,\ x < 0 \end{array}\right\}$$ is called modulus function.

(4) Integer part function Floor function.

(A) $f : R \rightarrow z$, f(x) = x the largest integer not exeeding x' is called integer part function.

It is denoted by [x] or [x].

$f(-3.1) = [-3.1] = [-3.1] = -4,\ f(\sqrt{7}) = \lfloor\sqrt{7}\rfloor = \lfloor\sqrt{7}\rfloor = 2$

(B) Celling funtion :

$f : R \rightarrow z,\ f(x) = x$ the smallest integer not less than x, is called celling funtion and denoted by [x].

$f(-3.1) = \lceil -3.1 \rceil = -2, \lfloor \sqrt{7} \rfloor = 3$

(5) Exponential Function

$f : R \rightarrow R^+, f(x) = a^x,\ a \in R^+$ is called exponential function.

N.B. : 1. For a = 1, R_f = {1} i.e, f is constant function.

2. for 0 < a < 1 then this function is decreasing function.

3. For a> 1 then this function is an increasing function.

4. Graph of any exponential function always passes through the point (0, 1)

(6) Logarithmic function :

$f : R^{+} \to R,\ f(x) = \log_a x,\ a \in R^{+} - \{1\}$,is logarithmic function.

N.B. :

1. Logarithmic function is an inverse function of an exponential function

Also exponential function (with R^{+}-{1} is an inverse function of logarithmic function.

i.e. Both are the inverse function of each other.

2. Working rules of :

(A) Exponential Function

For $a, b \in R^{+}\ \{1\}, x, y \in R$

$(i)\quad a^{x}, a^{y} = a^{x+y}$

$(ii)\ (ab)^{x} = a^{x} \cdot b^{x}$

$(iii)\ (a^{x})^{y} = a^{xy}$

$(iv)\ \left(\frac{a}{b}\right)^{x} = \frac{a^{x}}{b^{x}}$

$(v)\ \frac{a^{x}}{a^{y}} = a^{x-y}$

$(vi)\ \frac{1}{a^{x}} = a^{-x}$

$(vii)\ a \log_a x\ a = x\ \forall x \in R$

(B) Logarithmic function

For $a, b \in R^{+} - \{1\}, x, y \in R^{+}$

$(i)\ \log_a(xy) = \log_a x + \log y$

$(ii)\ \log_a\left(\frac{x}{y}\right) = \log_a x = \log_a y$

$(iii)\ \log_a x^{n} = n \log_a x,\ n \in R$

$(iv)\ \log_a x = \frac{\log_b x}{\log_b a}$

$(v)\ \log_a 1 = 0$

$(vi)\ \log_a a = 1$

(7) Polynomial function :

$f : A \to R,\ f(x) = a_n x^{n} + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_0;$

$a \in R,\ (i = 0, 1, 2, 3, ...n,\ a_n \neq 0)$ is called a Polynomial function

$(A \subset R,\ A \neq \phi,\ n \in N \cup \{0\})$

(8) Rational function :

$f : A \to R, f(x) = \frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomial function over (A on) and

$q(x) \neq 0,\ \forall x \in A,$ is called rational function.

(8) Signum Funtion:

$f : R \to \{-1, 0, 1\}$

$$f(x) = \begin{cases} 1 & , \quad x > 0 \\ 0 & , \quad x = 0 \\ -1 & , \quad x < 0 \end{cases}$$ is called signum function

N.B. : $f(x) = \begin{cases} \frac{|x|}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$

(10) Trigonometric functions : Inverese Trigonometric functions :

Function	Domian	Range	Function	Domian	Range
Sine	R	[-1, 1]	Sin^{-1}	[-1, 1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Cosine	R	[-1, 1]	Cos-1	[-1, 1]	$[0, \pi]$
Tangent	$R - \left\{\frac{(2k+1)\frac{\pi}{2}}{k \in z}\right\}$	R	tan-1	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Co-tangent	$R - \left\{\frac{k\pi}{k \in z}\right\}$	R	Cot-1	R	$(0, \pi)$
Secant	$R - \left\{(2k+1)\frac{\frac{\pi}{2}}{k \in z}\right\}$	R-(-1,1)	Sec-1	R-(-1, 1)	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
Cosecant	$R - \left\{\frac{k\pi}{k \in z}\right\}$	R-(-1,1)	Cosec-1	R-(-1, 1)	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(11) Even function :

If $f : A \to R, (A \subset R, A \neq \phi)$ is a function $x \in A \Rightarrow -x \in A$ yLku $f(-x) = f(x), \forall x \in A$, then f is called an even function.

(are all even functions define on their respective domain set $n \in z - \{0\}$)

(12) Odd function :

and $f : R \to R, (A \subset R, A \neq \phi)$ and $x \in A \Rightarrow -x \in A$ yLku $f(-x) = -f(x), \forall x \in A$, then f is called an odd function

and odd function define on their respective domain

- Limit of a fuction :

Let f(x) be a function define on a domain comtaining some interval but may be in the domain of f If for every f$\varepsilon > 0$ there exists some $\delta > 0$ there exists

$a-\delta > x < a+\delta,\ x \neq a,\ x \in D_f \Rightarrow \ell-\varepsilon < f(x) < \ell+\varepsilon$ whenever $x \to a$ we say left limit of

f(x) is *l* or $\lim_{x \to a} f(x) = \ell$.

- Right limit of a fuction :

If f(x) is function defined in some interval $(a-h, a), (h > 0)$ and for every $\varepsilon > 0$, there exists $\delta > 0$ such that $\ell - \varepsilon < f(x) < \ell + \varepsilon,\ \forall x \in (a-\delta, a)$ then we say tight limit of (x) is 1 as

$x \to a^+$ OR $\lim_{x \to a} f(x) = \ell$

- Algebra of Limits :

Let $\lim_{x \to a} f(x)$ exist and be equal to $\lim_{x \to a} g(x)$ exixt and be equal to,

Then (1) $\lim_{x \to a} \{f(x) + g(x)\}$exist and

$$\lim_{x \to a} \{f(x) + g(x)\} = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = \ell + m$$

(2) $\lim_{x \to a} (f(x)g(x))$ exist and

$$\lim_{x \to a} (f(x)\, g(x)) = \left\{\lim_{x \to a} f(x)\right\}\left\{\lim_{x \to a} g(x)\right\} = \ell m$$

(3) If $m \neq 0$ íkku $\lim_{x \to a} \frac{f(x)}{g(x)}$ exist and $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{\ell}{m}$

If $f(x) = c$ (C = constant) in a constant function then $\lim_{x \to a} f(x) = c$ or $\lim_{x \to a} c = c$

Theorem : 1 $\lim_{x \to a} x^n = a^n, n \in N$

Theorem : 2 If individual $\ell = 1, 2, 3, ...n$ then $\lim_{x \to a} f_i(x)$

$$\lim_{x \to a} \left(\sum_{i=1}^{n} f_i(x) \right)^{x \to a} \sum_{i=1}^{n} \lim_{x \to a} f_i(x)$$

Limt of a polynomial :

If $f(x) = c_n x^n + c_{n-1} x^{n-1} + ...+ c_0$, $x \in 0$, $c_i \in r$ $i = 0, 1, 2, ...n)$ g(x) is a polynomial of degree then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{f(a)}{g(a)} = h(a)$$

N.B. :

1. In a rational funtion $h(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$ f(x) and g(x) have a same factor (s) $(x - a)^k$ $(k \in N)$ with same index K then after cancellation of the factor (s) $(x-a)^k$, we have the limit by substituting x = a in the remaining part of the rational function.

$$\lim_{x \to a} h(x) = \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{(x-a)^k p(x)}{(x-a)^m q(x)}$$

$$= \lim_{x \to a} (x-a)^{k-m} \frac{p(x)}{q(x)}, k > m$$

= 0, If $k - m \in N$

$= \frac{p(a)}{q(a)}$ if k = m

= Limit does not exist, If k < m

2. $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n.\, a^{n-1}, (n \in N(x \neq a), x\ a \in R)$

N.B. : This result is true even for $n \in R$, while $x \in R^+, a \in R^+, x \neq a$

Rule of substitution (OR) Rule of Limit of a composite Function :

Suppose $\lim_{x \to a} f(x)$ exist and $\lim_{x \to a} f(x) = b$ and $\lim_{y \to b} g(y)$ exist and $\lim_{y \to b} g(y) = \ell$

then $\lim_{x \to a} g(f(x)) = \ell$

Two important rules :

1. If f(x) < g(x) in the same domain and both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then

$\lim_{x \to a} f(x) \le \lim_{x \to a} g(x)$

2. If $f_1(x) < f(x) < f_1(x), \forall x \in D_f = D_{f_1} = D_{f_2}$ and if $\lim_{x \to a} f_1(x)$ and $\lim_{x \to a} f_2(x)$ exist

and are $\lim_{x \to a} f_1(x) = \lim_{x \to a} f_2(x) = \ell$ exists and is equal to l.

Some important results of trignometric functions, limits.

1. $\cos x < \frac{\sin x}{x} < 1, \ \forall x, \ 0 < |x| < \frac{\pi}{2}$

(2) $|\sin x| \le |x|, \forall x \in R$

(3) $1 - \frac{x^2}{2} \le \cos x \le 1, \ \forall x \in R$

Limits:

(1) $\lim_{x \to a} |x| = 0$ then $\lim_{x \to 0} |f(x)| = 0 \ \lim_{x \to 0} f(x) = 0$

(2) $\lim_{x \to a} \sin x = 0$

(3) $\lim_{x \to a} \cos x = 1$

(4) $\lim_{x \to a} \sin x = \sin a$ and $\lim_{x \to a} \cos x = \cos a. \ (a \in R)$

(5) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(6) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

$x \to \infty$ and $x \to -\infty$ and infinite limit

(1) $\lim_{x \to \infty} f(x)$: If for every $\varepsilon > 0$, there exist $M \in R$ such that

$x > M, x \in R \Rightarrow |f(x) - \ell| < \varepsilon$ we say that f(x) =1

(2) $\lim_{x \to \infty} f(x)$: If for every $\varepsilon > 0$, there exist $M \in R$ $x < M$, $x \in R \Rightarrow |f(x) - \ell| < \varepsilon$,

we say that $\lim_{x \to \infty} f(x) = \ell$

Infinite limits:

(1) If $x \to 0_+$, $a^{\frac{1}{x}} \to \infty$; $(a > 1)$

(2) If $x \to 0_-$, $a^{\frac{1}{x}} \to 0$; $(a > 1)$

(3) If $x \to 0_+$, $a^{\frac{1}{x}} \to 0$; $(0 < a < 1)$

(4) If $x \to 0_-$, $a^{\frac{1}{x}} \to \infty$; $(0 < a < 1)$

Theorem : $\lim_{x \to a} f(x) = \ell$ and only if for every sequence $\{(a_n)\}$, $a_n (-d)f$, $\lim_{x \to \infty} a_n = a$

Impliess $\lim_{n \to \infty} f(a_n) = \ell$

Important limits :

(1) $\lim_{x \to \infty} r^n = 0$; $|r| < 1$

(2) $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$ (Where e = is an irrational number and $2 < e < 3$)

(3) $\lim_{h \to 0} \frac{a^h - 1}{h} = \log_e a$; $a \in R^+ - \{1\}$

(4) $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$

Continuity : Let f be a function defined on an interval (a, b) containing c. $c \in (a, b)$ containing $\lim_{x \to c} f(x)$ exists and is equal to f(c). then we say f(c), then we say f is continous at x = c.

If f is defined at isolated points, we say it is continous at that point consequently a function defined on a finite set $\{x_1, x_2, x_3, ...x_n\}$ is continous.

Continuity of a function on [a, b]

If f is defined on [a, b] , then f is continous on [a, b] if

(1) f is continous at every point (a, b)

(2) $\lim_{x \to a^+} f(x) = f(a)$

(3) $\lim_{x \to b^-} f(x) = f(b)$

N.B. : $f(x) = [x]$ yLku $f(x) = [x]$ are continous $\forall\ x \in R - z$ and discontinous for all $\forall\, n \in z$

Theorem : Let f and g be continous $x = c, c \in (a, b)$

(1) f + g is continous x = c

(2) kf continous x = c

(3) f - g continous x = c

(4) f x g continous x = c

(5) $\frac{k}{g}$ continous for x = c if $g(c) \neq 0$)

(6) $\frac{f}{g}$ continous for x = c if $g(c) \neq 0$)

Some important results of continuity :

1. A rational function is continous on its domain i.e.

$$\lim_{x \to a} h(x) = \lim_{x \to a} \frac{p(x)}{q(x)} = h(a).\ (q(a) \neq 0)$$

2. Sin and Cosine funtions are continous on R

3. Tageent and Secant and functions are continous $R - \left\{ \frac{(2k+1)\frac{\pi}{2}}{k \in z} \right\}$

4. Co-tangent and Cosecant functions are continous $R - \left\{ \frac{k\pi}{k \in z} \right\}$

Continuity of Composite functions :

Let $f : (a, b) \to (c, d)$ yLku $g : (c, d) \to (e, f)$ be two functions, so that gof is continous at $x_1 \in (a, b)$ and g is continous at $f(x_1) \in (c, d)$, then gof is continous at $x_1 \in (a, b)$

By the rule of limit of a composite function

$$\lim_{x \to x}, g o f(x) = \lim_{x \to x_1} g(f(x_1)) = g\left(\lim_{x \to x_1} f(x) = g(x_1)\right)$$

Question Bank

(1) $\lim_{x \to 2} \frac{4-8x+5x^2-x^3}{2x^3-9x^2+12x-4} = ?$

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) 3 (d) -3

(2) $\lim_{x \to 3} \frac{\sqrt{x^2+x+3}-\sqrt{4x+3}}{x^4-81} = ?$

(a) $\frac{1}{24\sqrt{3}}$ (b) $\frac{1}{72\sqrt{15}}$ (c) $\frac{1}{72\sqrt{3}}$ (d) $\frac{1}{24\sqrt{15}}$

(3) $\lim_{x \to \frac{\pi}{3}} \frac{1}{\pi \cot 2x - 4x \cot 2x} = ?$ ([x] = x)

(a) [-1.3] (b) $\left|[-0.75]\right|$ (c) [o. 75] (d) $\frac{[1.3]}{3}$

(4) $\lim_{x \to 0} \frac{sin 2x - \tan 2x}{x^3} = ?$

(a) 4 (b) -8 (c) -4 (d) 8

(5) $\lim_{x \to -\frac{\pi}{4}} \frac{\sin x \cdot \cos\frac{5\pi}{4} - \cos\frac{7\pi}{4}\cos x}{\pi+4x} = ?$

(a) $-\frac{1}{3}$ (b) $\frac{35}{4}$ (c) $-\frac{1}{4}$ (d) $-\frac{1}{35}$

(6) $\lim_{x \to \frac{\pi}{2}} \frac{\left\{2x \sin x \left((4k+1)\frac{\pi}{2}+\pi cosec\left((4K-1)\frac{\pi}{2}+x\right)\sin\left((4k-1)\frac{\pi}{2}-x\right)\right\}}{sec(2k\pi - x).cos\left((4k-1)\frac{\pi}{2}+x\right)} = ?$

(a) $\frac{\pi}{2}\left(Sin^2\frac{\pi}{2}+Cos^2\frac{\pi}{2}\right)$ (b) $\left|\tan^2\pi-Sec^2\pi\right|$

(c) $\frac{1}{2}\left(tan^2\pi + sec^2\pi\right)$ (d) Limit does not exist

(7) $\lim_{x\to 1}\{10\,(1 - x^{10})^{-1} - 9\,(1 - x^{9})^{-1}\} = ?$

(a) 0.5 (b) 0.05 (c) 45 (d) -45

(8) $\lim_{x\to\propto}\frac{\text{m Sin x - n Cos x}}{\text{x-}\propto} = ?$ (Where $m\sin\alpha - n\cos\alpha = 0$, m, n$\in$N, $\pi < \propto < \frac{3\pi}{2}$)

(a) $\sqrt{m^2+n^2}$ (b) $\sqrt{m^2-n^2}$ (c) $-\sqrt{m^2+n^2}$ (d) $-\frac{1}{\sqrt{m^2+n^2}}$

(9) $\lim_{x\to 1}\frac{x^{365} - 365x + 364}{(x-1)^2} = ?$

(a) 66,430 (b) 64,340 (c) 66,630 (d) 64,430

(10) $\lim_{x\to 0}\frac{(1+99x)^{100}-(1+100x)^{99}}{x^2} = ?$

(a) -4950 (b) 4950 (c) 9950 (d) -9900

(11) $\lim_{x\to\pi}\frac{25-\sqrt{626+Cosx}}{(\pi-x)^2} = ?$

(a) 0.1 (b) -0.02 (c) -0.01 (d) -0.1

(12) $\lim_{x\to 0}\frac{3}{x^3}sin\left(\pi^2+2x\right)-\frac{3}{x^3}sin\left(\pi^2+x\right)\ -\frac{1}{x^3}sin\left(\pi^2+3x\right)+\frac{1}{x^3}sin\left(\pi(1+x)\right) = ?$

(a) $Cos\pi^2$ (b) $-Cos\pi^2$ (c) $-\pi$ (d) π

(13) $\lim_{x\to 0}\frac{\tan\frac{x}{3} - \text{Sin}\frac{x}{3}}{x^3} = ?$

(a) $\frac{1}{27}$ (b) $\frac{1}{54}$ (c) $\frac{4}{27}$ (d) $\frac{5}{27}$

(14) $\lim_{x \to 0} \frac{\tan\left(\frac{\pi}{3}+2x\right) - 2\tan\left(\frac{\pi}{3}+2x\right) + \tan\frac{\pi}{3}}{x} = ?$

(a) $4\sqrt{3}$ (b) $-8\sqrt{3}$ (c) $-4\sqrt{3}$ (d) -8

(15) $\lim_{x \to \pi}$ (x - [x-3] - [3-x] = ?

Where x $\in$ (π-0.1, π+0.1) - {π}

(a) π (b) $-(\pi+1)$ (c) $\pi+1$ (d) $\pi-1$

(16) $\lim_{x \to 0} \frac{1-\cos\left(1-\cos\left(1-\cos\left(1-\cos\frac{x}{2}\right)\right)\right)}{x^{16}} = ?$

(a) $\frac{1}{2^{16}}$ (b) $\frac{1}{2^{31}}$ (c) $\frac{1}{2^{15}}$ (d) $\frac{1}{2^{32}}$

(17) If $\frac{a}{2}$ and $\frac{b}{2}$ be two distinct real roots of $lx^2+mx+n=0$ then

$\lim_{x \to \frac{a}{2}} \frac{1-\cos\left(\ell x^2+mx+n\right)}{(2x-a)^2} = ?$ (Where $l \neq 0$, a, b $\in$ R)

(a) $\frac{\ell^2}{8(a-b)^2}$ (b) $\frac{\ell^2}{32}(a^2-b^2)$

(c) $\frac{\ell^2}{32}(a-b)^2$ (d) $\frac{\ell^2}{16}(a^2-b^2)$

(18) If k^{th} term t_k, of the series is formulated as $t_k = \frac{k}{1+k^2+k^4}$ then

$\lim_{n \to \infty} \sum_{k=1}^{n} t_k =$......... is :

(a) 0 . 25 (b) 0 . 50

(c) 1 (d) Limit of the series does not exists

(19) $$\lim_{h \to 0} \frac{h^{\frac{7}{6}}\left\{a^2-(h-a)^2\right\}^{\frac{1}{3}}}{\left(2\sqrt{2ah-h^2}+2\sqrt{2ha}\right)^3} = ?$$

(a) $\frac{1}{128a^{\frac{4}{3}}}$ (b) $\frac{1}{128\sqrt{2}a^{\frac{5}{3}}}$ (c) $\frac{1}{128\sqrt[3]{2}a^{\frac{7}{6}}}$ (d) $\frac{1}{128\sqrt[6]{2}a^{\frac{7}{6}}}$

(20) $$\lim_{x \to \frac{\pi}{4}} \frac{16\sqrt{2} - (\text{Sim x} + \text{Cos x})^9}{1 - \text{Sim } 2\text{ x}} = ?$$

(a) $9\sqrt{2}$ (b) $18\sqrt{2}$ (c) $36\sqrt{2}$ (d) $16\sqrt{2}$

(21) $$\lim_{x \to 0} \left(\frac{\sum_{i=1}^{n} a_i^{x}}{n}\right)^{\frac{1}{x}} = ?$$ (Where $a_i \in R^{+}-\{1\}$ i = 1, 2, 3, ... n)

(a) $\left(\sum_{i=1}^{n} a\right)^3$ (b) $(a_1, a_2, a_3, ...a_n)^{\frac{1}{3}}$

(c) $\left(\sum_{i=1}^{n} a_i\right)^{\frac{1}{3}}$ (d) $(a_1, a_2, a_3, ...a_n)^3$

(22) $$\lim_{x \to \infty} \left(\frac{x^2 + 7x + 2013}{x^2}\right)^{7x} = ?$$

(a) e^7 (b) e^{14} (c) e^{21} (d) e^{49}

(23) $\lim_{x \to \infty} \left(\frac{\sum_{i=1}^{100} (x+i)^n}{x^n + 10^n} \right) = ?$ $(n \in N - \{1\})$

(a) n (b) 100 (c) 100n (d) 10n

(24) $\lim_{x \to 0+} \frac{\tan x + \tan^2 x + \tan^3 x + \tan^4 x + ...\infty}{\pi x}$ where $o < |x| < \frac{\pi}{4}$)

(a) $\frac{1}{\pi}$ (b) $\frac{1}{\pi^n}$ (c) π (d) 0

(25) $\lim_{x \to 0} \frac{\tan^{108}(107x)}{\log\left(1 + x^{108}\right)} = ?$

(a) $\frac{107}{108}$ (b) $(107)^{108}$ (c) $(107)^{-108}$ (d) $-\frac{107}{108}$

(26) $\lim_{x \to 0} \frac{\left(1+x^2\right)^{\frac{1}{3}} - (1\text{-}2x)^{\frac{1}{4}}}{x + x^2} = ?$

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

(27) $\lim_{n \to \infty} (0.2)^{\log_{\sqrt{5}} \left(\sum_{i=2}^{\infty} \left(\frac{1}{2}\right)^i \right)} = ...$

(a) 4 (b) $(0.2)^{\log_e 5}$ (c) $-\log_e (0.5)$ (d) $e\log_{(0.2)}^{5}$

(28) If $\lim_{x \to \infty} \frac{x^2(m-1)-(m+n)x-2013}{x+1} = 1$

then values of constant $m = ...$ and $n = ...$

(a) m = 1, n = o (b) m = 1, n = - 2

(c) m = 1, n = 1 (d) m = 1, n= - 1

(29) For the function $f(x) = \begin{cases} \dfrac{2^{-m} - x^{m}}{x^{-m} - 2^{m}} & ; x \neq 0.5 \\ + 0.0625 & \end{cases}$

If f is continous at x = 0.5 then the value of m = ...

(a) 0.5 (b) 2 (c) -2 (d) - 0.5

(30) If $2 - \delta < x < 2 + \delta$, $x \in D_f \Rightarrow 12.99 < f(x) < 13.01$ Where f(x) = 5x+3, limit $= l = 13$ then what would be the maximum value of δ ?(δ>o)

(a) 2×10^{-1} (b) 2×10^{-2} (c) 2×10^{-3} (d) 5×10^{-2}

(31) $\lim\limits_{n \to \infty} \dfrac{\sum\limits_{i=1}^{n} i^4}{n^5} = ?$

(a) 0.5 (b) 0.4 (c) 0.2 (d) 0.1

(32) $\lim\limits_{x \to 0} \dfrac{\left(\text{Sin } x^{o} + x \text{ Sin } x^{o}\right)}{\tan\left(\dfrac{x}{2}\right)^{o}} = ?$

(a) 2 (b) $(3)^{0}$ (c) 2^{0} (d) 3^{0}

(33) $\lim\limits_{y \to 0+} \dfrac{\sqrt[3]{y} + \sqrt[3]{y^2} - \sqrt[4]{y^3}}{\sqrt[3]{y} + \sqrt{y} + \sqrt[4]{y^3}}$

(a) -1 (b) Limit does not exist

(c) 1 (d) 0

(34) $\lim\limits_{x \to 0} \dfrac{\left\{\sum\limits_{i=1}^{4} x^{i}\right\} - 30}{x^3 - 8} = ?$

(a) $\dfrac{5}{2}$ (b) $\dfrac{8}{3}$ (c) $\dfrac{49}{12}$ (d) $\dfrac{43}{12}$

(35) $\lim\limits_{x \to 1} \dfrac{\sqrt{1-Cos2(x-1)}}{x-1} = ?$

(a) $\sqrt{2}$ (b) 1 (c)Limit does not exist (d) $-\sqrt{2}$

(36) $\lim\limits_{x \to \pi} \dfrac{\text{Sin } 3\,[x]}{[x]} = ?$ (Where [ ] = greatest integer part x $\in$ (π -o.01, π +0.01)

(a) 3 (b) $\dfrac{Sin3}{9}$ (c) $\dfrac{Sin9}{3}$ (d) Sin9

(37) $\lim\limits_{x \to 0} \dfrac{\tan\left(7x^3+6x^2-5x\right)}{x} = ?$

(a) -7 (b) -6 (c) -5 (d) -8

(38) $\lim\limits_{x \to 0} (\pi)^{e^x} (e)^{-\pi^x} = ?$

(a) $\dfrac{e}{\pi}$ (b) $\dfrac{\pi}{e}$ (c) 1 (d) Limit does not exist

(39) $\lim\limits_{x \to 1} \dfrac{Sin\left(n\left(\sqrt{x}-1\right)\right)}{\left(\sqrt[4]{x}-1\right)} =$ .... ($n \in R^+$, x > o)

(a) $\dfrac{n}{2}$ (b) 2 n (c) n (d) $\dfrac{n}{4}$

(40) $\lim\limits_{x \to 0} \dfrac{5 \tan x + 55 \text{ Sin } x - 555\, x}{5 \tan x - 55 \text{ Sin } x + 555\, x} = ?$

(a) $\dfrac{101}{99}$ (b) $-\dfrac{99}{101}$ (c) $-\dfrac{101}{99}$ (d) $\dfrac{99}{101}$

(41) ò $\lim\limits_{x \to 1} \dfrac{x^4-1}{x-1} = \lim\limits_{x \to m} \dfrac{m^3-x^3}{x^2-m^2}$ íkk m = ...

(a) $-\dfrac{4}{3}$ (b) $\dfrac{8}{3}$ (c) $-\dfrac{8}{3}$ (d) $-\dfrac{4}{3}$

(42) $\lim_{x \to 1} \dfrac{\sum_{i=1}^{n} (x^i - 1)}{x - 1} = ?$

(a) n (b) $\dfrac{n(n-1)}{2}$ (c) $\dfrac{n(n+1)}{2}$ (d) $\dfrac{n+1}{2}$

(43) $\lim_{x \to \frac{\pi}{4}} \dfrac{\sqrt{\cot x} - \sqrt[3]{\cot x}}{\text{cosec } x^2 - 2} = ?$

(a) $-\dfrac{1}{12}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{12}$ (d) 0

(44) $\lim_{x \to \frac{\pi}{2}} \dfrac{\sin(\cos x)\cos x}{\sin x - \text{cosec } x} = ?$

(a) 0 (b) 1 (c) Limit does not exist (d) -1

(45) $\lim_{\frac{1}{x} \to 0} \dfrac{(2+3x)^{40}\ (4+3x)^{5}}{(2-3x)^{45}} = ?$

(a) $\dfrac{40}{9}$ (b) - 35 (c) - 1 (d) $\dfrac{8}{9}$

(46) $\lim_{x \to 0} \dfrac{1 - \cos \frac{x}{2}}{\cos \frac{x}{3} - 1} = ?$

(a) $\dfrac{-3}{2}$ (b) $\dfrac{-2}{3}$ (c) $\dfrac{-9}{4}$ (d) $\dfrac{-4}{9}$

(47) $\lim_{\frac{1}{x} \to 0} x.\cos\left(\dfrac{\pi}{12x}\right)\sin\left(\dfrac{\pi}{12x}\right) = ?$ (x > 0)

(a) $\dfrac{\pi}{12}$ (b) $\dfrac{3}{\pi}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{12}{\pi}$

(48) $\lim_{x \to 0} \dfrac{(x - 1)^3 \; (1 - \cos 15\,x)}{(1 - \cos 5\,x)} = ?$

(a) 3 (b) -9 (c) -3 (d) 9

(49) $\lim_{x \to \frac{\pi}{4}} \dfrac{1 - \cot^3 x}{2 - \cot x - \cot^3 x} = ?$

(a) 0.25 (b) 0.50 (c) 0.75 (d) 0.66

(50) $\lim_{x \to 0} \dfrac{(2 - 3x)^{-5} - 2^{-5}}{x} = ?$

(a) $\dfrac{3}{32}$ (b) $\dfrac{10}{-64}$ (c) $\dfrac{15}{64}$ (d) $\dfrac{-32}{3}$

(51) $\lim_{x \to 0} \dfrac{\sqrt{5 + x^5} - \sqrt{5 - x^5}}{x^5} = ?$

(a) 5 (b) 25 (c) $\left(\sqrt{5}\right)^{-1}$ (d) $\sqrt[4]{5}$

(52) $\lim_{x \to \frac{\pi}{8}} \dfrac{\text{Sin } x - \text{Sin } \frac{\pi}{8}}{8x - \pi} = ?$

(a) $\dfrac{1}{16}\left(2+\sqrt{2}\right)$ (b) $\dfrac{1}{16}\sqrt{2-\sqrt{2}}$ (c) $\dfrac{1}{16}\left(2-\sqrt{2}\right)$ (d) $\dfrac{1}{16}\left(\sqrt{2+\sqrt{2}}\right)$

(53) $\lim_{x \to a} \dfrac{\sqrt[3]{2x + a} - \sqrt[3]{3x}}{\sqrt[3]{8a + x} - \sqrt[3]{9x}} = ?$

(a) $\dfrac{1}{8}\sqrt[3]{3}$ (b) $\dfrac{1}{8}\sqrt[3]{6}$ (c) $\dfrac{1}{8}\sqrt[3]{9}$ (d) $\dfrac{1}{8}\sqrt{3}$

(54) $\lim_{x \to \frac{\pi}{8}} (\sin 4x)^{\tan^2 4x} =$.........

(a) $e^{1/4}$ (b) $e^{-1/2}$ (c) $e^{-1/4}$ (d) $e^{1/2}$

(55) $\lim_{x \to 0} \left(\frac{1}{\sqrt[3]{8h^3 + h^4}} - \frac{1}{2h} \right) = ?$

(a) $\frac{1}{2}$ (b) $\frac{1}{68}$ (c) $-\frac{1}{12}$ (d) $-\frac{1}{48}$

(56) $\lim_{x \to -8} \frac{\sqrt[3]{x} + 2}{\sqrt{1-x} - 3} = ?$

(a) $\frac{1}{8}$ (b) $-\frac{1}{2}$ (c) $+\frac{1}{4}$ (d) $-\frac{1}{8}$

(57) $\lim_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}} = ?$

(a) -12 (b) 8 (c) -8 (d) 6

(58) $\lim_{\theta \to 0} 2\left(\frac{\sqrt{3}\ \text{Sin}\left(\frac{\pi}{6} + \theta\right) - \text{Cos}\left(\frac{\pi}{6} + \theta\right)}{\sqrt{3}\ \theta \left(\sqrt{3}\ \text{Cos}\theta - \text{Sin}\theta\right)} \right) = ?$

(a) $\frac{4}{3}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$

(59) $\lim_{x \to 0} \frac{(1 + 5x)^{\frac{3}{n}} - 1}{x} = ?$ $n \in R^+$

(a) $\frac{5}{n}$ (b) $\frac{10}{n}$ (c) $\frac{15}{n}$ (d) $\frac{1}{5n}$

(60) $\lim_{x \to \pi} \frac{1 + \text{Cos}\left((2m + 1)x\right)}{1 + \text{Cos}\left((2n - 1)x\right)} = ?$ (Where m, $n \in N-\{1\}$)

(a) $\left(\frac{2m + 1}{2n - 1}\right)^4$ (b) $\left(\frac{2m + 1}{2n - 1}\right)^2$ (c) $\left(\frac{2n - 1}{2m + 1}\right)^2$ (d) $\left(\frac{2n - 1}{2m + 1}\right)^4$

(61) $\lim_{h \to 5} \dfrac{(2h+5)^{\frac{5}{2}} - (15)^{\frac{5}{2}}}{h^3 - 125} = ?$

(a) $\sqrt{5}$ (b) $\sqrt{125}$ (c) $\sqrt{15}$ (d) $(15)^{\frac{5}{2}}$

(62) $\lim_{x \to \frac{-\pi}{4}} \dfrac{\text{Sn}3x - \text{Cos}3x}{4x + \pi} = ?$

(a) $-\dfrac{3}{2\sqrt{2}}$ (b) $\dfrac{3}{2\sqrt{3}}$ (c) $-\dfrac{3}{2\sqrt{3}}$ (d) $+\dfrac{3}{2\sqrt{2}}$

(63) $\lim_{x \to \pi} \dfrac{\sqrt{17 + Cosx} - 4}{(\pi - x)^2} = ?$

(a) $\dfrac{1}{8}$ (b) $\dfrac{1}{16}$ (c) $\dfrac{1}{24}$ (d) $\dfrac{1}{64}$

(64) $\lim_{x \to 1} \dfrac{\left(\sum_{i=1}^{3} (x+i)^2\right) - 29}{x-1} = ?$

(a) 9 (b) 12 (c) 18 (d) 30

(65) $\lim_{x \to 1} \dfrac{\left(\sum_{i=1}^{3} (x+i)^i\right) - 75}{x-1} = ?$

(a) 75 (b) 65 (c) 55 (d) 45

(66) If f(x) = $\dfrac{\left| x^3 - 3x^2 + 2x \right|}{x^3 - 3x^2 + 2x}$ then for which of the following set of the points a,

$\lim_{x \to a}$ f(x) does not exist

(a) {o} (b) {-1, 0, 1} (c) {0, 1, 2} (d) {-2, -1, 0, 1, 2}

(67) $\lim_{x \to 0} \dfrac{\left(x+\frac{\pi}{6}\right)^2 \sin\left(x+\frac{\pi}{6}\right) - \frac{\pi^2}{72}}{x} = ?$

(a) $\dfrac{\sqrt{3}\,\pi\left(\pi+4\sqrt{3}\right)}{72}$ (b) $\dfrac{\sqrt{3}\,\pi\left(\pi-4\sqrt{3}\right)}{72}$

(c) $\dfrac{\pi\left(\pi+4\sqrt{3}\right)}{72}$ (d) $\dfrac{\pi\left(\pi-4\sqrt{3}\right)}{24\sqrt{3}}$

(68) If $\lim_{x \to 0} \dfrac{Sin((n+1)x)+Sinx}{x} = \dfrac{1}{2}$ then value of n is :

(a) -2.5 (b) -0.5 (c) -1.5 (d) -1

(69) $\lim_{h \to 0} \dfrac{\sin\left(\frac{\pi}{4}+3h\right) - 3\sin\left(\frac{\pi}{4}+2h\right) + 3Sin\left(\frac{\pi}{4}+h\right)\frac{-1}{\sqrt{2}}}{h^3} = ?$

(a) $-\dfrac{\sqrt{3}}{2}$ (b) $-\dfrac{1}{\sqrt{2}}$ (c) $\dfrac{1}{\sqrt{2}}$ (d) $-\dfrac{1}{2\sqrt{2}}$

(70) $\lim_{x \to 1} \left\{m.5^{m-1}\left(5^m-(4+x)^m\right)^{-1} - n.5^{n-1}\left(5^m-(4+x)^m\right)^{-1}\right\} = ?$ (m, n$\in$N-{1}

(a) $\dfrac{m+n}{10}$ (b) $\dfrac{n-m}{10}$ (c) $\dfrac{m-n}{10}$ (d) $\dfrac{m^2-n^2}{10}$

(71) $\lim_{x \to \sqrt{2}} \dfrac{x^9 - 3x^8 + x^6 - 9x^4 - 4x^2 - 16x + 84}{x^5 - 3x^4 - 4x + 12} = ?$

(a) $11+\sqrt{2}$ (b) $11-\sqrt{2}$ (c) $\sqrt{2}-11$ (d) $11+2\sqrt{2}$

(72) $\lim_{x \to \frac{\pi}{3}} \dfrac{\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)}{2\cos\left(\frac{x}{2}-\frac{\pi}{2}\right)-1} = ?$

(a) $-\dfrac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\dfrac{1}{\sqrt{3}}$ (d) $\dfrac{-1}{2\sqrt{3}}$

(73) $\lim_{h \to \sqrt{10}} \dfrac{\left(\sqrt{5} - \sqrt{2}\right) - \sqrt{7 - 2h}}{h^2 - 10} = ?$

(a) $\dfrac{\left(\sqrt{5}-\sqrt{2}\right)}{\sqrt{10}}$ (b) $\dfrac{\left(\sqrt{5}+\sqrt{2}\right)}{6\sqrt{10}}$ (c) $\dfrac{\left(\sqrt{2}-\sqrt{5}\right)}{6\sqrt{10}}$ (d) $\dfrac{\left(\sqrt{5}+\sqrt{2}\right)}{\sqrt{10}}$

(74) $\lim_{x \to \frac{\pi}{3}} \dfrac{1-\sqrt{3}\cot x}{2\cos x - 1} = ?$

(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

(75) $\lim_{x \to 0} \dfrac{\left(x^{2n}+1\right)^{\frac{1}{2n}} - \left(x^n+1\right)^{\frac{1}{n}}}{x^n} = ?$

(a) $_n$-1 (b) $_n$(0.5) (c) $_n$(-0.5) (d) $_n$1

(76) If $f(x) = \begin{cases} \cos x - \dfrac{m\cos x}{\pi - 2x}, & x \neq \dfrac{\pi}{2} \\ -3, & x = \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ then value of m is :

(a) K = 3 (b) K = 6 (c) -3 (d) 6

(77) If $f(x) = \begin{cases} n\left(1 - x^2\right), & x > 3 \\ 3x + 1, & x \leq 3 \end{cases}$ is continuous at x = 3 then value of n is :

(a) 2.25 (b) 1.25 (c) -2.25 (d) -1.25

(78) If $f(x) = \begin{cases} \dfrac{\sin x}{k x}, & x < 0 \\ k, & x = 0 \\ \dfrac{\tan k x}{k^2 x}, & x > 0 \end{cases}$ is continuous at x = 0 then value of k is :

(a) K = 1 (b) K = 0 (c) K = $\pm$ 1 (d) K = $\pm$ 2

(79) If $f(x) = \begin{cases} m + 3nx & , x > 1 \\ 11 & , x = 1 \\ 5nx - 2m & , x < 1 \end{cases}$ is continuous at x = 1then m = ... and n = ...?

(a) m = 2, n = -3 (b) m = -2, n = 3

(c) m = 2, n = 3 (d) m = 3, n = 3

(80) If f (x) = $f(x) = \dfrac{2-(256+5x)^{\frac{1}{8}}}{(5x+32)^{1/5}-2}$ (x $\neq$ 0), then for f to be continous everywhere f(0) is equal to

(a) $\frac{2}{7}$ (b) $-\frac{7}{32}$ (c) $\frac{7}{64}$ (d) $-\frac{7}{64}$

(81) If f (x) = $\dfrac{\tan\left(\frac{\pi}{4} - x\right)}{\text{Cot } 2x}$ x $\neq \frac{\pi}{4}$ The value of $f\left(\frac{\pi}{4}\right)$ so that f is continuous at x = $\frac{\pi}{4}$ is :

(a) 0.50 (b) 0.25 (c) 0.75 (d) 1.25

(82) f(x) = $\begin{cases} \left(\frac{3}{x^2}\right)\sin 2x^2 & , x < 0 \\ \dfrac{x^2+2x+c}{1-3x^2} & , x \in [0,\infty) - \left\{\frac{1}{\sqrt{3}}\right\} \\ 0 & , x = \frac{1}{\sqrt{3}} \end{cases}$ then in order that f to be continuous at $x = 0$, value of c is :

(a) 2 (b) 4 (c) 6 (d) 8

(83) Let a function f be defined by f(x) = $\dfrac{x - |x|}{x}$, $x \neq 0$ and f(0) = 2, then f is:

(a) Continuous no where

(b) Continuous everywhere

(c) Continuous for all x except x = 1

(d)Continuous for all x except x = 0

(84) The value of k(k>0) for which the function

$$f(x)=\frac{\left(e^{x}-1\right)^{4}}{\operatorname{Sin}\left(\frac{x^{2}}{k^{2}}\right)\log\left(1+\frac{x^{2}}{2}\right)},$$

x $\neq$ o, f(o) = 8 may be continuous at x = o is:

(a) 1 (b) 2 (c) 4 (d) 3

(85) If = $\begin{cases} x + a\sqrt{2}\ \text{Sin}\ x & , \pi \le x < \frac{5\pi}{4} \\ 2x\ \text{Cot}\ x + b & , \frac{5\pi}{4} \le x < \frac{3\pi}{2} \\ a\ \text{Cos}\ 2x + b\ \text{Sinx} & , \frac{3\pi}{3} \le x \le 2\pi \end{cases}$

is continuous on [π, 2π], then a = ... and b = ...

(a) a = $\frac{5\pi}{2}$, b = $\frac{5\pi}{4}$ (b) a = - $\frac{5\pi}{2}$, b = -$\frac{5\pi}{4}$

(c) a = -$\frac{5\pi}{2}$, b = $\frac{5\pi}{4}$ (d) a = - $\frac{5\pi}{4}$, b = $\frac{5\pi}{2}$

(86) If f(x) = $\frac{5^{\frac{1}{x}}-5^{-\frac{1}{x}}}{5^{\frac{1}{x}}+5^{-\frac{1}{x}}}$, x $\neq$ o and $\lim_{x\to 0+}$ f(x) = a, $\lim_{x\to 0-}$ f(x) = b

then the value of a and b are :

(a) a = 1, b = -1 (b) a = 0, b = 1

(c) a = -1, b = 1 (d) a = 1, b = 0

(87) If f(x) = $\begin{cases} 1 + k\,x & , x \le 3 \\ 1 - k\,x^2 & , x > 3 \end{cases}$ is continuus at x=3 then the value of k is :

(a) k = o, k = 1 (b) k = o

(c) k = 1, k = -1 (d) k $\in$ R - {o, $\pm$ 1}

(88) The value p for which the function $f(x)=\dfrac{\left(4^x-1\right)^3}{\text{Sin}\left(\frac{x}{p}\right)\log\left(1+\left(\frac{x^2}{3}\right)\right)}, x\neq 0$,

f (x) = 12 $(\log 4)^3$, x = 0 may be continuous at x = 0 is :

(a) 1 (b) 2 (c) 3 (d) 4

(89) The value of m and n for which the function

$$f(x)=\begin{cases}\dfrac{\text{Sin}\,(m+1)\,x+\text{Sin}\,x}{x} & , x<0\\ n & , x=0\\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{\frac{3}{2}}} & , x>0\end{cases}$$

is continuous for $\forall x \in R$?

(a) m = $-\frac{3}{2}$, n = $\frac{1}{2}$ (b) m = $\frac{1}{2}$, n = $\frac{3}{2}$

(c) m = $\frac{1}{2}$, n = $-\frac{3}{2}$ (d) m = $\frac{5}{2}$, n = $\frac{1}{2}$

(90) $\lim\limits_{n\to\infty}\left((\log 2012)^n+(\log 2013)^n\right)^{\frac{1}{n}}$ = ?

(a) $\dfrac{\log 2012}{\log 2013}$ (b) log 2012 (c) $\dfrac{\log 2013}{\log 2012}$ (d) log 2013

(91) If f(x) = $\dfrac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}; x\neq 0$

is continuous at $x=0$ then f(0) = ...

(a) $a\sqrt{a}$ (b) $\sqrt{a}$ (c) $-\sqrt{a}$ (d) $-a\sqrt{a}$

(92) If $f(x) = \begin{cases} |x| \text{ Cos } \frac{1}{x} + 9x^2 & ; x \neq 0 \\ K & ; x = 0 \end{cases}$ is continuous at $x = 0$ then the value of K

(a) 9 (b) 6 (c) 0 (d) -9

(93) If $f(x) = \frac{\tan\left(\frac{\pi}{6} - x\right)}{\text{Cot } 3x}$; $x \neq \frac{\pi}{6}$, is continuous at $x = \frac{\pi}{6}$ then $f\left(\frac{\pi}{6}\right)$ = ...

(a) $\frac{1}{3\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2\sqrt{3}}{9}$ (d) $\frac{1}{6\sqrt{3}}$

(94) If $y = \frac{1}{t^2 + t - 2}$ yLk $t = \frac{1}{x-1}$ then y is discontinuous at $x \in$.....

(a) {1,2} (b) {1, -2} (c) $\{1, \frac{1}{2}, 2\}$ (d) $Z - \{1, \frac{1}{2}, 2\}$

(95) Let f be a non zero continuous funotion satistying f(x+y) = f(x) f(y), $\forall$x, y$\in$R, If f(z) =9 then f(30 = ?

(a) 1 (b) 27 (c) 9 (d) 6

(96) If $f(x) = \begin{cases} (\text{Sin } 2x)^{\tan^2 2x} & ; x \neq \frac{\pi}{4} \\ K & ; x = \frac{\pi}{4} \end{cases}$ is continuous $x \neq \frac{\pi}{4}$ then the value of K is :

(a) $e^{\frac{1}{2}}$ (b) $e^{-\frac{1}{2}}$ (c) e^2 (d) e^{-2}

(97) $\lim_{x \to \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 2}\right)^x$ = ?

(a) e^{-4} (b) e^2 (c) e^4 (d) e^{-2}

(98) If $f(x) = \begin{cases} \dfrac{e^{ax} - e^{x} - x}{x^2} & ; x \neq 0 \\ \dfrac{3}{2} & ; x = 0 \end{cases}$ is continuous function then the value of a is :

(a) 1 (b) -1 (c) 4 (d) 2

(99) If f(x) = ; x $\dfrac{\log\left(1 + x + x^2\right) + \log\left(1 - x + x^2\right)}{\sec x - \cos x}$; $x \neq 0$

is continuous at $x = 0$ then value of $f(0) = ...$

(a) 2 (b) -3 (c) 1 (d) -2

Hint

1. For the given rational poly f^n to find

$$\lim_{x\to a} h(x) = \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{(x-a)^k p(x)}{(x-a)^m q(x)} \qquad \lim_{x\to a}\frac{p(x)}{q(x)} = \frac{p(a)}{q(a)} \quad (K=m)$$

Given form

2. $$\lim_{x\to 3}\frac{\sqrt{x^2+x+3}-\sqrt{4x+3}}{x^4-81}$$

$$=\frac{\left\{\lim_{x\to 3}\frac{(x^2+x+3)^{1/2}-(15)^{1/2}}{(x^3+x+3)-15}\right\}\times \lim_{x\to 3}(x+4)-\left\{\lim_{x\to 3}\frac{(4x+3)^{1/2}-(15)^{1/2}}{(4x+3)-15}\right\}\times 4}{\left\{\lim_{x\to 3}\frac{x^4-3^4}{x-3}\right\}}$$

$$=\frac{\left(1/2\right)\cdot(15)^{-\frac{1}{2}}\cdot(3+4)-4\cdot\frac{1}{2}\cdot(15)^{\frac{-1}{2}}}{4\cdot 3^3}$$

$$=\frac{1}{72\sqrt{15}}$$

3. $$\lim_{x\to\frac{\pi}{2}}\frac{1}{\pi\cot 2x-4x\cot 2x} = \lim_{x\to\frac{\pi}{2}}\frac{1}{2\cot 2x\left(\frac{\pi}{2}-2x\right)}$$

New use $\lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1$

4. $$\lim_{x\to 0}\frac{\sin 2x-\tan 2x}{x^3}$$

$$=\lim_{x\to 0}\frac{\tan 2x(\cos 2x-1)}{x^3}$$

$$=-\left\{\lim_{x\to 0}\frac{\tan 2x}{2x}\right\}\times 4\times\left\{\frac{\sin x}{x}\right\}^2$$

5. $\lim_{x\to -\frac{\pi}{4}} \dfrac{\sin x\cdot\cos\frac{5\pi}{4}-\cos\frac{7\pi}{4}\cos x}{\pi+4x}$

$$=\lim_{x\to\frac{\pi}{4}} \frac{-\sin x\cdot\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\cos x}{4\left(\frac{\pi}{4}+x\right)}$$

$$= -\lim_{\left(\frac{\pi}{4}+x\right)\to 0} \frac{\sin\left(\frac{\pi}{4}+x\right)}{4\left(\frac{\pi}{4}+x\right)}$$

Ans. (c)

6. $\lim_{x\to\frac{\pi}{2}} \dfrac{\left(2x\ \sin\left(4k+1\right)\frac{\pi}{2}+x)\right)+\pi\ \mathrm{cosec}\left(\left(4K-1\right)\frac{\pi}{2}+x\right)\sin\left(\left(4k-1\right)\frac{\pi}{2}-x\right)}{\sec\left(2k\pi-x\right)\cdot\cos\left(\left(4k-1\right)\frac{\pi}{2}+x\right)}$

$$=\lim_{x\to\frac{\pi}{2}} \frac{2x\cos\ +\ \pi\left(-\sec x\right)\cdot\left(-\sin x\right)}{\sec x\cdot\sin x}$$

$$=\lim_{x\to\frac{\pi}{2}} \frac{2\times\cos^2 x}{\sin x}+\frac{\pi\sec x\cdot\sin x}{\sec\cdot\sin x}$$

= Ans (B) why ?

7. $\lim_{x\to 1}\left\{10\left(1-x^{10}\right)^{-1}-9\left(1-x^{9}\right)^{-1}\right\}$

$=\lim_{x\to 1}\left(\dfrac{10}{1-x^{10}}-\dfrac{9}{1-x^{9}}\right)$ $\left(\because \lim_{x\to 1}\dfrac{m}{1-x^m}-\dfrac{m}{1-x^m}-\dfrac{n}{1-x^n}=\dfrac{m-n}{2},\left(m,n\in N\right)\right)$

= Ans (A)

8. $\lim_{x\to\alpha}\dfrac{m\sin x-n\cos x}{x-\alpha}$

$= m\left(\lim_{x\to\alpha}\dfrac{m\sin x-\tan x.\cos x}{x-\alpha}\right),$ $\therefore\ \tan\ \alpha=\dfrac{n}{m}$

$=-\sqrt{m^2+n^2}$ $(\because ?)$

Ans (C)

9. $\lim_{x\to 1}\dfrac{x^{369}-365x+364}{\left(x-1\right)^2}$

$$=\lim_{x\to 1}\frac{x^{365}-1-365x+365}{(x-1)^2}$$

$$=\lim_{x\to 1}\frac{(x-1)\left(x^{364}+x^{363}+....+1\right)-365(x-1)}{(x-1)^2}$$

$$=\lim_{x\to 1}\frac{\left(x^{364}+x^{363}+....+1\right)-\left(1+1+1+...365\text{ times}\right)}{(x-1)}$$

= 66,463

= Ans (A)

10. $$=\lim_{x\to 0}\frac{(1+99x)^{100}-(1+100x)^{99}}{x^2}$$

= 4950

= Ans (B) (use of Binomial Thereom)

(Ý¿ÆÜ¹ß ÆÜÏÜÑÜÃÜß ÏÜ¹¹³Üß)

11. $$=\lim_{x\to \pi}\frac{25-\sqrt{626+\cos x}}{\left(\pi-x^2\right)}$$

$$=\lim_{x\to \pi}\frac{25-\sqrt{625-626-\cos x}}{\left(\pi-x^2\right)\left(25+\sqrt{626+\pi}\right)}$$

= Ans . (C) (why?)

12. $$=\lim_{x\to 0}\frac{3}{x^3}\text{Sin}\left(\pi^2+2x\right)-\frac{3}{x^3}\text{Sin}\left(\pi^2+x\right)\frac{1}{x^3}\text{Sin}\left(\pi^2+3x\right)+\frac{1}{x^3}\text{Sin}\left(\pi(1+\pi)\right)$$

$$=\lim_{x\to 0}\frac{1}{x^3}\left[3\left(\text{Sin}\left(\pi^2+2x\right)-\text{Sin}\left(\pi^2+x\right)\right)-\left(\text{Sin}\left(\pi^2+3x\right)\right)-\text{Sin}\left(\pi^2+\pi\right)\right]$$

$$=\lim_{x\to 0}\frac{1}{x^3}\left[3\left(2\text{Cos}...\text{Sin}....\right)-\left(2\text{Cos}...\text{Sin}...\right)\right]$$

$$=\lim_{x\to 0}\frac{1}{x^3}\text{Cos}...\left[3\text{Sin}...-\text{Sin}...\right]$$

$$=\lim_{x\to 0}\frac{1}{x^3}2\text{Cos}...\left[\left(3\text{Sin}...\right)-\left(\text{use sin 30 form}\right)\right]$$

= Ans (A)

13. $$=\lim_{x\to 0}\frac{\tan\frac{x}{3}-\text{Sin}\frac{x}{3}}{x^3}$$

$$= \lim_{x\to 0} \frac{\tan\frac{x}{3} - \left(1-\cos\frac{x}{3}\right)}{x^3}$$

$= \text{Ans (B)}$

14. $$= \lim_{x\to 0} \frac{\tan\left(\frac{\pi}{3}+2x\right) - 2\tan\left(\frac{\pi}{3}+x\right) + \tan\frac{\pi}{3}}{x}$$

$$= \lim_{x\to 0} \frac{\left(\frac{\sqrt{3}+\tan 2x}{1-\sqrt{3}\tan 2x}\right) - 2\left(\frac{\sqrt{3}+\tan 2x}{1-\sqrt{3}\tan 2x}\right) + \sqrt{3}}{x}$$

$$= \lim_{x\to 0} \frac{-8\tan 2x}{2x} \times \left(\frac{1+\sqrt{3}\ \tan 2x}{1-\sqrt{3}\tan 2x}\right)$$

= - 8 (Ans D)

15. $= \lim_{x\to\pi} \left(x-[x-3]-[3-x]\right) = ?$

$= x \to \pi \Rightarrow\ x\to\pi^+,\ x\to\pi^-$

$\Rightarrow\ x>\pi>3, 3<x<\pi\left(\therefore x\in ?\right)$

$\Rightarrow\ 1>x-3>0,\ x<x-3<1$

$\alpha-1<3-x<0\ \alpha-1<3-x<0$

$\Rightarrow [x-3]=0 \text{ and } [3-x]=-1$

$= \lim_{x\to\pi}\left(\pi-0-(-1)\right)$

$= \pi+1$ Ans (C)

16. $$\lim_{x\to 0} \frac{1-\cos\left(1-\cos\right)\left(1-\cos\left(1-\cos\frac{x}{2}\right)\right)}{x^{16}}$$

$$1-\text{Cos}\,2\theta = 2\text{Sin}^2\theta$$

$$1-\text{Cos}\frac{x}{2} = 2\text{Sin}\frac{x}{4}$$

$$1-\text{Cos}\frac{x}{4} = 2\text{Sin}\frac{x}{8}$$

$$1-\text{Cos}\frac{x}{8} = 2\text{Sin}\frac{x}{16}$$

= Ans (B) (?)

17. $\lim_{x\to\frac{a}{2}}\frac{1-\cos(lx+mx)}{(2x-a)^2}$

$$\lim_{x\to\frac{a}{2}}\frac{2Sin^2\left(\frac{\ell x^2+mx+n}{2}\right)}{4\left(x-\frac{a}{2}\right)^2} \qquad \text{(why?)}$$

$$=\lim_{x\to\frac{a}{2}}\frac{\ell^2}{2}\times\frac{1}{4}\times\left\{\frac{Sin\left(\frac{\ell\left(x-\frac{a}{2}\right)\left(x-\frac{b}{2}\right)}{2}\right)}{4\left(x-\frac{a}{2}\right)^2}\right\}^2\times\lim_{x\to\frac{a}{2}}\left(x-\frac{b}{2}\right)^2$$

$$=\frac{\ell^2}{8}\times(1)^2\times\frac{1}{4}(a-b)^2$$

$$=\frac{\ell^2}{32}(a-b)^2$$

= Ans (C)

18. $t_k=\frac{k}{k^4+k^2+1}=\frac{k}{(k^2+1)^2-k^2}=\frac{1}{2}\left[\frac{1}{k^2-k+1}-\frac{1}{k^2+k+1}\right]$

Now $\lim_{n\to\infty}\sum_{k=1}^{n}t_k$

$$=\frac{1}{2}\lim_{n\to\infty}\sum_{k=1}^{n}t_k\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{13}\right)+....+\left(\frac{1}{n^2-n+1}-\frac{1}{n^2+n}\right)\right]$$

$$=\frac{1}{2}\lim_{n\to\infty}\left[1-\frac{1}{n^2+n+1}\right]$$

= Ans (B) (why?)

19. $\lim_{h\to\infty}\frac{h^{\frac{7}{6}}\sqrt{2ah-h^2}}{8.h^{\frac{3}{2}}\left(\sqrt{2a-h}+\sqrt{2a}\right)^3}$

$$=\frac{1}{8}\lim_{h\to\infty}\frac{h^{\frac{3}{2}}}{h^{\frac{3}{2}}}\times\frac{\left(\sqrt[3]{2a-h}\right)}{\left(\sqrt{2a-h}+\sqrt{2a}\right)^{3}}$$

$$=\frac{1}{8}\times\frac{(2a)^{\frac{1}{3}}}{\left(2\sqrt{2a}\right)^{3}}$$

$$=\frac{\sqrt[3]{2}.a^{\frac{1}{3}}}{64.2\sqrt{2}\,a^{\frac{3}{2}}} = \text{Ans . (D) (why ?)}$$

20. $$\lim_{x\to\frac{\pi}{4}}\frac{16\sqrt{2}-(Sinx+Cosx)^{9}}{1-Sin2x}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{2^{\frac{9}{2}}-\left\{(8mx+Cosx)^{2}\right\}^{\frac{9}{2}}}{2-(1+Sin2x)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{(1+Sin2x)^{\frac{9}{2}}-2^{\frac{9}{2}}}{(1+Sin2x)-2}$$

= Ans (C) (why ?)

21. $$\lim_{x\to 0}\left(\frac{\sum_{i-1}^{n}a_i^x}{n}\right)^{\frac{1}{x}}$$

$$=\lim_{x\to 0}\left(\frac{a_1^x+a_2^x+...+a_n^x}{n}\right)^{\frac{1}{x}}$$

$$=\lim_{x\to 0}\left(\frac{1+\left(a_1^x-1\right)+\left(a_2^x-1\right)+...+\left(a_n^x-1\right)}{n}\right)^{\frac{1}{x}}$$

$$=\left(\frac{n}{1+\left(a_1^x-1\right)+\left(a_2^x-1\right)+...+\left(a_n^x-1\right)}\right)\times\frac{1}{n}\left(\frac{\left(a_1^x-1\right)}{x}+\frac{\left(a_1^x-1\right)}{x}\frac{\tan^x-11}{x}\right)$$

$$= e^{\frac{1}{n}} \{\log a_1 + \log a_2 + ... + \log a_n\} \qquad \left(\therefore \lim_{\infty \to 0} (1+\alpha)^{\frac{1}{\alpha}} = e \right)$$

$$= e^{\log_e} \ (a_1 \, a_2 \, a_3 \, ... a_n)^{\frac{1}{n}}$$

= Ans (B)

22. $$\lim_{x \to \infty} \frac{\left(x^2 + 7x + 2013\right)^{7x}}{x^2}$$

$$= \lim_{x \to \infty} \frac{\left(1 + \ 7x + \dfrac{2013}{x}\right)^{\frac{x}{\left(7+\frac{2013}{x}\right)} \times \frac{7+\left(\frac{2013}{x}\right)}{1} \times 7}}{x}$$

$$= e^{\left(\frac{7+0}{1}\right) \times 7} \text{ (why ?)}$$

= e^{49}

= Ans D

23. $$\lim_{x \to \infty} \frac{\left(\sum_{i=1}^{100} (x+i)^n \right)}{x^n + 10^n}$$

$$= \lim_{x \to \infty} \frac{\left\{ (x+1)^n + (x+2)^n + ... + (x+100)^n \right\}}{x^n + 10^n}$$

$$= \lim_{x \to \infty} \frac{\left\{ \left(1 + \dfrac{1}{x}\right)^n + \left(1 + \dfrac{2}{x}\right)^n + ... + \left(1 + \dfrac{100}{x}\right)^n \right\}}{1 + 10^n \left(\dfrac{1}{x}\right)^n}$$

$$= \frac{\left\{ (1+0)^n + (1+0)^n + ... + (1+0)^n \right\}}{1+0} \text{ (why ?)}$$

= 100 = Ans (B)

24. $$\lim_{x \to \infty} \frac{\tan \alpha + \tan^2 x + ... \infty}{\pi x} \left(\therefore 5_n = \frac{aC_1 - r^n}{1-r} \right), |r| < 1$$

$$\lim_{x \to \infty} \left(\frac{\tan x}{1 - \tan x} \right) \lim_{n \to \infty} S_n = \frac{a}{1-r}$$

$=\frac{1}{\pi}$ = Ans (A)

25. $\lim_{x\to 0}\frac{\tan^{108}(107x)}{\log(1+x^{108})}$

$=\lim_{x\to 0}\left\{\frac{\tan(107x)}{(107x)}\right\}^{108}\times\frac{(107x)^{108}}{\log(1+x^{108})}$

$=(107)^{108}\times\frac{1}{\log_e e}$

$=(107)^{108}$

= Ans (B)

26. $\lim_{n\to\infty}=\frac{(1+x^2)^{\frac{1}{3}}-(1+2x)^{\frac{1}{4}}}{1+x^2}$

$$=\lim_{x\to\infty}\frac{\frac{x^2(1+x^2)^{\frac{1}{3}}-1}{x^2}-\frac{(1-2x)^{\frac{1}{4}}-1(-2x)}{(-2x)}}{x+x^2}$$

Ans = $\frac{1}{2}$

27. $\lim_{x\to\infty}(0.2)\log_{\sqrt{5}}\left(\sum_{i=2}^{\infty}\left(\frac{1}{2}\right)^{i}\right)$

$=\lim_{n\to\infty}\left(\frac{1}{5}\right)\log_{\sqrt{5}}\left\{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\ldots\infty\right\}$

(For infineste G.S.)

$\lim_{n\to\infty}S_n=\frac{a}{1-r}1r_1<1$

Ans . (A)

28. $\lim_{x\to\infty}\frac{x^2(m-1)}{x+1}-\frac{(m+n)x}{x+1}-\frac{2013}{x+1}$

$=\lim_{x\to\infty}\frac{x(m-1)}{1+\frac{1}{x}}-\frac{(m+n)x}{1+\frac{1}{x}}-\frac{2013}{(x+1)}$

Here limit exists and it is equal to 1

$\therefore m-1=0 \Rightarrow m=1$

$$-\frac{(m+n)}{1+o}=1\left(\because \lim_{x\to\infty}\frac{1}{x}=0\right)$$

$\therefore m=1, n=-2 \Rightarrow n=-2$

29. Here f is continuous

at x = 0.5 = $\frac{1}{2}$

$$\therefore \lim_{x\to\frac{1}{2}} f(x)=f\left(\frac{1}{2}\right)=-0.0625$$

$$\therefore \lim_{x\to 0.5}\frac{2^{-m}-x^{m}}{x^{-m}-2^{m}}=-0.0625$$

$$\therefore \lim_{x\to\frac{1}{2}}\frac{x^{m}-\left(\frac{1}{2}\right)^{m}}{x^{-m}-\left(\frac{1}{2}\right)^{-m}}=-0.0625$$

$$\therefore (0.5)^{2m}=\frac{5^4}{10^4}=(0.5)^4$$

$\therefore m=2$ Ans (B)

30. f (x) = 5x + 3 and $2-\delta<x<2+\delta, x\in f \Rightarrow 12.99 \Rightarrow < f(x)<13.\& l=13$

12.99 < f (x) < 13.01

$\Rightarrow 13-0.01<5x+3<13+0.01$

$\Rightarrow 10-0.01<5x<10+0.01$

$\Rightarrow 2-0.02<x<2+0.002$

Comparing with C

$2-\delta<x<2+\delta$

$\underline{\delta=0.002=2\times 10^{-3}}$ Ans (C)

31. $$\lim_{n\to\infty}\frac{\sum_{i=1}^{n} i^4}{n^5}$$

$$=\lim_{n\to\infty}\frac{n}{30}\frac{(n+1)(6n^3+9n^2+n-1)}{n^5} \qquad =\frac{1}{5}=0.2 = \text{Ans (C)}$$

32. $$\lim_{x\to 0}\frac{\left(Sinx^{\circ}+xSinx^{\circ}\right)}{\tan\left(\frac{x}{2}\right)^{\circ}}$$

$$\lim_{x\to 0}\frac{Sin\frac{\pi x}{180}+xSin\frac{\pi x}{180}}{\tan\left(\frac{\pi x}{360}\right)}$$

= Ans (A)

33. $$\lim_{y\to 0^{+}}\frac{\sqrt[3]{y}+\sqrt[3]{y^{2}}-\sqrt[4]{y^{3}}}{\sqrt[3]{y}+\sqrt{y}+\sqrt[4]{y}^{3}}$$

= 1 = Ans (C)

34. $$\lim_{x\to 2}\frac{\left(x^{1}+x^{2}+x^{3}+x^{4}\right)-\left(2^{1}+2^{2}+2^{3}+2^{4}\right)}{(x-2)\left(x^{2}+2x+4\right)}$$

35. $$\lim_{x\to 1}\frac{\sqrt{1-Cos(x-1)}}{x-1}$$

36. $$\lim_{x\to \pi}\frac{Sin[x]}{[x]}$$

$x\to\pi^{+}$

$\to x>\pi\geq 3.14$ $\qquad$ $x\to\pi^{-}$

$\Rightarrow [x]=3$ $\qquad$ $\Rightarrow 3<x<\pi$

$\Rightarrow [x]=3$

37. $$\lim_{x\to 0}\frac{\tan\left\{\left(7x^{3}+6x-5\right)\right\}}{x(7x^{2}+6x-5)}\times\lim_{x\to 0}(7x^{2}+6x-5)$$

= − 5

Ans . (C)

38. $$\lim_{x\to 0}(\pi)^{e^{x}}(e)^{-\pi x}$$

$$=\frac{\lim_{x\to 0}(\pi)e^{x}}{\lim_{x\to 0}(e)^{\pi x}}$$

Ans (B)

39. $=\lim\limits_{x\to I}\dfrac{Sin\left(\left(n\sqrt{x}-1\right)\right)}{n\left(\sqrt{x}-1\right)}\times\lim\limits_{x\to1}\left(\sqrt[4]{x}+1\right)\times n$

= 2n = Ans (B)

40. $=\lim\limits_{x\to0}\dfrac{5\tan x+55\,Sinx-555x}{5Sinx-55\tan x+555x}$

= Ans (B)

41. $\lim\limits_{x\to1}\dfrac{x^4-1^4}{x-1}=-\lim\limits_{x\to m}\dfrac{x^3-m^3}{x^2-m^2}$

m = Ans (C)

42. $\lim\limits_{x\to1}\dfrac{\sum\limits_{i=1}^{n}(x^i-1)}{x-1}$

$=\left\{\lim\limits_{x\to1}\dfrac{x^1-1^1}{x-1}+\lim\limits_{x\to1}\dfrac{x^2-1^2}{x-1}+...+\lim\limits_{x\to1}\dfrac{x^n-1^n}{x-1}\right\}$

$=\dfrac{n}{2}\left(n+1\right)$ = Ans (c)

43. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{\cot x}-\sqrt[3]{\cot x}}{Co\sec^2 x-2}=\dfrac{1}{12}$ = Ans (C)

Taking cot x = t^6

$x\to\dfrac{\pi}{4}\Rightarrow t\to1$

44. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{Sin\,(Cosx).Cosx}{Sinx-Co\sec x}$

$=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{Sin\,(Cosx).Cosx.Sinx}{-Co\,s^2\,x}$

= Ans (D)

45. $=\lim\limits_{\frac{1}{x}\to0}\dfrac{(2+3x)^{40}.(4+3x)^5}{(2-3x)^{45}}$

As x > 0 and $\dfrac{1}{x}\to0$

$\Rightarrow x\to\infty$

$=\dfrac{3^{45}}{-3^{45}}=-1$, Ans (C)

46. $\lim_{x\to 0} \frac{1-Cos\frac{x}{2}}{Cos\frac{x}{3}-1}$

$\lim_{x\to 0} \frac{2Sin^2\frac{x}{4}}{-2Sin^2\frac{x}{6}}$

= Ans (C)

47. $\lim_{\frac{1}{x}\to 0^+} xCos\left(\frac{\pi}{12x}\right)Sin\left(\frac{\pi}{12x}\right)$

As $\frac{1}{x}\to 0^+ \Rightarrow \infty$

$=\lim_{x\to\infty}\left\{Cos\left(\frac{\pi}{12}\left(\frac{1}{x}\right)\right)\right\}\times\left\{\lim_{x\to\infty}\frac{Sin\left(\frac{\pi}{12}\left(\frac{1}{x}\right)\right)}{\left(\frac{\pi}{12}\right)\left(\frac{1}{x}\right)}\times\frac{\pi}{12}\right.$

$=\frac{\pi}{12}$ Ans . (A)

48. $1-Cos\,x=2Sin^2\frac{x}{2}$ use formula

Ans (B)

49. Factrorise

Ans (C)

50. $\lim_{x\to 0}\left\{\frac{(2-3x)^{-5}-2^{-5}}{(2-3x)-2}\right\}(-3)$

Ans (C)

51. Conjugate Surd

Ans (C)

52. $=\lim_{x\to\frac{\pi}{8}}\frac{Sinx-Sin\frac{\pi}{8}}{8x-\pi}$

$$=\lim_{x\to\frac{\pi}{8}} \frac{2Cos\left(\frac{x}{2}+\frac{\pi}{8}\right).Sin\left(\frac{x}{2}-\frac{\pi}{18}\right)}{16\left(\frac{x}{2}-\frac{\pi}{16}\right)}$$

$$=\frac{1}{8}Cos\left(\frac{\pi}{8}\right)$$

= Ans (D)

53. $\lim_{x\to a}\frac{\sqrt[3]{2x+a}-\sqrt[3]{3x}}{\sqrt[3]{8a+x}-\sqrt[3]{9x}}$

$$=\frac{1}{8}\times\frac{3^{\frac{4}{3}}.a^{\frac{2}{3}}}{3^{\frac{2}{3}}.a^{\frac{2}{3}}}$$

= Ans (C)

54. $\lim_{x\to\frac{\pi}{8}}\left(Sin4x\right)^{\tan^2 4x}$

$$=\lim_{x\to\frac{\pi}{8}}\left(1-Cos^2 4x\right)\frac{Sec^2 4x-1}{2}$$

$$=\left\{\lim_{x\to\frac{\pi}{8}}\left(1+\left(-Cos^2 4x\right)\right)^{\frac{-1}{\cos^2 4x}}\right\}^{\frac{-1}{2}}\left\{\lim_{x\to\frac{\pi}{8}}\left(1+\left(-Cos^2 4x\right)\right)\right\}^{\frac{-1}{2}}$$

$= e^{-\frac{1}{2}}$ = Ans (B)

55. $=\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{(8+h)^{\frac{1}{3}}}-\frac{1}{(8)^{\frac{1}{3}}}\right\}$

$$=\lim_{h\to 0}\frac{1}{h}\,\frac{(8+h)^{-\frac{1}{3}}-8^{\frac{-1}{3}}}{(8+h)-8}$$

$$=\frac{-1}{3}(8)^{\frac{-4}{3}}$$

$=-\frac{1}{48}$ Ans (D)

56. $\lim_{x\to -8} \frac{\sqrt[3]{x}+2}{\sqrt{1-x}-3}$

$$=\left\{\frac{\lim_{x\to -8}\frac{x^{\frac{1}{3}}-(-8)^{\frac{1}{3}}}{x-(-8)}}{\lim_{(1-x)\to 9}\frac{(1-x)^{\frac{1}{2}}-9^{\frac{1}{2}}}{(1-x)-9}}\right\}$$

$$=\frac{2}{3}\times\frac{3}{4}$$

$$=\frac{-1}{2}$$ Ans (B)

57. $\lim_{x\to 2} \frac{2^x+2^{3-x}-6}{2^{\frac{-x}{2}}-2^{1-x}}$

$$=\lim_{x\to 2}\frac{(2^x)^2-6.2^x+8}{2^{\frac{-x}{2}}-2}$$

$$=\lim_{x\to 2}\frac{l^2-6l+8}{\sqrt{t}-2}$$ $(2^x = t)$

= 8 Ans (B)

58. $\lim_{\theta\to 0} 2\frac{\sqrt{3}\,Sin\left(\frac{\pi}{6}+\theta\right)-Cos\left(\frac{\pi}{6}+\theta\right)}{\sqrt{3}\theta\left(\sqrt{3}Cos\theta-\frac{1}{2}Sin\theta\right)}$

$$=\frac{4(1)}{\sqrt{3}\left(\sqrt{3}-0\right)}$$

$$=\frac{4}{3}$$ = Ans (A)

59. $\lim_{x\to 0}\frac{(1+5x)^{\frac{3}{n}}-1}{x}$

$$=5\lim_{(1+5x)\to 1}\frac{(1+5x)^{\frac{3}{n}}-1^{\frac{3}{n}}}{(1+5x)-1}$$

$$==5\left(\frac{3}{n}\right)(1)^{\frac{3}{n}-1}$$

$$=\frac{15}{n} \quad = \text{Ans (C)}$$

60. $$=\lim_{x\to\pi}\frac{Sin^2\,(2m+1)x}{Sin^2\,(2n-1)x}\times\frac{1-\cos((2n-1)x)}{1-\cos((2m+1)x)}$$

$$=\left(\frac{2m+1}{2n-1}\right)^2 \quad = \text{Ans (B)}$$

61. $$\lim_{h\to5}\frac{(2h+5)^{\frac{5}{2}}-(15)^{\frac{5}{2}}}{h^3-5^3}$$

$$=\frac{n\left(\frac{m}{2}\right)-m\left(\frac{n}{2}\right)}{mn}.5^{m+n-3-n+2}=\frac{m-n}{10}$$

$$=2\left\{\frac{\lim\limits_{(2h+5)\to15}\dfrac{(2h+5)^{\frac{5}{2}}-(15)^{\frac{5}{2}}}{(2h+5)-15}}{\left\{\lim\limits_{h\to5}\dfrac{h^3-5^3}{h-5}\right\}}\right\}$$

$$=\frac{1}{15}(15)^{\frac{3}{2}}$$

$$=\sqrt{15}= \text{Ans (C)}$$

62. $$\lim_{x\to-\frac{\pi}{4}}\frac{Sin3x-Cos3x}{4x+\pi}$$

$$\lim_{x\to-\frac{\pi}{4}}\frac{(38\sin x-4\sin^3 x)-(4C\,os^3 x-3Cosx)}{4\left(x+\frac{\pi}{4}\right)}$$

Ans . (A)

63 $$\lim_{x\to\pi}\frac{(17+\cos x)^{\frac{1}{2}}-16^{\frac{1}{2}}}{(\pi-x)^2}$$

$$\lim_{x\to\pi}\frac{(17+\cos x-16)}{(\pi-x)^2}\times\frac{1}{(17+\cos x)^{1/2}+16^{1/2}}$$

$$\left\{\lim_{\pi-x\to 0}\frac{\sin(\pi-x)}{(\pi-x)}\right\}^2\times\frac{1}{(1-(-1))\left((17-1)^{1/2}+4\right)}$$

$$=(1)\times\frac{1}{2\times 8}$$

$$=\frac{1}{16}\ \ =\text{Ans. (B)}$$

64. $$\lim_{x\to 1}\frac{(x+1)^2-2^2}{(x+1)-2}+\lim_{x\to 1}\frac{(x+2)^2-3^2}{(x+2)-3}+\frac{(x+3)^2-4^2}{(x+3)-4}$$

$$=2(2)^1+2(3)^1+2(4)^1$$

= 18 = Ans. (C)

65. $$\lim_{x\to 1}\frac{(x+1)^1-2^1}{(x+1)^1-2^1}+\lim_{x\to 1}\frac{(x+2)^2-3^2}{(x+2)-3}+\lim_{x\to 1}\frac{(x+3)^3-4^3}{(x+3)-4}$$

= 55 = Ans . (C)

66. $$f(x)=\frac{|x|\ |x-1|}{x\cdot(x-1)(x-2)}$$

It is clear that

$$\lim_{x\to 0}\frac{|x|}{x}\quad\text{does not exist}$$

$$\lim_{x\to 1}\frac{|x-1|}{(x-1)}\ \text{does not exists.}$$

and $\lim_{x\to 2}\frac{|x-2|}{(x-2)}$ does not exist.

$\therefore$ Requiud Ans. (C)

67. $$\lim_{x\to 0}\frac{\left(x+\frac{\pi}{6}\right)^2\cdot\sin\left(x+\frac{\pi}{6}\right)-\frac{\pi^2}{72}}{x}$$

try your self.

Ans. (A)

68. $$\left(\lim_{x\to 0^-}\frac{\sin(n+1)x}{(n+1)\cdot x}\right)(n+1)+\lim_{x\to 0^-}\frac{\sin x}{x}=\frac{1}{2}$$

$$\therefore (n+1)(1)+1=\frac{1}{2}$$

$$(n+2)=\frac{1}{2}$$

$$n=\frac{1}{2}-2=-3/2=-1.5 \quad \text{Ans. (C)}$$

69. $$\lim_{h\to 0}\frac{Sin\left(\frac{\pi}{4}+3h\right)-3Sin\left(\frac{\pi}{4}+2h\right)+3Sin\left(\frac{\pi}{4}+h\right)-\frac{1}{\sqrt{2}}}{h^3}$$

(Solution of this example applying the method of ex. 12)
= Ans (B)

70. $$\lim_{x\to 1} m.5^{m-1}\left(5^m-(4+x)^m\right)^{-1}-n.5^{n-1}(5^n-(4+x)^{n-1})$$

Suppose
x = 1 + h
As $x\to 1 \Rightarrow h\to 0$

$$=\lim_{h\to 0}\left\{\frac{m.5^{m-1}}{(5^m-(5+h)^m)}-\frac{n.5^{n-1}}{(5^n-(5+h)^n}\right\}$$

$$=\lim_{h\to 0}\frac{m.5^{m-1}\,5^n-(5+h)^n-n.5^{n-1}(5^m-(5+h)^m)}{(5^m-(5+h)^m.(5^n-(5+h)^n)}$$

$$=\frac{n\left(\frac{m}{2}\right)5^{m+n-3}-m\left(\frac{n}{2}\right)^{5n-2}}{mn.5^{n-1}.5^{m-1}}$$

$$\left(\therefore \lim_{h\to 0} \& \lim it\ of\ a\ polynomial\ function\right)$$

$$=\frac{n\left(\frac{m}{2}\right)-m\left(\frac{n}{2}\right)}{mn}.5^{m+n-3-n+2}=\frac{m-n}{10} \quad = \text{Ans (C)}$$

$$=\frac{\sqrt{2}}{4}\lim_{x\to\frac{-\pi}{4}}\left\{\frac{\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x}{\left(x+\pi/4\right)}\right\}\cdot\lim_{x\to\frac{-\pi}{4}}(3-4+4\sin x\cos x)$$

$$=\frac{1}{2\sqrt{2}}(1)\times\left(-1+4\left(-\frac{1}{\sqrt{2}}\right)\cdot\left(\frac{1}{\sqrt{2}}\right)\right)$$

$$= \frac{1}{2\sqrt{2}} = \text{Ans .(A)}$$

71. $$\lim_{x\to\sqrt{2}} \frac{x^9 - 3x^8 + x^6 - 9x^4 - 4x^2 - 16x + 84}{x^5 - 3x^4 - 4x + 12}$$

$\left(x-\sqrt{2}\right)$ is the factor of the numerator and denominator both.

$\therefore \left(x+\sqrt{2}\right)$ must be a factor, as the co-efficients are rational.

$\therefore \left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right) = x^2 - 2$ is also a factor.

$$\lim_{x\to\sqrt{2}} \frac{\left(x^2-2\right)\left(x^7+3x^6+2x^5-5x^4+4x^3-19x^2+8x-42\right)}{\left(x^2-2\right)\left(x^3-3x^2+2x-6\right)}$$

$$= \frac{32\sqrt{2}-124}{4\sqrt{2}-12}$$

$$= \frac{-77-7\sqrt{2}}{-7}$$

$$= 11+\sqrt{2} = \text{Ans. (A)}$$

72. $$\lim_{x\to\pi/3} \frac{\sin\left(x/2 - \pi/6\right)}{2\cos\left(x/2 - \pi/2\right) - 1}$$

$$\lim_{x\to\frac{\pi}{3}} \frac{\sin\frac{X}{2}\cdot\frac{\sqrt{3}}{2} - \cos\frac{x}{2}\cdot\frac{1}{2}}{2\left(\cos\frac{x}{2}\cdot 0 + \sin\frac{x}{2}\cdot 1\right) - 1}$$

$$= -\frac{1}{2}\lim_{x\to\frac{\pi}{3}} \frac{\left(3\sin^2 x/2 - \cos^2 x/2\right)}{\left(1-2\sin x/2\right)\cdot\left(\sqrt{3}\sin x/2 + \cos x/2\right)}$$

$$= \frac{1}{2}\lim_{x\to\frac{\pi}{3}} \frac{\left(1+2\sin x/2\right)}{\left(\sqrt{3}\sin x/2 + \cos x/2\right)}$$

$$=\frac{1}{2}\times\frac{\left(1+2\left(\frac{1}{2}\right)\right)}{\left(\sqrt{3}\cdot\left(\frac{1}{2}\right)+\frac{\sqrt{3}}{2}\right)}$$

$$=\frac{1}{\sqrt{3}}=\text{Ans. (c)}$$

73. $$\lim_{h\to\sqrt{10}}\frac{\left(\sqrt{5}-\sqrt{2}\right)-\sqrt{7-2h}}{h^2-10}$$

$$=\frac{-\left\{\lim_{h\to\sqrt{10}}\frac{(7-2h)^{1/2}-\left(7-2\sqrt{10}\right)^{1/2}}{(7-2h)-\left(7-2\sqrt{10}\right)}\right\}\times(-2)(h-\sqrt{10})}{(h-\sqrt{10})(h+\sqrt{10})}$$

$$=\frac{2\cdot\frac{1}{2}\left(7-2\sqrt{10}\right)^{-\frac{1}{2}}}{\left(\sqrt{10}+\sqrt{10}\right)}$$

$$=\frac{\left(\sqrt{5}+\sqrt{2}\right)}{6\sqrt{10}}=\text{Ans. (B)}$$

74. $$\lim_{x\to\frac{\pi}{3}}\frac{1-\sqrt{3}\cot x}{2\cos x-1}$$

$$=\lim_{x\to\frac{\pi}{3}}\frac{\left(\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x\right)}{\left(\cos x-\cos\frac{\pi}{3}\right)\times\sin x}$$

$$=\lim_{x\to\frac{\pi}{3}}\left.\frac{\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}+\frac{x}{2}\right)\cdot\sin x}\right\}$$

$$=-\frac{\cos 0}{\sin\pi/3\cdot\sin\pi/3}$$

$= -\frac{4}{3}$ = Ans . (B)

75. $\lim_{x\to 0} \frac{\left(x^{2n}+1\right)^{\frac{1}{2n}} - \left(x^n+1\right)^{\frac{1}{n}}}{x^n}$

$= \left\{\lim_{x\to 0} x^n\right\} \times \left\{\lim_{\left(x^{2n}+1\right)\to 1} \frac{\left(x^{2n}+1\right)^{\frac{1}{2n}} - 1^{\frac{1}{2n}}}{\left(x^{2n}+1\right)-1}\right\} - \left\{\lim_{\left(x^{n}+1\right)\to 1} \frac{\left(x^{n}+1\right)^{\frac{1}{n}} - 1^{\frac{1}{n}}}{\left(x^{n}+1\right)-1}\right\}$

$= \frac{-1}{n}$ Ans. (A)

76. Since f is continuous at $x = \frac{\pi}{2}$,

(y°k f y $x = \frac{\pi}{2}$ ykøk¤ Mkíkík A)

$\therefore \lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

77. Since f is continuous at x = 3

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} f(x) = f(3).$

78. Since f is continuous at x = 0

$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x)$

79. Since f is continuous at x = 1

$\lim_{x\to 1^+} f(x) = f(1) = \lim_{x\to 1^-} f(x)$

80. Here f is continuous $\forall n \in R$

$\therefore \lim_{x\to 0} f(x) = f(0)$.

81. f is continuous at $x = \pi/4$ if ,

$\lim_{x\to\pi/4} f(x) = f\left(\pi/4\right).$

82. Since f is continuous (Mkíkík) at x = 0 .

$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x)$

83. $f(x) = \frac{x - |x|}{x}$ $n \neq 0$

$$\therefore \lim_{x\to 0^+} f(x) = \frac{x-x}{x} = 0 \quad \& \quad \therefore \lim_{x\to 0^-} \frac{x-(-x)}{x} = 2$$

84. Let f is continuous at x = 0

$$\therefore \lim_{x\to 0} f(x) = f(0)$$

85. Since f is continuous on $[\pi, 2\pi]$ and $\frac{5\pi}{4}$. and $\frac{3\pi}{2} \in [\pi, 2\pi]$, f is also continuous at $x = \frac{5\pi}{4}$ and $x = \frac{3\pi}{2}$.

86. $f(x) = \frac{5^{1/x} - 5^{-1/x}}{5^{1/x} + 5^{-1/x}}$, $x \neq 0.$

87. Since is continuous at x = 0

$$\therefore \lim_{x\to 3^+} f(x) = f(3) = \lim_{x\to 3^-} f(x)$$

88. Let f be the continuous at x = 0

$$\therefore \lim_{x\to 0} f(x) = f(0)$$

89. Since f is continuous for every $x \in R$,
so it is continuous at x = 0

$$\therefore \lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x)$$

90. $\lim_{x\to\infty}\left(\left(\log 2012\right)^n + \left(\log 2013\right)^n\right)^{\frac{1}{n}}$

$$\lim_{x\to\infty} \log\left(2013\right)\left\{\left(\frac{\log 2012}{\log 2013}\right)^n + 1\right\}^{\frac{1}{n}}$$

91. f is continuous at x = 0

$$\lim_{x\to 0} f(x) = f(0)$$

93. Since f is continuous at x = 0

$$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x)$$

94. here f is continuous at $x = \frac{\pi}{6.}$

$$\lim_{x\to\frac{\pi}{6}} f(x) = f\left(\frac{\pi}{6}\right)$$

95. Let $t = f(x) = \frac{1}{x-1}$ is discontinuous at x = a,

$$y=g(t)=\frac{1}{t^2+t-1}=\frac{1}{(t+2)(t-1)}$$

Now t = - 2 then $-2=\frac{1}{x-2}\Rightarrow x=\frac{1}{2}$

If t = 1 then $1=\frac{1}{x-2}\Rightarrow x=2.$

Thus composite function is discontinuous at x = 1 , $1/2$, 2

96. Here $f(x+y)=f(x)f(y),\quad \forall x,\ y\in R$

Any non zero continuous function satisfying the given functional equation is of the form a^x for some $a\in R^+-\{1\}$ since f(2) = 9, so $a^2=9$

$\therefore a=3.\quad f(x)=a^x=3^x$

$\therefore f(3)=3^3=27$ Ans. (B)

97. Since

f(x) is contionuous at $x=\frac{\pi}{4}$

$$\lim_{x\to\frac{\pi}{4}} f(x)=f\left(\frac{\pi}{4}\right)$$

$$\therefore f\left(\frac{\pi}{4}\right)=\lim_{x\to\frac{\pi}{4}}(\sin\ 2x)\tan^2\ 2x$$

$$=\lim_{x\to\frac{\pi}{4}}\left(1-\cos^2 2x,\right)^{\frac{\sec^2 2x-1}{2}}$$

$$=\left\{\lim_{x\to\frac{\pi}{4}}\left(1+\left(-\cos^2 2x\right)\right)^{-\frac{1}{\cos^2 2x}}\right\}^{-\frac{1}{2}}\times\left\{\left(1-\cos^2 2x\right)^{-\frac{1}{2}}\right\}$$

98. (P. T. O.)

For Example (98)

99. Here is continuous function at x = 0

$$\therefore \lim_{x\to 0} f(x)=f(0)$$

$$\therefore \lim_{x\to 0}\frac{e^{ax}-e^x-x}{x^2}=\frac{3}{2}$$

$$\therefore \lim_{x\to 0}\frac{a\cdot e^{ax}-e^x-1}{2x}=\frac{3}{2}\qquad\left(\because \lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}.\right)\left(For\ \frac{0}{0}\right)$$

For the existences of the

limit. $\lim_{x \to 0} a \cdot e^{ax} - e^{x} - 1 = 0$

100. Since f is continuous at x = 0

$\lim_{x \to 0} f(x) = f(0)$

Answers

1	B	26	B	51	C	76	D
2	B	27	A	52	D	77	D
3	C	28	B	53	C	78	C
4	C	29	B	54	B	79	C
5	C	30	C	55	D	80	C
6	B	31	C	56	B	81	A
7	A	32	A	57	B	82	C
8	C	33	C	58	A	83	D
9	A	34	C	59	C	84	B
10	B	35	C	60	B	85	C
11	C	36	C	61	C	86	A
12	A	37	C	62	A	87	B
13	B	38	B	63	B	88	D
14	D	39	B	64	C	89	A
15	C	40	B	65	C	90	D
16	B	41	C	66	C	91	C
17	C	42	C	67	A	92	C
18	B	43	C	68	C	93	C
19	D	44	D	69	B	94	C
20	C	45	C	70	C	95	B
21	B	46	C	71	A	96	B
22	D	47	A	72	C	97	B
23	B	48	B	73	B	98	C
24	A	49	C	74	B	99	D
25	B	50	C	75	A		

Unit-8

Differentiation And Application Of Darivative

Important Point

1. ***Derivative of a Function***

Let y=f(x) be a function defined on the interval[a,b]. For a small increment δx in x, let the corresponding

increment in the value of y be δy .Then, $y = f(x)$ and $y+\delta y = f(x+\delta x)$

On subtraction, we get

$\delta y = f(x+\delta x) - f(x)$ $\therefore$ $\frac{\delta y}{\delta x} = \frac{f(x+\delta x)-f(x)}{\delta x}$ so, $\lim_{\delta x \to o}$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x\_ = f(x)}{\delta x}$$

The above limit, if it exists, is called the derivative or diffrential coeficient of y with respect to x and is written

as $\frac{\delta y}{\delta x}$ or $f'(x)$. $\therefore$ $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x}$

2. **Derivative at Point**

The value of obtained by putting x=a, is called the derivative of $f(x)$ at x=a and it is denoted

by or $f'(a)$ or $\left(\frac{dy}{dx}\right)_{x=a}$

3. **Some Standard Derivatives**

1. $\frac{d}{dx}(\sin x) = \cos x$

2. $\frac{d}{dx}(\cos x) = -\sin x$

3. $\frac{d}{dx}(\tan x) = \sec^2 x$

4. $\frac{d}{dx}(\cot x) = -co\sec^2 x$

5. $\frac{d}{dx}(\sec x) = \sec x \tan x$

6. $\frac{d}{dx}(co\sec x) = -co\sec x \cot x$

7. $\frac{d}{dx}(e^x) = e^x$

8. $\frac{d}{dx}(a^x) = e^x \log_e a, a > 1$

9. $\frac{d}{dx}(\log_e x) = \frac{1}{x}, x > 0$

10. $\frac{d}{dx}(x^n) = nx^{n-1}$

11. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, -1 < x < 1$

12. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}, -1 < x < 1$

13. $\frac{d}{dx}(\tan^{-1} x) = -\frac{1}{1+x^2}, -\frac{\pi}{2} < x < \frac{\pi}{2}$

14. $\frac{d}{dx}(\sec^{-1} x) = -\frac{1}{\sqrt{x^2-1}}, |x| > 1$

15. $\frac{d}{dx}(co\sec^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}, |x| > 1$

16. $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}, -\pi < x < \pi$

3(A). <u>Some Rules for Diffrentiation</u>

1. The derivative of a constant function is zero, i.e. $\frac{d}{dx}(c) = 0$

2. The derivative of constant times a function is constant times the derivative of the function.

i.e. $\frac{d}{dx}[c.f(x)] = c.\frac{d}{dx}[f(x)]$

3. The derivative of the sum or diffrence of two functions is the sum or diffrence of their deriva tives. i.e.

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x) \pm \frac{d}{dx}[g(x)]$$

4. Product rule of differentiation:
The derivative of the product of two functions =
(first function)X(derivative of second function) + (second function)X(derivative of first function)

i.e. $\frac{d}{dx}[f(x).g(x)] = f(x).\frac{d}{dx}[g(x)] + g(x).\frac{d}{dx}[f(x)]$

5. Quotient rule of diffrentiation :

The derivative of the quotient of two functions $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x).\frac{d}{dx}[f(x)] - f(x).\frac{d}{dx}[g(x)]}{[g(x)]^2}$

6. <u>Derivative of a function of a function (Chain rule):</u>

y=f(t) and t=g(x), then $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$

if y=f(u), where u=g(y) and y=h(x), then, $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx}$

7. **Derivative of Parametric Functions**

x=f(t) and y=g(t) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{f'(t)}{g'(t)}$ and, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$

8. **Logarithmic Diffrentiation :**

1. $\log_e(mn) = \log_e m + \log_e n$
2. $\log_e\left(\frac{m}{n}\right) = \log_e m - \log_e n$
3. $\log_e(m)^n = n\log_e m$
4. $\log_e e = 1$
5. $\log_n m = \frac{\log_e m}{\log_e n}$
6. $\log_n m . \log_m n = 1$

9. **Another method for finding derivative of a Logarithmic Diffrentiation :**

If $y = [f(x)]^{g(x)}$, then to find $\frac{dy}{dx}$

Method 1:

Express $y = [f(x)]^{g(x)} = e^{g(x)\log f(x)}][\because a^x = e^{x\log a}]$ diffrentiate w.r.t. x to obtain $\frac{dy}{dx}$

Method 2:

Evaluate A=Diffrential coefficient of y treating f(x) as constant Evaluate B=Diffrential coefficient of treating g(x) as constant. $\frac{dy}{dx} = A + B$

10. **Diffrentiation of Inverse Trigonometric Function**

Note: Some important substitutions to reduce the function to a simpler form :

Expression	Substitition
$\sqrt{a^2 - x^2}$	Put $x = a \sin\theta$ or $x = a \cos\theta$
$\sqrt{x^2 - a^2}$	Put $x = a \sec\theta$ or $x = a \operatorname{cosec}\theta$
$\sqrt{a^2 + x^2}$	Put $x = a \tan\theta$ or $x = a \cot\theta$
$\frac{a-x}{a+x}$ or $\frac{a+x}{a-x}$	Put $x = a \tan\theta$
$\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	Put $x = a \cos\theta$

11. **Some useful Trigonometric and inverse Trigomentric Transformations**

1. $1 + \cos mx = 2\cos^2\frac{mx}{2}$

2. $1 - \cos mx = 2\sin^2\frac{mx}{2}$

3. $\sin mx = \dfrac{2\tan\dfrac{mx}{2}}{1+\tan^2\dfrac{mx}{2}}$

4. $\cos mx = \dfrac{1-\tan^2\dfrac{mx}{2}}{1+\tan^2\dfrac{mx}{2}} = \dfrac{\cot^2\dfrac{mx}{2}-1}{\cot^2\dfrac{mx}{2}+1}$

5. $\tan\left(\dfrac{\pi}{4}+x\right)=\dfrac{1+\tan x}{1-\tan x}$

6. $\tan\left(\dfrac{\pi}{4}-x\right)=\dfrac{1-\tan x}{1+\tan x}$

7. $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\dfrac{x+y}{1-xy}\right),$ if x,y>0 and xy<1

8. $\tan^{-1}x+\tan^{-1}y=\pi+\tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$ if x,y>0 and xy>1

9. $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\dfrac{x-y}{1+xy}\right), if x,y>0$

10. $\sin^{-1}+\cos^{-1}x=\dfrac{\pi}{2}=\tan^{-1}x+\cot^{-1}x=\sec^{-1}x+\cos ec^{-1}x$

11. $\sin^{-1}x\pm\sin^{-1}y=\sin^{-1}(x\sqrt{1-y^2}\pm y\sqrt{1-x^2}, if x,y\geq 0\, and\; x^2+y^2\leq 1$

12. $\sin^{-1}x\pm\sin^{-1}y=\pi-\sin^{-1}(x\sqrt{1-y^2}\pm y\sqrt{1-x^2}), if x,y\geq 0\, and\; x^2+y^2>1$

13. $\cos^{-1}x\pm\cos^{-1}y=\cos^{-1}(xy\mp\sqrt{1-x^2}\sqrt{1-y^2}), if\; x,y>0\, and\; x^2+y^2\leq 1$

14. $\cos^{-1}x\pm\cos^{-1}y=\pi-\cos^{-1}(xy\mp\sqrt{1-x^2}\sqrt{1-y^2}), if\; x,y>0\, and\; x^2+y^2>1$

15. $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)=2\tan^{-1}x$ 16. $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)=2\tan^{-1}x$

17. $\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}=\dfrac{1}{2}\cos^{-1}x$

12. ***(I) Tangents and Normals***

Geomatrical Meaning of Derivative at a point

- The derivative of a function f(x) at a point x = a is the slope of the tangent to the curve y = f(x) at the point [a,f(a)].

- Consider a curve y = f(x) and a point P(x,y) on this curve . If tangent to the curve at P(x,y) makes an angle with the positive direction of X-axis , then at the point

$P(x,y):\dfrac{dy}{dx}=\tan\theta=m=$ gradient or slope of tangent to the curve at $p(x,y)$.

Equation of tangent

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} (x - x_1)$$

Equation of Normal

$$y - y_1 = \frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}} (x - x_1)$$

13. Angle of Intersection of two curves

Let y = f(x) and y = g(x) be two curves intersecting at a point $P(x_1, y_1)$. Then , the angle of intersection of these two curves is defined as the angle between the tangents to the two curves at their point of intersection .

- If θ is the required angle of information of intersection , then $\theta = \pm(\theta_1 - \theta_2)$,

where θ_1 and θ_2 are the inclinations of tangents to the curves $y = f(x)$ and $y = g(x)$ respectively at the point P .

14. Length of tangent , Length of Normal ,Subtangent and Subnormal

(i) Length of the tangent $= PT = |y \operatorname{cosec} \theta| = \left|y\sqrt{1+\cot^2\theta}\right| = \left|\frac{y\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\frac{dy}{dx}}\right|$

(ii) Length of the Normal $= PN = |y \sec \theta| = \left|y\sqrt{1+\tan^2\theta}\right| = \left|y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right|$

(iii) Subtangent $= TM = |y \cot \theta| = \left|\frac{y}{\left(\frac{dy}{dx}\right)}\right|$

(iv) Subnormal $= MN = |y \tan\theta| = \left|y\left(\frac{dy}{dx}\right)\right|$

15. *(II) Increasing and Decreasing Functions (Monotonicity)*

Increasing Function

- A function $f(x)$ is said to be an increasing function on an interval I ,if

$$x_1 < x_2 \Rightarrow f(x_1) \le f(x_2), \forall\ x_1, x_2 \in I$$

Strictly Increasing Function

- A function $f(x)$ is said to be a strictly increasing function on an interval I ,if

$$x_1 < x_2 \Rightarrow f(x_1) < f(x_2), \forall\ x_1, x_2 \in I$$

Decreasing Function

- A function $f(x)$ is said to be a decreasing function on an interval I ,if

$$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2), \forall\ x_1, x_2 \in I$$

Strictly Decreasing Function

- A function $f(x)$ is said to be a stictly decreasing function on an interval I ,if

$$x_1 < x_2 \Rightarrow f(x_1) > f(x_2), \forall\ x_1, x_2 \in I$$

16. *(III) Maxima and Minima of Functions*

Rolle's Theorem

- If a function f defined on $[a,b]$, is

(i) continuous on $[a,b]$,

(ii) derivable on (a,b) and

(iii) $f(a) = f(b)$,

then there exists atleast one real number c between a and $b(a < c < b)$ such that $f'(c) = 0$

Lagrange's Mean Value Theorem

- If a function f defind on $[a,b]$, is (i) continuos on $[a,b]$ and (ii) derivable on (a,b),then there exists atleast one real number c between a and $b(a < c < b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

QUESTION BANK

(1) If $y \quad 3x^{\frac{3}{2}}(x \quad 1)$ then $\left(\frac{dy}{dx}\right)_{(x \quad 1)}$ = ________

(a) 6 (b) 3 (c) 1 (d) 0

(2) If $y \quad \log\sqrt{\frac{1 \quad \cos ax}{1 \quad \cos ax}}$ then $\left(\frac{dy}{dx}\right)_{(x \quad 1)}$ = ________

(a) a cosec*a* (b) –a cosec**a** (c) cosec*a* (d) –cosec*a*

(3) If $y = \cos^n x \cdot \sin nx$ and $\left(\frac{dy}{dx}\right) = n \cdot \cos^{n-1} x \times \cos B$ then B = ________

(a) $(n - 1)x$ (b) $(n + 1)x$ (c) nx (d) $(1 - n)x$

(4) If $y \quad \frac{\sin^2 x}{1 \quad \cot x} \quad \frac{\cos^2 x}{1 \quad \tan x}$ and $\left(\frac{dy}{dx}\right)_{x \quad \frac{}{4}}$ = ________

(a) 0 (b) + 1 (c) $\frac{1}{2}$ (d) $\left(\frac{1}{2}\right)$

(5) If $f(x) = \quad x^n$ then $\quad$ = ________

(a) $\frac{f'(1)}{n}$ (b) $\frac{f'(\ )}{n}$ (c) $\frac{f'(n)}{n}$ (d) 1

(6) If $f(x) = Ax + B$ and $f(0) = f'(x) = 2$ then $f(1)$ = ________

(a) 4 (b) 2 (c) 1 (d) –4

(7) For $\frac{d}{dx}\log\left[e^x\left(\frac{x \quad 4}{x \quad 4}\right)^{\frac{3}{4}}\right]$ at $x = 5$ then $\frac{dy}{dx}$ = ________

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $-\frac{3}{2}$ (d) $-\frac{5}{3}$

(8) If $y \quad \sin\frac{x}{2}\left(\frac{1}{\cos\frac{x}{2}\cos x} \quad \frac{1}{\cos x\cos\frac{3x}{2}} \quad \frac{1}{\cos\frac{3x}{2}\cos 2x}\right)$ then $\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} =$ ________

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) -1 (d) 1

(9) If $x^2e^y + 2xye^x + 23 = 0$ then $\frac{dy}{dx} =$ ________

(a) $2xe^{y-x} + 2y(x + 1)$ (b) $2xe^{x-y}$ - $3y(x + 1)$

(c) $\frac{2xe^{y \quad x} \quad 2y(x \quad 1)}{x(xe^{y \quad x} \quad 2)}$ (d) $2xe^{y-x} - y(x+1)$

(10) $f(x) = |\,[x]x\,|$, $-1 \quad x \quad 2$ then

(a) continuous at $x = 0$ (b) discontinuous at $x = 0$

(c) differntiable at $x = 0$ (d) continuous at $x = 2$

(11) If $f(x) = x \cdot \cot^{-1} x$ then $f'(1) =$ ________

(a) $\frac{}{4} \quad \frac{1}{2}$ (b) $\frac{}{4} \quad \frac{1}{2}$ (c) $\frac{}{4} \quad \frac{1}{3}$ (d) $\frac{}{4} \quad 1$

(12) $f(x) = |x - 2|$ and $g(x) = f(f(x))$, $x > 20$ then $g'(x) =$ ________

(a) 1 (b) 0 (c) 9 (d) 18

(13) If f is an even function and $f'(x)$ is define than $f'(\ ) + f'(-\ ) =$ ________

(a) 0 (b) $\quad 0$ (c) $\quad 0$ (d) > 0

(14) If $y \quad \tan^{\ 1}\left(\frac{\sqrt{1 \quad x^2} \quad \sqrt{1 \quad x^2}}{\sqrt{1 \quad x^2} \quad \sqrt{1 \quad x^2}}\right)$ and $z = \cos^{-1}x^2$ then $\frac{dy}{dx} =$ ________

(a) $\frac{1}{2}$ (b) $\frac{x}{\sqrt{1-x^4}}$ (c) $\frac{}{4}$ (d) $\frac{}{4} \quad \frac{1}{2}$

(15) If $y \quad \sqrt{\frac{3x^2 \quad x \quad 1}{x}}$ then $\left(\frac{dy}{dx}\right)_{(x \quad 1)} =$ ________

(a) $\frac{1}{\sqrt{5}}$ (b) $\sqrt{5}$ (c) 5 (d) $\frac{1}{5}$

(16) If $y \quad \frac{1}{\sqrt[3]{x}} \quad \log_5 x \quad 8$ then $\frac{dy}{dx}$ = ________

(a) $-\frac{1}{3}x^{\frac{-2}{3}} + \frac{1}{x}\log_5 e$ (b) $-\frac{1}{3}x^{\frac{-2}{3}} + \frac{1}{x}\log_5 x$

(c) $-\frac{1}{3}x^{\frac{-2}{3}} + \frac{1}{x}\log_x 5$ (d) $\frac{1}{3}x^{\ \frac{4}{3}} \quad \log_e 5$

(17) If $y = (x^2 + 7x + 2)\ (e^x - \log x)$ and

$\frac{dy}{dx} \quad (x^2 \quad Ax \quad B)\left(e^x \quad \frac{1}{x}\right) \quad (e^x \quad \log x)\ (Cx \quad D)$ then A + B – C – D = ________

(a) 0 (b) 7 (c) 2 (d) 9

(18) If $xy + x \cdot e^{-y} + y \cdot e^x = x^2$ and $\frac{dy}{dx} = \left(\frac{A + y + e^{-y} - 2x}{B + e^x - x}\right)$ then A + B = ________

(a) $ye^x + xe^{-y}$ (b) $ye^x - xe^{-y}$ (c) $ye^{-x} + xe^{-y}$ (d)

$ye^{\ x} \quad xe^{e^{\ y}}$

(19) If $f(x) = x^n$ then the value of $f(x)$ is

$f(1) \quad \frac{f'(1)}{1!} \quad \frac{f''(1)}{2!} \quad \frac{f'''(1)}{3!} \quad \ldots \quad \frac{(\ \ 1)^n f^n(1)}{n!}$ = _____

(a) 2^n (b) 2^{n-1} (c) 0 (d) 1

$(x) = n! \qquad f^n(1) = n!$

(20) If $f''(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) \quad \left[f\left(\frac{x}{2}\right)\right]^2 \quad \left[y\left(\frac{x}{2}\right)\right]^2$ then F(10) = ________

(a) 5 (b) 10 (c) 0 (d) 15

(21) If $x \quad \tan\left(\frac{1}{b}\log t\right)$ and $A\frac{d^2t}{dx^2} + (B - a)y_1 = 0$ then A + B = ________

(a) $(1 - x)^2$ (b) $(1 + x)^2$ (c) $(x - 1)^3$ (d) $1 - x$

(22) If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$ then $f'(1) =$ ________

(a) -1 (b) 1 (c) $\log_e 2$ (d) $-\log_e 2$

(23) If $y = b\tan^{-1}\left(\frac{x}{a} + \tan^{-1}\frac{y}{x}\right)$ and $\frac{dy}{dx} = \frac{\frac{1}{a} - \frac{y}{A}}{\frac{1}{b}\sec^2\frac{y}{b} + \frac{x}{A}}$ then A = ________

(a) $x^2 + a^2$ (b) $x^2 - a^2$ (c) $y^2 - x^2$ (d) y

(24) If $x = \tan\theta + \cot\theta$, $y = 2\log(\cot\theta)$ then $\frac{dy}{dx} =$ ________

(a) $-\tan 2\theta$ (b) $\tan 2\theta$ (c) $\sin 2\theta$ (d) $\cos 2\theta$

(25) $\frac{d}{dx}\left[\log(1+\sin x) + \log\left(\sec\left(\frac{\pi}{4} - \frac{x}{2}\right)\right)^2\right] =$ ________

(a) 0 (b) $4\left(\frac{\cos x - \tan x}{\sin x - \cos x}\right)$

(c) $\log_e 2$ (d) $-\log_e 2$

(26) If $y = f(f(f(x)))$ and $f(0) = 0$, $f'(0) = 1$ then $\left(\frac{dy}{dx}\right)_{x=0} =$ ________

(a) 0 (b) 1 (c) -1 (d) 2

(27) If $y = x\tan\frac{x}{2}$ and $A\frac{dy}{dx} - B = x$ then $\frac{B}{A} =$ ________

(a) $\cot\frac{x}{2}$ (b) $\tan\frac{x}{2}$ (c) $\tan x$ (d) $\cot x$

(28) If $f: (-\infty, -1] \cup [1, \infty) \to (0, \pi)$ and $f(x) = \sec^{-1}x$ then $f'(x) =$ ________

(a) $\frac{1}{x\sqrt{x^2-1}}$ (b) $\frac{1}{x\sqrt{x^2+1}}$

(c) $\frac{1}{|x|\sqrt{x^2-1}}$ (d) $\frac{1}{|x|\sqrt{x^2+1}}$

(29) If $y \quad \cos^{1}\left(\frac{3x \quad 4\sqrt{1 \quad x^2}}{5}\right)$ then $\frac{dy}{dx}$ = ________

(a) $\frac{1}{\sqrt{1 \quad x^2}}$ (b) $\frac{2}{\sqrt{1 \quad x^2}}$ (c) $\frac{5}{3\sqrt{1 \quad x^2}}$ (d) $\frac{3}{5\sqrt{1 \quad x^2}}$

(30) Derivative of $\sin^{-1}(3x - 4x^3)$ *with respect to* $\sin^{-1}x$ is

(a) 3, $|x| < 1$ (b) 3, $|x| \leq \frac{1}{2}$ and –3, $\frac{1}{2} < |x| < 1$

(c) –3, $|x| < 1$ (d) – 3, $|x| \leq \frac{1}{2}$ and 3, $\frac{1}{2} < |x| < 1$

(31) Derivative of function $f(x) \quad \frac{x^2}{1 \quad \sin^2 x}$ is

(a) Even function (b) Odd function (c) Not define (d) Incresing Function

(32) Approximate value of $(1.0002)^{3000}$ is ________

(a) 1.2 (b) 1.4 (c) 1.6 (d) 1.8

(33) If $f'(x) > 0$ and $g'(x) < 0 \quad x \quad R$ then

(a) $f(g(x)) > f(g(x + 1))$ (b) $f(g(x)) \quad f(g(x + 1))$

(c) $g(f(x)) \not> g(f(x + 1))$ (d) $g(f(x)) > g(f(x - 1))$

(4) If $y \quad 10^{\log_{10}\left(\sin^{1}\frac{1 \quad x^2}{1 \quad x^2}\right)} \quad e^{\log\left(\sin^{1}x \quad \sec^{1}\frac{1}{x}\right)}$ then $\frac{dy}{dx}$ = ________

(a) $\frac{2}{1 \quad x^2}$ (b) $\frac{2}{1 \quad x^2}$ (c) $\frac{2}{1 \quad x^2}$ (d) 0

(35) If $x \quad \sqrt{\frac{1 \quad t^2}{1 \quad t^2}}$ and $y \quad \frac{\sqrt{1 \quad t^2} \quad \sqrt{1 \quad t^2}}{\sqrt{1 \quad t^2} \quad \sqrt{1 \quad t^2}}$ then $\left(1 \quad x^2\right)\frac{dy}{dx}$ = ________

(a) – 1 (b) 1 (c) – 2 (d) 2

(36) If $y \quad f\left(\frac{2x \quad 1}{x^2 \quad 1}\right)$ and $f'(x) = \sin x^2$, $\frac{dy}{dx} \quad \frac{2(x^2 \quad x \quad 1)}{A} \cdot \sin B$ then AB = ________

(a) $(2x - 1)^2$ (b) $(x - 1)^2$ (c) $(x^2 + 1)^2$ (d) $(2x + 1)^2$

(37) If $f''(x) = -f(x)$ and $f'(x) = g(x)$, $m(x) = (f(x))^2 + (g(x))^2$ then find $m(20)$. Where, $m(10) = 22$

(a) 22 (b) 11 (c) 0 (d) 5

(38) If $y^2 = p(x)$ is polynominal function more than 3 degree then $2\frac{d}{dx}\left(y^3 y_2\right) =$ ________

(a) $p'(x) \cdot p''(x)$ (b) $p(x) \cdot p''(x)$ (c) $p'(x) \cdot p'''(x)$ (d) $p(x) \cdot p'''(x)$

(39) In which interval $f(x) = \sin^{-1}\left(2x\sqrt{1 \quad x^2}\right)$ is strictly incresing ?

(a) $[-1, 1]$ (b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (c) $[0, 1]$ (d) $[-1, 0]$

(40) If $x = \frac{e^{2y} + 1}{e^{2y} - 1}$ then $\frac{dy}{dx}$ = ________

(a) $1 + x^2$ (b) $x^2 - 1$ (c) $1 - x^2$ (d) x^2-1

(41) If $f(x) + 1 + nx + \frac{n(n \quad 1)}{2}x^2 \quad \frac{n(n \quad 1)(n \quad 2)}{6}x^3$ ..... x^n then $f''(1) =$ ______

(a) $n(n - 1).2^{n-2}$ (b) $n(n - 1)2^n$ (c) $n(n - 1).2^{n-1}$ (d) $(n - 1).2^{n-1}$

(42) Equation of the curve is $2x = 2a\cos \quad + b\cos 2$ and $2y = 2a\sin \quad + b\sin 2$. If $\frac{d^2y}{dx^2} \quad 0$ then the value is

(a) $\sin \quad \frac{2a^2 \quad b^2}{5ab}$ (b) $\tan \quad \frac{3a^2 \quad 5b^2}{4ab}$

(c) $\cos \quad \frac{a^2 \quad 2b^2}{3b}$ (d) $\cos \quad \frac{(a^2 \quad 2b^2)}{3ab}$

(43) The rate of change of $\sqrt{x^2 \quad 16}$ with respect to $\frac{x}{x \quad 1}$ at $x = 3$ is

(a) 2 (b) $\frac{11}{5}$ (c) $\frac{12}{5}$ (d) -3

(44) If two variables x and y and $x > 0$, $xy = 1$ then minimum value of $x + y$ is ________

(a) 1 (b) 2 (c) $2\frac{1}{2}$ (d) $3\frac{1}{3}$

(45) f and g are two differentiable function and fog = I (Identify function) and $g'(a) = 2$, $g(a) = b$ then $f'(x)$ = ________

(a) 2 (b) $\frac{1}{2}$ (c) –2 (d) $-\frac{1}{2}$

(46) Derivative of $y = \frac{1}{3}\left[\log(x+1) - \log\sqrt{x^2 - x + 1}\right] + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}}$ is

$\frac{dy}{dx}\quad \frac{1}{3A}\quad \frac{B}{3(x^2\quad x\quad 1)}$ then AB = ________

(a) $-x^2 - x - 2$ (b) $x^2 - x + 2$ (c) $-x^2 + x + 2$ (d) $x^2 + x + 2$

(47) $\frac{d}{dx}\left(3^{\log_{10}|\text{cosec}\quad 1\, x|}\right)$ = ________

(a) $-\frac{3^{\log_{10}|\text{cosec}^{-1}x|}}{\text{cosec}^{-1}x}\left(\frac{1}{x\sqrt{x^2-1}}\log_{10}3\right)$ (b) $-\frac{3^{\log_{10}|\text{cosec}^{-1}x|}}{\text{cosec}^{-1}x}\left(\frac{1}{|x|\sqrt{x^2-1}}\log_{10}3\right)$

(c) $-\frac{3^{\log_{10}|\text{cosec}^{-1}x|}}{\text{cosec}^{-1}x}\left(\frac{1}{|x|\sqrt{x^2-1}}\log_{3}10\right)$ (d) None of these

(48) If $y = \sum_{r=1}^{x}\tan^{-1}\frac{1}{1+r+r^2}$ then $\frac{dy}{dx}$ = ________

(a) $\frac{1}{1\quad x^2}$ (b) $\frac{1}{1\quad (1\quad x)^2}$ (c) 0 (d) $\frac{1}{1-(x+1)^2}$

(49) If $(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)^{-1} + \sin^{-1}\left(\frac{1\quad x}{1\quad x}\right)$ then $\frac{dy}{dx}$ = ________

(a) $\frac{-8}{\log_e 2} - \frac{32}{16+\pi^2}$ (b) $\frac{8}{\log_2}\quad \frac{8}{4}$ (c) $\frac{8}{\log_2}\quad \frac{8}{4}$ (d) $\frac{8}{\log_e 2} - \frac{32}{16+\pi^2}$

(50) If $x = a(1 - \cos^3\theta)$, $y = a\sin^3$ and $\left(\frac{d^2y}{dx^2}\right)_{\overline{6}}\quad \frac{A}{a}$ then A = ________

(a) $\frac{27}{32}$ (b) $\frac{32}{27}$ (c) $\frac{32}{27}$ (d) $\frac{27}{32}$

(51) If $e^{y} \quad \frac{e^2}{x^2}$ and $\frac{d^2y}{dx^2} \quad \frac{A}{x^2}$ then A = ________

(a) –2 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{2}$

(52) If $y = x^x$ and $\frac{d^2y}{dx^2} - \frac{y}{x} = \frac{1}{\alpha} \cdot \left(\frac{dy}{dx}\right)^2$ then $\quad$ = ________

(a) x^y (b) x^x (c) y^x (d) x

(53) If $x^{13}y^7 = (x + y)^{20}$ then $xy_1 - y$ = ________

(a) 7 (b) 13 (c) 20 (d) 0

(54) If $x \quad 235 \left[\cos t \quad \frac{1}{10^{10}} \log \tan^{10^{10}} \frac{t}{2}\right], y \quad 235 \cos t$ then $\frac{dy}{dx}$ = ________

(a) $\cot^2 t$ (b) $\tan^2 t$ (c) $\cos^2 t$ (d) $\sin^2 t$

(55) The slope of the tangent at (2, –1) for the curve $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ is ________

(a) $\frac{6}{7}$ (b) –6 (c) $\frac{22}{7}$ (d) $\frac{7}{6}$

(56) Equation of the tangent of the curvey $y \quad 1 \quad e^{\frac{x}{2}}$ when intersect to y–axis than = ________

(a) $x + y = 0$ (b) $x + 2y = 0$ (c) $2x + y = 0$ (d) $x - y = 0$

(57) The length of line segnment when tangent of the curve intersect to both axes for curve $\left(\frac{x}{a}\right)^{\frac{2}{3}} \quad \cos^2$ and $\left(\frac{y}{a}\right)^{\frac{2}{3}} \quad \sin^2$ is

(a) a (b) $|a|$ (c) a^2 (d) a^3

(58) For curve $y = f(x)$, $\frac{dy}{dx} \quad 2x$ than angle made by the tangent at (1, 1) with $\overline{OX}$ is

(a) $\frac{\ }{4}$ (b) $\frac{\ }{3}$ (c) $\tan^{-1}2$ (d) $\tan^{-1}\frac{1}{2}$

(59) curve $y = \frac{3}{2}\sin 2$, $x = e$ · sin · 0 < 2 for which value of tangent is parallel to X–axis ?

(a) 0 (b) $\frac{}{2}$ (c) $\frac{}{4}$ (d) $\frac{}{6}$

(60) Function $f(x)$ $\left|x \quad \frac{1}{2}\right|$ $|x \quad 1|$ $\tan x$ how many points are not differenciable in (0, 2) ?

(a) 1 (b) 2 (c) 3 (d) 4

(61) $f(x) = (x - a)^m (x - b)^n$, x $[a, b]$ is the satisfing the Roll's condition than c = ... (a, b).

(a) $\frac{mb \quad na}{m \quad n}$ (b) $\frac{mb \quad na}{m \quad n}$ (c) $\frac{mb \quad na}{m \quad n}$ (d) $\frac{mb \quad na}{m \quad n}$

(62) If function $f(x) = ax^3 + bx^2 + 11x - 6$, x [1, 3] is satisfing Roll's condition $f'\left(2 \quad \frac{1}{\sqrt{3}}\right)$ 0 than a = ________, b = ________

(a) 1, –6 (b) –2, 1 (c) –1, $\frac{1}{2}$ (d) –1, 6

(63) If $f(x) = 1 + 2\sin x + 3\cos^2 x$, 0 x $\frac{2}{3}$ then

(a) Minimum value of x $\frac{}{2}$

(b) Maximum value of x $\sin^{1}\frac{1}{\sqrt{3}}$

(c) Minimum value of x $\frac{}{6}$

(d) Maximum value of x $\sin^{1}\frac{1}{6}$

(64) Equation of the tangent for the curve $y = a \log \sec\frac{x}{a}$ at $x = a$ is ________

(a) $(y - a\log\sec 1)\tan 1 = x - a$

(b) $(x - a)\tan 1 = (y - a)\log\sec 1$

(c) $(x - a)\cos 1 = ((y - a)\log\sec 1)\tan 1$

(d) None of these

(65) If $\cos\frac{x}{2} \cdot \cos\frac{x}{x^2} \cdot \frac{x}{2^3}$ .... $\frac{\sin x}{x}$ then $\frac{1}{2^2}\tan\frac{x}{2}$ $\frac{1}{2^2}\tan\frac{x}{2^2}$ $\frac{1}{2^3}\tan\frac{x}{2^3}$ ..... = ______

(a) $\frac{1}{x}$ $\cot x$ (b) $\frac{1}{x}$ $\tan x$ (c) $x - \tan x$ (d) $x - \cot x$

(66) If $y = (x \log x)^{\log \log x}$ for $\frac{dy}{dx}$ $\quad A^{\log\log x \quad 1}$ (B + (log$x$ + x) log log$x$) then AB = ________

(a) $x \log x$ (b) $x(\log x)^2$ (c) $x^2 \cdot \log x$ (d) log x

(67) Prove that the condition that $x\cos \quad + y\sin \quad = p$ touches the curve $x^m y^n = a^{m+n}$ is $p^A m^n n^m = A^A \cdot a^A \cos^m \quad \cdot \sin \quad$ is then A = ________

(a) $m + n$ (b) $m - n$ (c) $n - m$ (d) $m^2 - n^2$

(68) For every x, $x \quad$ R $f(x) = (a + 2)x^3 - 3ax^2 + 9ax - 1$ the function is decreasing then $a \quad$ ________

(a) (– , –3) (b) (–3, –2) (c) (3,) (d) (– , –3)]

(69) In which interval $f(x) = \sin^4 x + \cos^4 x$, $x \quad \left[0, \frac{\pi}{2}\right]$ is increasing function

(a) $\left(\frac{}{4}, \frac{}{2}\right)$ (b) $\left(0, \frac{}{4}\right)$ (c) $\left(0, \frac{}{2}\right)$ (d) $\left[\frac{}{4}, \frac{}{2}\right]$

(70) In the interval $\left[0, \frac{}{4}\right]$, the function $f(x) = \tan^{-1}(\sin x + \cos x)$ is ________

(a) Decreasing (b) Increasing
(c) Both (d) Even Function

(71) The point on the curve $y^2 \quad 12\left[x \quad 2\sin\frac{x}{2}\right]$ at which the tangent is parellal to x – axis is lie on

(a) a line (b) a parallal
(c) a circle (d) an ellipse

(72) The minimum value of $\frac{(\quad x)(\quad x)}{(\quad x)}$, $x \quad c$ is ________

(a) $\left(\sqrt{\quad} \quad \sqrt{\quad}\right)^2$ (b) $\left(\sqrt{\quad} \quad \sqrt{\quad}\right)^2$

(c) $\left(\sqrt{\quad} \quad \sqrt{\quad}\right)^2$ (d) $(\alpha - \beta)^2$

$x \quad \sqrt{(\quad)\ (\quad)}$

(73) If A > 0, B > 0 and A + B = $\frac{\ }{3}$ than the maximum value of tanA · tanB is ________

(a) $\frac{1}{3}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) 3

(74) If the curve $y = a^x$ and $y = b^x$ intersect each other at angle then tan = ________

(a) $\frac{a \quad b}{1 \quad ab}$ (b) $\frac{\log a \quad \log b}{1 \quad \log a \cdot \log b}$

(c) $\frac{a \quad b}{1 \quad ab}$ (d) $\frac{\log a \quad \log b}{1 \quad \log a \cdot \log b}$

(75) $\frac{1 + x \tan x}{x}$ is maximum at ________

(a) $x = \sin x$ (b) $x = \cos x$ (c) $x \quad \frac{\ }{3}$ (d) $x = \tan x$

(76) Let $f(x) \quad \sqrt{x \quad 1} \quad \sqrt{x \quad 24 \quad 10\sqrt{x \quad 1}}$, $1 < x < 26$ be real valued function Then $f'(x)$ = ________ for $1 < x < 26$ is

(a) 0 (b) $\frac{1}{\sqrt{x \quad 1}}$ (c) $2\sqrt{x-1}$ (d) $\sqrt{x-1}$

(77) The Roll's theorem is applicable in the interval –1 x 1 for the function

(a) $f(x) = x$ (b) $f(x) = x^2$ (c) $f(x) = 2x^3 + 3$ (d) $f(x) = |x|$

(78) If the rate of change of value of a sphere is equal to the rate of change of its radius, then its radius is equal to = ________

(a) 1 unit (b) $\sqrt{2}$ unit (c) $\frac{1}{\sqrt{2}}$ unit (d) $\frac{1}{2\sqrt{\ }}$ unit

(79) Two measarment of a cylinder are varying in such a way that the volume is kept constant. If the rates of change of the radius (r) and high (h) are equal in magnitute but opposite is sign then

(a) $r = 2h$ (b) $h = 2r$ (c) $h = r$ (d) $h = 4r$

(80) The point on the curve $y = (x - 2)(x - 3)$ at which the tangent makes an angle of 225^o with possitive direction of x – axis has co–ordinates

(a) (0, 3) (b) (3, 0) (c) (–3, 0) (d) (0, –3)

(81) If the curves $2x^2 + 3y^2 = 6$ and $ax^2 + 4y^2 = 4$ intersect orthogonally than = ________

(a) 2 (b) 1 (c) 3 (d) -3

(82) At any point of a curve (sub tangent) (sub normal) is equal to the square of the

(a) Slope of the tangent at the point (b) Slope of the normal at the point

(c) abscissa of the point (d) ordinate of the point

(83) The length of the normal to the curve $y \quad s\left(\frac{e^{\frac{x}{5}} \quad e^{\frac{x}{5}}}{2}\right)$ at any point varies as the

(a) sbscissa of the point (b) ordinate of the point

(c) Square of the abscissa of the point (d) Square of the ordinate of the point

(84) If $m = \tan$ is the slope of the tangent to the curve $_ey = 1 + x^2$ than

(a) $|\tan \quad| > 1$ (b) $|\tan \quad| < 1$ (c) $\tan \quad < 1$ (d) $|\tan \quad| \quad 1$

(85) In $a + b + c = 0$, than the equation $3ax^2 + 2bx + c = 0$ has, in the interval (0, 1)

(a) at least one root (b) at most one root (c) no root (d) Exactly one root exist

(86) In (0, 1) mean value theorm is not applicable to

(a) $f(x) = \left(\frac{1}{2} - x\right), x < \frac{1}{2}$ (b) $f(x) = \frac{\sin x}{x}, x = 0$

(c) $f(x) = x|x|$ (d) $f(x) = |x|$

(87) Let $f(x)$ satisfy the requirement of lanrange's mean value theorm in [0, 2]. If $f(0) = 0$

and $|f'(x)| \quad \frac{}{2} \quad x, x \quad [0, 1]$ for all $x \quad [0, 2]$ then

(a) $f(x) \quad 2$ (b) $|f(x)| \quad 1$

(c) $f(x) = 2x$ (d) $f(x) = 3$ for at least one x in (0, 2)

(88) The function $f(x) = 4\log(x - 2) - x^2 + 4x + 1$ increasing on the interval

(a) (2, 3) (b) (1, 2) (c) (2, 4) (d) (1, 3)

(89) If $f(\ ) \quad \frac{}{\sin}$ and $g(\ ) \quad \frac{}{\tan}$ where $0 < \quad 1$ than in this interval

(a) both $f(\ )$ and $g(\ )$ are increasing function

(b) both $f(\ )$ and $g(\ )$ are decreasing function

(c) $f(\ )$ is an increasing function

(d) $g(\ )$ is an increasing function

(90) The interval of increase of the function $f(l) = l - e^{l} + \tan\frac{112\pi}{7}$

(a) (0,   )   (b) $(-\infty, 0)$ (b) (1,   )   (d) (– , 1)

(91) If $f(x) = \frac{a^2 \quad 1}{a^2 \quad 1} x^3 - 3x + 5\log_e 2$ is a decreasing function of x for all x R then the set of possible values of a is

(a) [–1, 1]   (b) [1,   ]   (c) (– , –1]   (d) (– , –1)

(92) If $0 < x < \frac{}{2}$ than

(a) $\cos x > 1 - \frac{2x}{}$ (b) $\cos x < 1 - \frac{2x}{}$ (c) $\cos x > \frac{2x}{}$ (d) $\cos x < \frac{2x}{}$

(93) The minimum value of $\sec^2 x +$ $\mathrm{cosec}^2 x -$ – equation maximum value of $a\sin^2$ $+ b\cos^2$ where $a > b > 0$,

(a) $a = b$ (b) $a = 2b$ (c) $a = 3b$ (d) $a = 4b$

(94) Derivative of $\sec^{-1}\left(\frac{1}{2x^2 \quad 1}\right)$ $w.r.t. \sqrt{1 \quad 3x}$ at $x \quad \frac{1}{3}$ is

(a) 0   (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 3

(95) $f(x) \quad |x| \quad \left|x \quad \frac{1}{2}\right| \quad |x \quad 3| \quad \left|x \quad \frac{5}{2}\right|$ minimum value is ________

(a) 0   (b) 2   (c) 4   (d) 6

(96) The function $f(x) \quad \left(\frac{e^{2x} \quad 1}{e^{2x} \quad 1}\right)$ is

(a) Increasing   (b) Decreasing

(c) Even   (d) Strictly increasing

(97) If $f(x) = |x - 1|$ any $g(x) = f(f(f(x))$ the for $x > 2$ $g'(x)$ is equal to

(a) 1 for all $x > 2$ (b) 1 for $2 < x < 3$

(c) –1 for $2 < x < 3$ (d) Not defined

(98) The second order derivative of $a \sin^3$ with respect to $a\cos^3$ at $\frac{}{4}$.

(a) $\frac{4\sqrt{2}}{3a}$ (b) 2   (c) $\frac{1}{12a}$ (d) 0

(99) The equation of the normal to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$.

(a) $x + y = 2$ (b) $x + y = 1$

(c) $x - y = 1$ (d) $x^2 - y^2 = 2$

(100) If $y \quad \tan\ ^{1}\frac{1}{x^2 \quad x \quad 1} \quad \tan\ ^{1}\frac{1}{x^2 \quad 3x \quad 3} \quad \tan\ ^{1}\frac{1}{x^2 \quad 5x \quad 7}$ ..... to n terms

then $\frac{dy}{dx}$ = ________

(a) $\frac{1}{1 \quad (x \quad n)^2} \quad \frac{1}{1 \quad x^2}$ (b) $\frac{1}{1 \quad (x \quad n)^2} \quad \frac{1}{1 \quad x^2}$

(c) $\frac{2}{1 \quad (x \quad n)^2} \quad \frac{1}{1 \quad x^2}$ (d) $\sum n$

Hints

(1) $y = 3x^{3/2}(x-1)$ $\quad 3x^{\frac{5}{2}} \quad 3x^{\frac{3}{2}}$

(2) $y \quad \frac{1}{2}\log\left(\frac{1 \quad \cos ax}{1 \quad \cos ax}\right)$, $\frac{dy}{dx} \quad \frac{1}{2}\left(\frac{a\sin x}{1 \quad \cos ax} \quad \frac{a\sin ax}{1 \quad \cos ax}\right)$

(3) $y = \cos^n x \sin nx$, Using Multiplication Rule

(4) $y \quad \frac{\sin^3 x}{\sin x \quad \cos x} \quad \frac{\cos^3 x}{\sin x \quad \cos x}$

$$y \quad \frac{1}{\sin x \quad \cos x}(\sin^3 x \quad \cos^3 x) \;, y \quad 1 \quad \frac{1}{2}\sin 2x \quad \frac{dy}{dx} \quad \cos 2x$$

(5) $f(x) = \quad x^n \frac{d}{dx}(f(x)) \quad \frac{d}{dx}(x^n) \therefore \quad f'(1) = \quad \cdot n \quad f'(x) = \quad \cdot n \cdot x^{n-1}$

(6) $f(x) = Ax + B$ $f'(x) = A$ and $f(0) = f'(0) = 2$

$\therefore$ A = 2, B = 2 So, $f(1) = 2 + 2 = 4$

(7) $\frac{d}{dx}\log\left(e^x\left(\frac{x \quad 4}{x \quad 4}\right)^{\frac{3}{4}}\right)$

$$\frac{d}{dx}\left(x \quad \frac{3}{4}\log(x \quad 4) \quad \frac{3}{4}\log(x \quad 4)\right) \qquad \frac{dy}{dx} \quad \frac{x^2 \quad 10}{x^2 \quad 16}$$

(8) $y \quad \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}\cos x} \quad \frac{\sin\frac{x}{2}}{\cos x\cos\frac{3x}{2}} \quad \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}\cos 2x}$

$$\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}\cos x} \quad \frac{\sin\left(x \quad \frac{x}{2}\right)}{\cos x\cos\frac{x}{2}} \quad \tan x \quad \tan\frac{x}{2}$$

same way , $\dfrac{\sin\frac{x}{2}}{\cos x \cos\frac{3x}{2}} \quad \tan\dfrac{3x}{2} \quad \tan x$ and $\dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}\cos x} \quad \tan 2x \quad \tan\dfrac{3x}{2}$

(9) $x^2e^y + 2xy \cdot e^x + 23 = 0$ diff w r to x

$$\frac{dy}{dx} \quad \frac{2xe^y \quad 2xye^x \quad 2ye^x}{x^2e^y \quad 2xe^x}$$,Nr & Dr by x

(10) $f(x) = |\,[x]x\,| = |\,x\,| \cdot |[x]| \qquad F[0] = 0$ using L.H.D. and R.H.D.

(11) $f(x) = x \cdot \cot^{-1} x$ using Product Rules

(12) $f(x) = |\,x - 2\,|$ and $g(x) = f(f(x))$, Now $x > 20$ for $f(x) = x - 2$

$g(x) = f(f(x)) = f(x - 2) = |\,x - 4\,|$, Now, $x > 20$ for $g'(x) = 0$

(13) even functions derivative is odd functions,$f'(\ ) + f'(-\ ) = 0$

(14) $z = \cos^{-1}x^2$ suppose $x^2 = \cos^2 z$,

$$y \quad \tan^{1}\left(\frac{1 \quad \tan\frac{z}{2}}{1 \quad \tan\frac{z}{2}}\right) \qquad y \quad \frac{}{4} \quad \frac{z}{2}$$

(15) $y \quad \sqrt{\dfrac{3x^2 \quad x \quad 1}{x}} \quad \sqrt{3x \quad 1 \quad \dfrac{1}{x}} \qquad \left(\dfrac{dy}{dx} \quad \sqrt{ax \quad b}\right)$

using Darivative of composite function

(16) $y \qquad \dfrac{1}{\sqrt[3]{x}} \quad \log_5 x \quad 8 \quad x^{\frac{1}{3}} \quad \log_5 x \quad 8$

$$\frac{dy}{dx} \quad \frac{1}{3}x^{\frac{1}{3}\ 1} \quad \frac{1}{x \cdot \log 5} \quad \frac{1}{3}x^{\ \frac{2}{3}} \quad \frac{1}{x}\log_5 e$$

(17) $y = (x^2 + 7x + 2)\,(e^x - \log x)$ Apply Product Rule

(18) $xy + x \cdot e^{-y} + y \cdot e^x = x^2$using derivative,

$$\frac{dy}{dx} \quad \frac{(ye^x \quad y \quad e^{\ y} \quad 2x)}{x \quad xe^{\ y} \quad e^x} \quad \frac{(ye^x \quad y \quad e^{\ y} \quad 2x)}{(\quad xe^y \quad e^x \quad x)}$$

A = ye^x, B = $-\ xe^{-y}$

(19) $f(x) = x^n$ $f(1) = 1,$ $f'(x) = nx^{n-1}$ $f'(1) = 1$

$f''(x) = n(n-1)x^{n-2}$ $f''(1) = n(n-1)$

$f^n(x) = n!$ $f^n(1) = n!$

$$1 \quad \frac{n}{1!} \quad \frac{n(n \quad 1)}{2!} \quad \ldots\ldots\ldots \quad 0$$

(20) $\Rightarrow F(x) = \left[f\left(\frac{x}{2}\right)\right]^2 + \left[g\left(\frac{x}{2}\right)\right]^2$

$$= \left[f\left(\frac{x}{2}\right)^2 + \left(f'\left(\frac{x}{2}\right)\right)\right]^2$$

$$\therefore F'(x) = F\left(\frac{x}{2}\right).f'\left(\frac{x}{2}\right) + f'\left(\frac{x}{2}\right).f''\left(\frac{x}{2}\right)$$

(21) $x = \tan\left(\frac{1}{b}\log t\right) \therefore b\tan^{-1}x = \log t \therefore \frac{b}{1+x^2} = \frac{1}{t}\cdot\frac{dt}{dx}$ *then after we get taking second Darivative* $(1 + x^2)y_2 + (2x - b)y_1 = 0$

(22) $f(x) \quad \cot^{\ 1}\left(\frac{x^x \quad x^{\ x}}{2}\right),\ f'(x) \quad \frac{1}{1 \quad \left(\frac{x^x \quad x^{\ x}}{2}\right)^2}\ \frac{d}{dx}\left(\frac{x^x \quad x^{\ x}}{2}\right)$

$$f'(x) \quad \frac{2}{4 \quad (x^x \quad x^{\ x})^2}\left(e^{x\log x}\cdot\frac{d}{dx}(x\cdot\log x) \quad e^{\ x\log x}\frac{d}{dx}(\quad x\log x)\right)$$

(23) $y \quad b\tan^{\ 1}\left(\frac{x}{a} \quad \tan^{\ 1}\frac{y}{x}\right)$

$$\tan\frac{y}{b} \quad \frac{x}{a} \quad \tan^{\ 1}\frac{y}{x},\ \frac{1}{b}\sec^2\frac{y}{b}\cdot\frac{dy}{dx} \quad \frac{1}{a} \quad \frac{x\cdot\frac{dy}{dx} \quad y}{x^2 \quad y^2}\ \ldots\ldots\ldots$$

(24) x= tan + cot Same way, x $\frac{\sin}{\cos} \quad \frac{\cos}{\sin},\ \frac{dy}{d\theta} = -4\,\mathrm{cosec}\,4\theta$

$$\frac{\sin^2 \quad \cos^2}{\sin \quad \cdot\cos} \qquad \frac{dy}{dx} \quad \ldots.. \qquad \frac{2}{\sin 2}$$

(25) $\frac{d}{dx}\left[\log(1+\sin x) + \log\left(\sec\left(\frac{\pi}{4} - \frac{x}{2}\right)\right)^2\right] \quad \frac{\cos x}{1 \quad \sin x} \quad \tan\left(\frac{}{4} \quad \frac{x}{2}\right)$

$$\frac{\cos^2\frac{x}{2} \quad \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} \quad \sin\frac{x}{2}\right)^2} \quad \frac{\cos\frac{x}{2} \quad \sin\frac{x}{2}}{\cos\frac{x}{2} \quad \sin\frac{x}{2}} \quad 0$$

(26) $y = f(f(f(x))), \ \frac{dy}{dx} \quad x \cdot \sec^2\frac{x}{2} \cdot \frac{1}{2} \quad \tan\frac{x}{2}$

$= f'(f(f(x)) \cdot f'(f(x)) \cdot f'(x) \qquad f'(0) = 1$

$\left(\frac{dy}{dx}\right)_{x=0} = f'(f(f(0))) \cdot f'(f(0)) \cdot f'(0)$

(27) $y \quad x\tan\frac{x}{2}$

$(1 \quad \cos x)\frac{dy}{dx} \quad x \quad \tan\frac{x}{2}(1 \quad \cos x), \ \frac{dy}{dx}$

$x \cdot \sec^2\frac{x}{2} \cdot \frac{1}{2} \quad \tan\frac{x}{2} \qquad (1 \quad \cos x)\frac{dy}{dx} \quad x\sin x \quad \frac{x}{2}\frac{1}{\cos^2\frac{x}{2}} \quad \tan\frac{x}{2}$

$\frac{x}{2}\frac{1}{(1 \quad \cos x)} \quad \tan\frac{x}{2}$, B = sin$x$, A = 1 + cos$x$

(28) $f(x) = \sec^{-1}x, f'(x) \quad \frac{1}{|x|\sqrt{x^2 \quad 1}}$

(29) $y \quad \cos^{\ 1}\left(\frac{3x \quad 4\sqrt{1 \quad x^2}}{5}\right), \ y \quad \cos^{\ 1}\left(\frac{3}{5}x \quad \frac{4}{5}\sqrt{1 \quad x^2}\right)$

Taking , $\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5}$

(30) $y = \sin^{-1}(3x - 4x^3) \qquad = 3\sin^{-1}x, \ |x| \quad \frac{1}{2} = \quad - 3\sin^{-1}x, \ \frac{1}{2} < x \quad 1$

$= - \quad - 3\sin^{-1}x, \ -1 \quad x < \frac{1}{2}$

$z = \sin^{-1}x$ then y $= 3z, |x| \; \frac{1}{2} = \; - 3z, \frac{1}{2} < x < 1$

$= - \; - 3z, -1 \; x < \frac{1}{2}$

(31) Try your self

(32) $y = x^{3000}$, $x = 1$ and $x + \; x = 1.002$

$$\frac{dy}{dx} = 3000x^{2999} \Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 3000$$

Now using formula $y \; \frac{dy}{dx} \cdot \; x$

(33) $f'(x) > 0$ and $g'(x) < 0$

f is a incresing and g is a Decrising function

$f(x - 1) < f(x) < f(x + 1)$ and

$y(x - 1) > g(x) > g(x + 1)$ ....

(34) $y \; \sin^{1}\left(\frac{1 \; x^2}{1 \; x^2}\right) \; \frac{}{2}$ Differentiate using Chain Rule

(35) $x \; \sqrt{\frac{1 \; t^2}{1 \; t^2}} \qquad x^2 \; \frac{1 \; t^2}{1 \; t^2}$

Now taking $y \; \frac{1 \; x}{1 \; x}$ and then find $\frac{dy}{dx}$

(36) $\frac{dy}{dx} \; \sin\left(\frac{2x \; 1}{x^2 \; 1}\right)^2 \left(\frac{2(x^2 \; x \; 1)}{(x^2 \; 1)^2}\right)$

AB $(x^2 \; 1)^2 \cdot \frac{(2x \; 1)^2}{(x^2 \; 1)^2} \; (2x \; 1)^2$

(37) $f''(x) = f(x)$ and $f'(x) = g(x)$

$m(x) = (f(x))^2 + (g(x))^2$

$m'(x) = 2f(x) \cdot f'(x) + 2g(x) \cdot g'(x)$

$m'(x) = 0$ ($\because f'(x) = g(x), f''(x) = g'(x)$)

(38) $y^2 = p(x) \; 4y^3y_2 = 2y^2 \cdot p''(x) - p'(x)^2, \; 2yy_1 = p'(x)$

$$y_2 \quad \frac{2y^2 \cdot p''(x) \quad p'(x)^2}{2y^3}$$

(39) $f(x) \quad \sin^{1}\left(2 \quad \sqrt{1 \quad x^2}\right), \; f'(x) \quad \frac{2(1 \quad 2x^2)}{|1 \quad 2x^2|\sqrt{1 \quad x^2}} \; |x| \quad 1$

Here $f'(x) > 0$then $1 - 2x^2 < 0$

$2x^2 < 1 \qquad 1 \quad 1 < \frac{1}{\sqrt{2}}, \; \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ strictly incrising

(40) Taking componendo any dividendo So we get $\frac{x \quad 1}{x \quad 1} \quad e^{2y}$

taking log both sides, $\log(x + 1) - \log(x - 1) = 2y$

$$\frac{dy}{dx} = \frac{-1}{x^2 - 1} = \frac{1}{1 - x^2}$$

(41) $f(x) = (1 + x)^n, \; f'(x) = n(1 + x)^{n-1}, \; f''(x) = n(n - 1)(1 + x)^{n-2}$

$f''(1) = n(n - 1)2^{n-2}$

(42) $x = a\cos \quad + \frac{b}{2} \cos 2 \qquad$ Now $\frac{d^2y}{dx^2} \quad 0$

$y = a\sin \quad + \frac{b}{2} \sin 2 \qquad \frac{dy}{dx} \quad 0$

$$\frac{dy}{dx} \quad \frac{a\cos \quad b\cos 2}{a\sin \quad b\sin 2}$$

(43) $y \quad \sqrt{x^2 \quad 16}$, $z = \frac{x}{x+1}$

$$\frac{dy}{dx} = x(x^2 + 16)^{-\frac{1}{2}}$$

$$\frac{dz}{dx} \quad \frac{1}{(x \quad 1)^2}$$

(44) How $xy = 1, \quad y \quad \frac{1}{x}$ Now $x + y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 + 2 \geq 2$

For all $x > 0$, $x + y \quad 1$, Min. value is 2

(45) $fog(x) = I(x)$

$f(g(x)) = x,\ f'(g(x)).g'(x) = 1$

$f'(g(a)).g'(a) = 1$

$f'(b) \cdot 2 = 1$

(46) $y \quad \frac{1}{3}\left(\log(x \quad 1) \quad \frac{1}{2}\log(x^2 \quad x \quad 1)\right)$

$$\frac{dy}{dx} \quad \frac{1}{3}\cdot\frac{1}{x \quad 1} \quad \frac{1}{6}\cdot\frac{1}{x^2 \quad x \quad 1}(2x \quad 1) \quad \frac{1}{\sqrt{3}}\cdot\frac{1}{1 \quad \left(\frac{2x \quad 1}{\sqrt{3}}\right)^2}$$

Now simplifyng $\frac{dy}{dx} \quad \frac{1}{3(x \quad 1)} \quad \frac{2 \quad x}{3(x^2 \quad x \quad 1)}$

(47) Now $\frac{d}{dx}\left(3^{\log_{10}|\text{cosec} \quad ^1 x|}\right)$

$3^{\log_{10}|\text{cosec} \quad ^1 x|} \cdot \log_3 \cdot \frac{d}{dx}\log_{10}|\text{cosec} \quad ^1 x|$, Now simplifing

(48) $y = \sum_{r=1}^{x}\tan^{-1}\frac{1}{1+r+r^2} \quad \sum_{r \quad 1}^{n}\tan \quad ^1\left(\frac{r \quad 1 \quad r}{1 \quad r(r \quad 1)}\right)$

Now taking darivative

(49) $y = (\log_{\cos x}\sin x)(\log_{\sin x}\cos x)^{-1}$ and $x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Now $\frac{dy}{dx} \quad \frac{dy}{dx} \quad \frac{dy}{dx}, \frac{dy}{dx} = \frac{d}{dx}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$y = \left(\frac{\log \ \sin x}{\log \ \cos x}\right)^2 \qquad \therefore \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{-8}{\log_e 2}$

$\left(\frac{dy}{dx}\right)_{\left(x \quad \frac{}{4}\right)} \quad \frac{8}{\log 2} \quad \frac{32}{16 \quad ^2}$

(50) $x = a\,(1 - \cos^3\ )$

$$\frac{dx}{d} = 3a\cos^2\ \cdot \sin$$

$y = a\sin^3$ $\qquad \frac{dy}{d} = 3a\sin^2\ \cdot \cos$

$$\frac{dy}{dx} \quad \frac{\frac{dy}{d}}{\frac{dx}{d}} \quad \frac{3a\sin^2\ \cos}{3a\cos^2\ \sin} \quad \left(\frac{d^2y}{dx^2}\right) \quad \sin^2 \qquad \frac{dy}{dx} \quad \left(\frac{d^2y}{dx^2}\right)_{a\ \ \overline{6}} \quad \text{..........}$$

(51) $e^y \quad \frac{e^2}{x^2} \qquad y = \log_e 2 - \log x^2,\ y = 2 - 2\log_e x$

taking $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ respectively

(52) $y = x^x,\ \log y = x \cdot \log x$

$$\frac{d^2y}{dx^2} \quad \frac{y}{x} \quad \frac{1}{x^x} \cdot \left(\frac{dy}{dx}\right)^2$$

$$\frac{dy}{dx} = x^x\,(1 + \log x)$$

(53) $x^{13} \cdot y^7 = (x + y)^{20},\ 13\log x + 7\log y - 20\log(x + y)$

$$\frac{13}{x} \quad 7\frac{1}{y} \cdot \frac{dy}{dx} \quad 20\frac{1}{x \quad y}\left(1 \quad \frac{dy}{dx}\right) \text{...........}$$

(54) $x \quad 235\left(\cos t \quad \log\tan\frac{t}{2}\right) \qquad \frac{dx}{dt} \quad 235 \cdot \frac{\cos^2 t}{\sin t}$

$y = 235\cos t \qquad \frac{dy}{dt} \qquad 235\sin t$

$$\frac{dy}{dx} \qquad \tan^2 t$$

(55) Here $x = 2$ and $y = -1$

$2 = t^2 + 3t - 8$ and $-1 = 2t^2 - 2t - 5$

$(t - 2)(t + 5) = 0$ and $(t - 2)(t + 1) = 0$

$$\left(\frac{dy}{dt}\right)_{t\;2} \quad \frac{6}{7}$$

(56) the tangent Intersect to Y–axis, tan x = 0, point of intersection is (0,0)

the tangent is $y - 0 = \left(\frac{1}{2} \quad e^{o}\right)(x - 0)$, $x + 2y = 0$

(57) $\left(\frac{x}{a}\right)^{\frac{2}{3}} = \cos^2\theta, \left(\frac{y}{b}\right)^{\frac{2}{3}} = \sin^2\theta$

$\left(\frac{x}{a}\right)^{\frac{1}{3}} = \cos\theta, \left(\frac{y}{b}\right)^{\frac{1}{3}} = \sin\theta$

$x = a\cos^3\theta,\ y = a\sin^3\theta$

$\frac{dy}{dx} = \tan\theta$

(58) $y = f(x) \quad \left(\frac{dy}{dx}\right)_{(1,\,1)} \quad 2,\ \frac{dy}{dx} = f'(x)$, tan = 2, $2x = f'(x)$, $= \tan^{-1}2$

(59) If curve is parallel to x–axis then $\frac{dy}{d} \quad 0$ and $\frac{dx}{d} \quad 0 \quad \cos 2 \quad = 0$

$\frac{d}{d\theta}\left(\frac{3}{2}\sin 2\theta\right) = 0 \quad 2\theta = (2k + 1)\frac{\pi}{2}$, k $\in$ z

$2 \quad \frac{}{2} \quad \frac{}{4}$

(60) $|x|$ is not differentiable at $x = 0$

$\left|x \quad \frac{1}{2}\right|, |x \quad 1|$ it also not differantiable at $x = \frac{1}{2}$, 1

$x \quad \partial 2x \quad 1 \int \frac{}{2}, x \quad 1 \qquad f(x)$ not differentiable $x = 0$, $, \frac{}{2}$

(61) $f(x) = (x - a)^m (x - b)^n \quad f'(x) = (x - a)^{m-1} \cdot (n - b)^{n-1} [x(m + n) - (mb + na)]$

$f'(x) = 0$, $x = a$ or $x = b$ or $x \quad \frac{mb \quad na}{m \quad n}$

(62) Getting Equation from $f(1) = f(3)$ So, we get, $26a + 18b = -22$

$f'(x) = 3ax^2 + 2bx + 11$

$f'\left(2 \quad \frac{1}{\sqrt{3}}\right) \quad 0 \qquad 6a + b = 0 \qquad \frac{a}{b} \quad \frac{1}{6} \qquad a = 1, b = -6$

(63) $f'(x) = 1 + 2\sin x + 3\cos^2 x, 0 \quad x \quad 2\frac{}{3}$

$f'(x) = 0 \qquad 2\cos x\,(1 - 3\sin x) = 0 \qquad \cos x = 0$ or $\sin x = \frac{1}{3}$

$x \quad \frac{}{2}$ or $x \quad \sin^{1}\frac{1}{3}$

(64) $y \quad a\log\sec\frac{x}{a}$ Now using slope – point equation

$\frac{dy}{dx} \quad \tan\frac{x}{a}, \left(\frac{dy}{dx}\right)_{x=a} = \tan 1$

(65) $\cos\frac{x}{2} \cdot \cos\frac{x}{2^2}$ ..... $\frac{\sin x}{x}$, Taking log on both side

$\log\cos\frac{x}{2} + \log\cos\frac{x}{2^2} + \log\cos\frac{x}{2^3} + ... \infty = \log\sin x - \log x$, Now diff w.r to x

(66) $y = (x \cdot \log x)^{\log(\log x)}$, Taking log both side, $\log y = \log(\log x)\,[\log x + \log(\log x)]$

(67) $n^m \cdot y^n = a^{m+n}$, $m\log x + n\log y = (m+n)\log x$

$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} \qquad \frac{my_1}{nx_1}$ then using slope–point and compare

(68) $f(x) = (a+2)x^3 - 3ax^2$

$f(x) = (a+2)x^3 - 3ax^2$

$(a+2) < 0$ and $(4a^2 - 4(a+2)\,3a^2 < 0$

(69) $f(x) = -\sin 4x \qquad < 4x < 2 \qquad \sin 4x < 0$, f is incresing in $\left(\frac{}{4}, \frac{}{2}\right)$

$-\sin 4x > 0$

(70) $f(x) = \tan^{-1}(\sin x + \cos x)$, $f'(x) = \dfrac{\cos x\,(1 \quad \tan x)}{1 \quad (\sin x \quad \cos x)^2}$

$0 < x < \dfrac{}{4}$ $\quad f'(x) > 0$, In $(0, \dfrac{}{4})$, increasing function

(71) $y^2 \quad 12 \left(x \quad 2\sin\dfrac{x}{2} \right) \qquad y\dfrac{dy}{dx} = 6\left(1 + \cos\dfrac{x}{2}\right)$

tangent are paralll to x–axis $then \dfrac{dy}{dx} = 0$

$$1 \quad \cos\frac{x}{2} \quad 0 \quad \cos\frac{x}{2} \quad 1, \quad x = 0, 2\pi$$

(72) $y \quad \dfrac{(\quad x)(\quad x)}{(\quad x)} \quad f''(x) \quad \dfrac{2(\quad)(\quad)}{(x \quad)^3}$

$f'(x) \quad \dfrac{(\quad)(\quad)}{(x \quad)^2} \quad 1, f''(x) > 0$

$x \quad \sqrt{(\quad) \quad (\quad)}$

(73) Assuming tanA tanB = tanA $\tan\left(\dfrac{\pi}{3} - A\right) = Z$

Now $\dfrac{dZ}{dA} = \tan A \sec^2\left(\dfrac{\pi}{3} - A\right) + \tan\left(\dfrac{\pi}{3} - A\right)\sec^2 A$

Now , So $A = \dfrac{\pi}{6} \quad \therefore \left.\dfrac{d^2Z}{dA^2}\right]_{A=\frac{\pi}{6}} < 0 \therefore Z = \dfrac{1}{3}$

(74) $y \quad a^x \quad \left(\dfrac{dy}{dx}\right)_{(0,1)} \quad \log a \quad m_1, \; y \quad b^x \quad \left(\dfrac{dy}{dx}\right)_{(0,1)} \quad \log b \quad m_2$

Now $\tan\alpha = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

(75) $y \quad \dfrac{1 \quad x\tan x}{x}, \; \dfrac{dy}{dx} \quad \dfrac{1}{x^2} \quad \sin^2 x$

$\frac{1}{x}$ $\tan x$ For max. value $\frac{dy}{dx}$ 0 $x = \cos x$

(76) Using Roll's Theorm

(77) The roll's theroem is applicable for f(x) = x^2

(78) y $\frac{4}{3}$ r^3 $\frac{dy}{dt}$ 4 r^2 $\frac{dr}{dt}$ but $\frac{dy}{dt}$ $\frac{dr}{dt}$ 4 r^2 1

Now find the value of r

(79) $v =$ $r^2 h$ First Find $\frac{dv}{dt}$ and gets $\frac{dv}{dt} = 0$, So we gets r = 2h

(80) $y = (x - 2)(x - 3)$, Now m = 225°

$\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$ $2x$ 5

$x = 3$ and $y = 0$

(81) If two curves are intersect ortho gonally they using condition

$a^2 - b^2 - c^2 - f^2$

(82) Using the formaua of normal and sub normal

(83) First find $\frac{dy}{dx}$ and put up the curve in formula of the light of normal is

y $\sqrt{1 \left(\frac{dy}{dx}\right)^2}$

(84) Here $_e y = 1 + x^2$ Here A^m G^m

$y = \log(1 + x^2)$ $\frac{1 \quad |x|^2}{2}$ $|x|$, $|m| = \frac{2|x|}{1 + |x|^2}$ $|\tan\theta|$ 1

(85) $f'(x) = 3ax^2 + 2bx + c$ it's a dariving of $f(x) = ax^3 + bx^2 + cx$, x (0, 1)

Now $f(0) = 0$ and $f(1) = a + b + c = 0$

(86) $f(x) = \frac{1}{2} - x, \quad x < \frac{1}{2}$

$= \left(\frac{1}{2} - x\right)^2, \; x \geq 2$

For x $\frac{1}{2}$ L.H.D. and R.H.D. are not same

(87) $f'(c) \quad \frac{f(x) \quad f(0)}{x \quad 0}$ $\qquad \left|\frac{f(x)}{x}\right| \quad |f'(c)| \quad \frac{1}{2}$

$f'(c) \quad \frac{f(x)}{x}$ $\qquad |f(x)| \quad \frac{|x|}{2}$

$|f(x)| \quad \frac{|x|}{2}$

(88) $f(x) = 2\log(x-2) - x^2 + 4x + 1,\ f'(x) \quad \frac{2(x \quad 1)(x \quad 3)(x \quad 2)}{(x \quad 2)^2}$

$f'(x) > 0 \qquad (x-1)(x-2)(x-3) < 0$

(89) $f'(\ )$ and $g'(\ )$ find first

$m(\ ) = \sin \quad - \quad \sin$ $\quad m'(\ )$ and $h'(\ )$

$h(\ ) = \tan \quad - \quad \cdot \sec^2$ $\qquad$ Find

(90) $f(l) = l - e^l + \tan\frac{112\pi}{7}$ $\qquad -1 + e^l < 0,\ f'(l) > 0 = l = e^l > 0 \qquad l < 0$

In $l \in \quad , 0)$ Incrising function

(91) $f(x) = \frac{a^2 \quad 1}{a^2 \quad 1}x^3 - 3x + 5\log_e 2,\ f'(x) = \frac{a^2 \quad 1}{a^2 \quad 1}(3x^2) - 3$

$\frac{a^2 \quad 1}{a^2 \quad 1} \quad 0,\ a \quad [-1, 1],$

(92) Using $\cos x > 1 - \frac{2x}{\quad}$

(93) $y = \quad \tan^2 x + \quad \cot^2 x$ and $Z = a\sin^2 \quad + b\cos^2$

$$y = \left(\sqrt{\alpha}\tan x - \sqrt{\beta}\cot x\right)^2 + 2\sqrt{\alpha\beta} \geq 2\sqrt{\alpha\beta}$$

$$y_{\min} = 2\sqrt{\alpha\beta} \qquad \therefore\ \frac{d^2 Z}{d\theta^2} = 2(a-b)\cos^2\theta$$

$$y_{\min} = z_{\max} = 2\sqrt{ab} = a$$

(94) $y \quad \sec^{\ 1}\left(\frac{1}{2x^2 \quad 1}\right)$ $\qquad \frac{dy}{dx} = \frac{\sqrt{1+3x}}{\sqrt{1x^2}}$

$y = 2\cos^{-1}x, \quad \left(\frac{dy}{dz}\right)_{x \;\; \frac{1}{3}} \quad 0$

$Z = \sqrt{1+3x} \qquad \therefore \frac{dz}{dx} = \frac{3}{2}\cdot\frac{1}{\sqrt{1+3x}}$

(95) minimum value is 6

(96) $f(x) \quad \frac{e^{2x} \quad 1}{e^{2x} \quad 1} \qquad f'(x) \quad \frac{4\cdot e^{2x}}{(1 \quad e^{2}x)^{2}} \quad 0$ So $f(x)$ is a Increasing function

(97) For $x > 2$ $f(x) = |x-1| = x-1, f(f(x)) = f(x-1) = |(x-1)-1|$

$f(f(x)) = f(x-2)$

$= |(x-2)-1|$

$= x-3, \; x \quad 3$

$= 3-x, \; e^{x} \quad 3$

(98) $y = a\sin^{3}$, $x = a\cos^{2}$

$\frac{dy}{d} = 3a\sin^{2} \cdot \cos \qquad \frac{dx}{d} = -3a\cos^{-2} \quad \sin$

$\frac{dy}{dx} = -\tan\alpha \qquad \frac{d^{2}y}{dx^{2}} = -\sec^{2}\alpha \bullet \frac{d\alpha}{dx}$

(99) $y = (1+x)^{y} + \sin^{-1}(\sin^{2}x)$ If $x = 0$ then $y = 1$

$\left(\frac{dy}{dx}\right)_{(0,1)} \quad 1$ Eq of Normal is $y - 1 = -1(x - 0)$

$x + y = 1$

(100) $y \quad \tan^{\;1}\left(\frac{(x \quad 1) \quad x}{1 \quad x(x \quad 1)}\right) \quad \tan^{\;1}\left(\frac{(x \quad 2) \quad (x \quad 1)}{1 \quad (x \quad 2)(x \quad 1)}\right)$

$\tan^{\;1}\left(\frac{(x \quad 3) \quad (x \quad 2)}{1 \quad (x \quad 3)(x \quad 2)}\right) \quad .....$

$y = \tan^{-1}(x+n) - \tan x \qquad \frac{dy}{dx} \quad \frac{1}{1 \quad (x \quad n)^{2}} \quad \frac{1}{1 \quad x^{2}}$

Answers

1	b	2	a	3	b	4	a	97	c
5	a	6	a	7	a	8	d	98	a
9	c	10	a	11	a	12	b	99	b
13	a	14	b	15	a	16	a	100	b
17	a	18	b	19	c	20	a		
21	b	22	a	23	a	24	b		
25	a	26	b	27	b	28	c		
29	a	30	b	31	b	32	c		
33	a	34	c	35	c	36	a		
37	a	38	d	39	b	40	c		
41	a	42	d	43	c	44	b		
45	b	46	a	47	b	48	a		
49	a	50	b	51	c	52	b		
53	d	54	b	55	a	56	b		
57	a	58	c	59	c	60	d		
61	a	62	d	63	a	64	b		
65	a	66	b	67	a	68	b		
69	a	70	b	71	b	72	b		
73	b	74	b	75	b	76	a		
77	b	78	d	79	a	80	b		
81	a	82	d	83	d	84	d		
85	a	86	a	87	b	88	a		
89	b	90	b	91	a	92	a		
93	d	94	a	95	d	96	a		

Unit - 9

Indefinite And Definite Integration

Important Points

1. If $\frac{d}{dx}[F(x)+c] = f(x)$ then $\int f(x)\,dx = F(x)+c$

$\int f(x)dx$ is indefinite integral of f(x) w.r.to x where c is the arbitrary constant.

Rules of indefinite Integration

1 If f and g are integrable function on [a,b] and f+g is also integrable function on [a,b], then

$\int (f(x)+g(x))dx = \int f(x)\,dx + \int g(x)\,dx$.

If $f_1, f_2, \ldots, f_n$ an integrable function on [a,b] then

$\int (f_1(x) + f_2(x) + f_3(x) + \ldots + f_n(x))dx = \int f_1(x)dx + \int f_2(x)dx + \ldots + \int f_n(x)dx.$

2 (i) If f is integrable on [a, b] and k is the real constant then, kf is also integrable then

$\int kf(x)dx = k\int f(x)dx$

(ii) $\int [k_1 f_1(x) + k_2 f_2(x) + \ldots\ldots\ldots + k_n f_n(x)]\,dx$

$= k_1 \int f_1(x)dx + k_2 \int f_2(x)dx + \ldots\ldots\ldots + k_n \int f_n(x)\,dx$

3 If f and g are integrable functions on [a, b] then

$\int (f(x)-g(x))dx = \int f(x)\,dx - \int g(x)dx$

- Important formulae

1 $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c;\ n\in R - \{-1\};\ x\in R^+$

If n = 0 then $\int dx = x + c$

2 $\int \frac{1}{x}\,dx = \log|x| + c;\ x \in R - \{0\}$

3 (i) $\int a^x\,dx = \frac{a^x}{\log_e a} + c;\ a\in R^+ - \{1\},\ x \in R$

(ii) $\int e^x dx = e^x + c; \forall\, x\in R$

4 $\int \sin x\,dx = -\cos x + c\ ,\ \ \forall\ x \in R$

5 $\int \cos x \, dx = \sin x + c \ , \quad \forall \ x \in R$

6 $x = \tan x + c \ , \quad x \neq (2k-1)\dfrac{\pi}{2}, \ k \in Z$

7 $\int cosec^2 x \, dx = - \cot x + c \ , \ x \neq k\pi, k \in Z$

8 $\int \sec x \tan x \, dx = \sec x + c \ , \ x \neq (2k-1)\dfrac{\pi}{2}, \ k \in Z$

9 $\int cosec\, x . \cot x d\, x = -cosec\, x + c, \ x \neq k\pi, k \in Z$

10 $\int \dfrac{1}{x^2 + a^2} dx = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + c \ , \ a \in R - \{0\}, \ x \in R$

$= -\dfrac{1}{a}\cot^{-1}\dfrac{x}{a} + c, \ a \in R - \{0\}, \ x \in R$

11 $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + c, \ a \in R - \{0\}, \ x \neq \pm a$

12 $\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a}\log\left|\dfrac{x+a}{x-a}\right| + c \ , \ a \in R - \{0\}, \ x \neq \pm a$

13 $\int \dfrac{dx}{\sqrt{x^2 \pm k}} = \log\left| x + \sqrt{x^2 \pm k}\right| \ c \ , \quad |x| > |k|$

14 $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\dfrac{x}{a} + c \ , \quad x \in (-a, a), \ a > 0$

$= -\cos^{-1}\dfrac{x}{a} + c, \quad x \in (-a, a); \ a > 0$

15 $\int \dfrac{1}{|x|\sqrt{x^2 - a^2}} dx = \dfrac{1}{a}\sec^{-1}\dfrac{x}{a} + c \ , \ |x| > |a| > 0$

$= -\dfrac{1}{a} cosec^{-1}\dfrac{x}{a} + c \ , \quad |x| > |a| > 0$

16. $\int \dfrac{1}{a + bx^2} dx = \dfrac{1}{\sqrt{ab}}\tan^{-1}\left(\sqrt{\dfrac{b}{a}}\, x\right) + c, \ (a, b > 0)$

Method of substitution

* If $g : [\alpha, \beta] \to R$ is continuous and differentiable on (α, β)

and $g'(t)$ is continuous and non zero on (α, β) if $R_g \subset [a,b]$

and $f : [a, b] \to R$ is continuous and $x = g(t)$ then $\int f(x)\, dx = \int [f\, g(t)\, g'(t)] dt$

* If $\int f(x)\, dx = F(x)$ +c then $\int f(ax+b)\, dx = \frac{1}{a} F(a\alpha + b) + C,$

wherc $f : I \to R$ is continuous $(a \neq 0)$

* $\int f(x)^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c, \quad (n \neq -1,\ f(x) > 0,\ f'(x) \neq 0)$

* $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c,$ (f and f′ are continuous $f'(x) \neq 0,\ f(x) \neq 0$)

* $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$ (f and f′ are continuous $f'(x) \neq 0,\ f(x) \neq 0$)

17. $\int \tan x\, dx = \log|\sec x| + c$

$= -\log|\cos x| + c \qquad x \neq \frac{k\pi}{2}, k \in z$

18. $\int \cot x\, dx = \log|\sin x| + c, \qquad x \neq \frac{k\pi}{2}, k \in z$

$= -\log|\operatorname{cosec} x| + c$

19. $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x|\, c, \; x \neq \frac{k\pi}{2}, k \in z$

$= \log\left|\tan \frac{x}{2}\right| + c$

20. $\int \sec x\, dx = \log|\sec x + \tan x| + c, \quad x \neq \frac{k\pi}{2}, k \in z$

$= \log\left|\tan \frac{\pi}{4} + \frac{x}{2}\right| + c, \qquad x \neq \frac{k\pi}{2}, k \in z$

Integrals	Substitutions		
(i) $\sqrt{x^2 + a^2}$	$x = a \tan\theta$	or	$x = a \cot\theta$
(ii) $\sqrt{x^2 - a^2}$	$x = a \sec\theta$	or	$x = a \operatorname{cosec}\theta$
(iii) $\sqrt{a^2 - x^2}$	$x = a \sin\theta$	or	$x = a \cos\theta$

(iv) $\sqrt{\dfrac{a-x}{a+x}}$ $\qquad x = a\cos 2\theta$

(v) $\sqrt{2ax - x^2}$ $\qquad x = 2a\sin^2\theta$

(vi) $\sqrt{2ax - x^2} = \sqrt{a^2 - (x-a)^2}$ $\qquad x - a = a\sin\theta$ or $a\cos\theta$

For the integrals :

$\dfrac{1}{a + b\cos x}$, $\dfrac{1}{a + c\sin x}$ and $\dfrac{1}{a + b\cos x + c\sin x}$, taking $\tan\dfrac{x}{2} = t$

*** <u>Integration by parts</u>**

$$\int u\ v\ dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$$

21. $\int\sqrt{x^2+a^2}\ dx = \dfrac{x}{2}\sqrt{x^2+a^2} - \dfrac{a^2}{2}\log\left|\ x + \sqrt{x^2+a^2}\ \right| + c$

22. $\int\sqrt{x^2-a^2}\ dx = \dfrac{x}{2}\sqrt{x^2-a^2} - \dfrac{a^2}{2}\log\left|\ x + \sqrt{x^2+a^2}\ \right| + c$

23. $\int\sqrt{a^2-x^2}\ dx = \dfrac{x}{2}\sqrt{a^2-x^2} + \dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a} + c \quad (a > 0)$

24. $\int e^{ax}\sin bx\ dx = \dfrac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c$

25. $\int e^{ax}\cos bx\ dx = \dfrac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$

26. $\int e^{ax}\sin bx\ dx = \dfrac{e^{ax}}{\sqrt{a^2+b^2}}\sin(bx - \theta) + c$

$$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}\ ;\ \sin\theta = \frac{b}{a^2+b^2}\ ;\ \theta\in(0,\ 2\pi)$$

27. $\int e^{ax}\cos bx\ dx = \dfrac{e^{ax}}{\sqrt{a^2+b^2}}\cos(bx - \theta) + c$

$$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}\ ;\ \sin\theta = \frac{b}{a^2+b^2}\ ;\ \theta\in(0,\ 2\pi)$$

28. $\int e^x \left(f(x) + f'(x) \, dx = e^x \ f(x) + c \right)$

Definite Integration

Limit of a Sum

1. $\int_a^b f(x)dx = \lim_{h \to 0} h \sum_{i=1}^{n} f(a+ih)$

2. $\int_a^b f(x)dx \ \lim_{n \to \infty} \frac{b-a}{n} \sum_{i=1}^{n} f\left[a + i\left(\frac{b-a}{n} \right) \right] Where\, h = \frac{b-a}{n}$

Fundamental theorem of definite Integration

If f is continuous on [a, b] and F is differentiable on (a, b) such that

$\forall x \in (a,b) \ if \ \frac{d}{dx}(F(x)) = f(x) \ then \int_a^b f(x)\, dx = F(b) - F(a)$

Rules of definite Integration

1 If f and g are continuous in [a, b] then $\int_a^b [f(x) + g(x)]\, dx = \int_a^b f(x)dx + \int g(x)dx$

2 If f is continuous on [a, b] and k is real constant, then $\int_a^b k\, f(n)\, dx = k\int_a^b f(x)\, dx$

3 If f is continuous on the [a, b] and a < c < b then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

4 $\int_a^b f(x)dx = -\int_b^a f(x)dx$

5 $\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(u)du$

Theorems

1 If f is even and continuous on the [-a, a] then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

2 If f is odd and continuous on the [-a, a] then $\int_{-a}^{a} f(x)dx = 0$

3 If f is continuous on [0, a] then $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

4 If f is continuous on [a, b] then $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

5 If f is continuous on [0, 2a] then $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

Application of Integration

1 The area A of the region bounded by the curve y=f(x), X - axis and the lines

x = a, x = b is given by A = |I|, where I = $\int_a^b f(x)dx$ or I = $\int_a^b ydx$

2 The area A of the region bounded by the curve x = g(y) and the line y = a and y = b given

by A = |I| Where I = $\int_a^b g(y)dy$ or I = $\int_a^b ydx$

3 If the curve y = f(x) intersects X - axis at (c, 0) only and a < c < b then the area of the region bounded by y = f(x), x = a, x = b and X - axis is given by

A = $|I_1|+|I_2|$ where $I_1 = \int_a^c ydx$, $I_2 = \int_c^b ydx$

4 If two curves y = $f_1(x)$ and y = $f_2(x)$ intersect each other at only two points for
x = a and x = b (a $\neq$ b) then the area enclosed by them is given by

A = |I| and I = $\int_a^b \left(f_1(x) - f_2(x) \right) dx$

5 If the two curves $x = g_1(y)$ and $x = g_2(y)$ intersect each other at only two points for
y = a and y = b (a $\neq$ b) then the area enclosed by them is given by

A = |I| where I = $\int_a^b \left(g_1(y) - g_2(y) \right) dy$

Question Bank

(Indefinite Integration)

(1) $\int \frac{dx}{1+\tan x} =$ ________ $+c$

(a) $\log |\sec x+\tan x|$ (b) $2\sec^2 \frac{x}{2}$

(c) $\log |x+\sin x|$ (d) $\frac{1}{2}\left[x+\log |\sin x+\cos x|\right]$

(2) $\int \frac{e^x+1}{e^x-1}dx =$ ________ $+c$

(a) $2\log \left|e^{\frac{x}{2}}-e^{\frac{-x}{2}}\right|$ (b) $2\log \left|e^{\frac{x}{2}}+e^{\frac{-x}{2}}\right|$

(c) $2\log |e^x-1|$ (d) $\log |e^x+1|$

(3) $\int \frac{e^{5\log x}-e^{3\log x}}{e^{4\log x}-e^{2\log x}}dx =$ ________ $+c$

(a) $e\cdot 2^{-2x}$ (b) $e^3 \log_e x$ (c) $\frac{x^3}{3}$ (d) $\frac{x^2}{2}$

(4) $\int \frac{dx}{x\left(x^n+1\right)} =$ ________ $+c$

(a) $\frac{1}{n}\log \left|\frac{x^n+1}{x^n}\right|$ (b) $\frac{1}{n}\log \left|\frac{x^n}{x^n+1}\right|$

(c) $\frac{1}{n}\log |x^n+1|$ (d) $\frac{1}{n}\log \left|\frac{x^n-1}{x^n}\right|$

(5) $\int \frac{\log(x+1)-\log x}{x(x+1)}dx =$ ________ $+c$

(a) $\log x-\log(x+1)$ (b) $\log(x+1)-\log x$

(c) $-\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2$ (d) $-\left[\log\left(\frac{x+1}{x}\right)\right]^2$

(6) $\int e^{\cot^{-1}x}\left(1-\frac{x}{1+x^2}\right)dx =$ ________ $+c$

(a) $\frac{1}{2}xe^{\cot^{-1}x}$ (b) $\frac{1}{2}e^{\cot^{-1}x}$ (c) $xe^{\cot^{-1}x}$ (d) $e^{\cot^{-1}x}$

(7) $\int \frac{\tan x}{\sqrt{\cos x}} dx =$ __________ $+ c$

(a) $\frac{-2}{\sqrt{\cos x}}$ (b) $-\frac{1}{\sqrt{\cos x}}$ (c) $\frac{-2}{3\sqrt{\cos x}}$ (d) $\frac{-3}{2\sqrt{\cos x}}$

(8) $\int e^{4\log x}\left(x^5+1\right)^{-1} =$ __________ $+ c$

(a) $\frac{1}{5}\log\left(x^4+1\right)$ (b) $-\log\left(x^4+1\right)$ (c) $\log\left(x^4+1\right)$ (d) $\frac{1}{5}\log\left(x^5+1\right)$

(9) $\int \cos ec^3 x \ dx =$ __________ $+ c$

(a) $-\frac{1}{2}\cos ec \ x \ \cot x + \frac{1}{2}\log \ \left|\cos ecx + \cot x\right|$ (b) $-\frac{1}{2}\cos ec \, x \cot x$

(c) $\frac{1}{2}\cos ec \ x \cot x + \frac{1}{2}\log \ \left|\cos ecx + \cot x\right|$ (d) $\frac{1}{2}\cos ec \ x \cot x - \frac{1}{2}\log \ \left|\cos ecx + \cot x\right|$

(10) If $\int \frac{2^{1/x^2}}{x^3} dx = k 2^{1/x^2} + c$ then $k =$ ________

(a) $-\frac{1}{2\log_e 2}$ (b) $-\log \ 2$ (c) -2 (d) $-\frac{1}{2}$

(11) $\int (x-1)e^{-x} dx =$ __________ $+ c$

(a) xe^x (b) $-xe^{-x}$ (c) $-xe^x$ (d) xe^{-x}

(12) $\int \left(\sin(\log x) - \cos(\log x)\right) dx =$ ________ $+ c$

(a) $\sin(\log x) - \cos(\log x)$ (b) $-x \ \sin(\log x)$

(c) $-x \ \cos(\log x)$ (d) $\sin(\log x) + \cos(\log x)$

(13) $\int (x+4)(x+3)^7 dx =$ __________ $+ c$

(a) $\frac{(x+3)^9}{9} - \frac{(x+3)^8}{8}$ (b) $\frac{(x+3)^8(8x+33)}{72}$ (c) $\frac{(x+3)^8(8x+33)}{72}$ (d) $\frac{(x+3)^8}{8}$

(14) $\int \frac{dx}{(x+3)\sqrt{x+2}} =$ __________ $+ c$

(a) $2\tan^{-1}\sqrt{x+2}$ (b) $2\tan^{-1}\sqrt{x^2+3}$ (c) $2\tan^{-1} x$ (d) $2\tan^{-1}\sqrt{x^2+2}$

(15) $\int \frac{e^x}{e^x+2+e^{-x}} =$ __________ $+ c$

(a) $-\frac{1}{2}\left(e^{2x}+1\right)$ (b) $-\frac{1}{2}\left(e^{2x}+1\right)^{-1}$ (c) $-\left(e^{2x}+1\right)$ (d) $-\left(e^{2x}+1\right)^{-1}$

(16) If $\int \frac{\cos x}{\sqrt{\sin^2 x + 2\sin x + 1}} dx = A \log \sqrt{\sin x + 1} + c$ then $A =$ ________

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) -2

(17) $\int \frac{dx}{e^x + 1} =$ ________ $+ c$

(a) $-\log \left|\frac{e^x + 1}{e^x}\right|$ (b) $-\log \left|\frac{e^x}{e^x + 1}\right|$ (c) $\log \left|\frac{e^x + 1}{2e^x}\right|$ (d) $\log \left|\frac{e^{2x}}{e^x + 1}\right|$

(18) $\int \frac{\cos^8 x - \sin^8 x}{1 - 2\sin^2 x \cos^2 x} dx =$ ________ $+ c$

(a) $-\frac{\cos 2x}{2}$ (b) $-\frac{\sin 2x}{2}$ (c) $\frac{\cos 2x}{2}$ (d) $\frac{\sin 2x}{2}$

(19) $\int \frac{1}{1 + (\log x)^2} d(\log x) dx =$ ________ $+ c$

(a) $\frac{\tan^{-1}(\log x)}{x}$ (b) $\tan^{-1}(\log x)$ (c) $\frac{\tan^{-1}}{x}$ (d) $\tan^{-1} x$

(20) If $\int \frac{1 + \cos 8x}{\cot 2x - \tan 2x} dx = A \cos 8x + C$ then A = ________

(a) $\frac{1}{16}$ (b) $-\frac{1}{8}$ (c) $-\frac{1}{16}$ (d) $\frac{1}{8}$

(21) If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \log(9e^{2x} - 4) + c$ then $A =$ ______ and $B =$ ______

(a) $\frac{3}{2}, \frac{-35}{36}$ (b) $\frac{-3}{2}, \frac{-35}{36}$ (c) $\frac{-3}{2}, \frac{35}{36}$ (d) $\frac{3}{2}, \frac{35}{36}$

(22) If $\int \frac{dx}{sm^6 x + \cos^6 x} = K \tan^{-1}\left(\frac{\tan 2x}{2}\right) + c$ then $K =$ ________

(a) $\frac{1}{2}$ (b) -1 (c) 1 (d) $-\frac{1}{2}$

(23) If $\int \frac{\sqrt{x}}{\sqrt{1 - x^{3/2}}} dx = P\sqrt{1 - x^{3/2}} + c$ then $P =$ ________

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{-4}{3}$ (d) $\frac{-3}{4}$

(24) $\int \frac{\sec x dx}{\sqrt{\sin(2x + \alpha) + \sin \alpha}} =$ __________ $+ c$

(a) $\sqrt{2\sec\alpha(\tan x-\tan\alpha)}$ (b) $\sqrt{2\sec\alpha(\tan x+\tan\alpha)}$

(c) $\sqrt{2\sec\alpha(\cot x+\cot\alpha)}$ (d) $\sqrt{2\sec\alpha(\cot x-\cot\alpha)}$

(25) If $\int \frac{x^4+1}{x^6+1}dx=\tan^{-1}x+P\tan^{-1}x^3+c$ then $P=$ ________

(a) 3 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) -3

(26) $\int \frac{\log x-1}{(\log x)^2}dx=$ __________ $+c$

(a) $x\log x$ (b) $-x\log x$ (c) $\frac{x}{\log x}$ (d) $\frac{-x}{\log x}$

(27) $\int \frac{e^x\log(ex^x)}{x}dx=$ _________ $+c$

(a) $\frac{e^x}{x}\log x^x$ (b) $e^x\log x^x$ (c) $e^x x\log x$ (d) $\log(xe^x)$

(28) If $\int x\operatorname{cosec}^2 x dx=P\cdot x\cot x+Q\log|\sin x|+c$ then $P+Q=$ ________

(a) 1 (b) 2 (c) 0 (d) -1

(29) If $\int x^6\log x dx=Px^7\log x+Qx^7+c$ then $P+Q=$ __________

(a) $\frac{6}{49}$ (b) $-\frac{1}{49}$ (c) $\frac{1}{49}$ (d) $-\frac{6}{49}$

(30) $\int\left[\log(\log x)+\frac{1}{\log x}\right]dx=$ __________ $+c$

(a) $\frac{x}{\log(\log x)}$ (b) $x+\log(\log x)$ (c) $\log(\log x)+\frac{1}{x}$ (d) $x\log(\log x)$

(31) $\int\left(\frac{x^2+1}{x^2}\right)e^{\frac{x^2-1}{x^2}}dx=$ ____________ $+c$

(a) $e^{x-\frac{1}{x}}$ (b) $e^{x+\frac{1}{x}}$ (c) $e^{\frac{1}{x}-x}$ (d) $e^{-x-\frac{1}{x}}$

(32) $\int \frac{(x^2-1)dx}{\left(x^4+3x^2+1\right)\tan^{-1}\left(\frac{x^2+1}{x}\right)} =$ ____________ $+ c$

(a) $\log\left|\tan^{-1}\left(x+\frac{1}{x}\right)\right|$ (b) $\log\left|\tan^{-1}\left(x-\frac{1}{x}\right)\right|$

(c) $\tan^{-1}\left(x+\frac{1}{x}\right)$ (d) $\tan^{-1}\left(x-\frac{1}{x}\right)$

(33) $\int \cos x \; d\left(\sin x\right) =$ ___________ $+ c$

(a) $\frac{\sin 2x}{2} - x$ (b) $\frac{1}{2}\left(\frac{\sin 2x}{2} - x\right)$ (c) $\tan^{-1}\left(x+\frac{1}{x}\right)$ (d) $\tan^{-1}\left(x-\frac{1}{x}\right)$

(34) $\int \frac{e^x + xe^x}{\cos^2\left(xe^x\right)} dx =$ ___________ $+ c$

(a) $\log\left|e^x + xe^x\right|$ (b) $\sec\left(xe^x\right)$ (c) $\tan\left(xe^x\right)$ (d) $\cot\left(xe^x\right)$

(35) If $\int \sin^3 x dx = A\cos^3 x + B\cos x + c$ then $A - B =$ ______

(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

(36) $\int \frac{dx}{e^x + e^{-x}} =$ __________ $+ c$

(a) $\log\left|e^x + e^{-x}\right|$ (b) $\tan^{-1}\left(e^x\right)$

(c) $\log\left|e^x + 1\right|$ (d) $\tan^{-1}\left(e^{-x}\right)$

(37) $\int e^{2x + \log x} dx =$ __________ $+ c$

(a) $\frac{1}{4}(2x-1)e^{2x}$ (b) $\frac{1}{2}(2x-1)e^{2x}$

(c) $\frac{1}{4}(2x+1)e^{2x}$ (b) $\frac{1}{4}(2x+1)e^{2x}$

(38) $\int \frac{x-\sin x}{1-\cos x}dx =$ ________ $+c$

(a) $x\tan\frac{x}{2}$ (b) $-x\cot\frac{x}{2}$ (c) $\cot\frac{x}{2}$ (d) $-\cot\frac{x}{2}$

(39) $\int \frac{5+\log x}{(6+\log x)^2}dx =$ ________ $+c$

(a) $\frac{\log x}{x}$ (b) $\frac{x}{\log x+6}$ (c) $\frac{\log x+6}{x}$ (d) $x(\log x+6)$

(40) If $\int \frac{dx}{5+4\cos x} = P\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$ then P: ________

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(41) $\int \frac{\log x}{x^2}dx =$ ________ $+c$

(a) $\frac{-1}{x}(\log_e x+1)$ (b) $\frac{1}{x}(\log_e x+1)$ (c) $\log_e x+1$ (d) $-(1+\log_e x)$

(42) If $\int \frac{(-\sin x+\cos x)dx}{(\sin x+\cos x)\sqrt{\sin x\cos x+\sin^2 x\cos^2 x}} = -\operatorname{cosec}^{-1}[f(x)]+c$ then $f(x)=$ __

(a) $\sin 2x+1$ (b) $1-\sin 2x$ (c) $\sin 2x-1$ (d) $\cos 2x+1$

(43) If $\int \frac{\cos x dx}{\sin^3 x+\cos^3 x} = -\frac{1}{6}\log\left|\frac{z^2-z+1}{(z+1)^2}\right| - \frac{1}{\sqrt{3}}\tan^{-1}\frac{2z-1}{\sqrt{3}}+c$ then $z =$ ____

(a) $\tan x$ (b) $\cot x$ (c) $\sin x$ (d) $\cos x$

(44) $\int \sqrt{1+\sec x}dx =$ ________ $+c$

(a) $-2\sin^{-1}(2\cos x+1)$ (b) $-\sin^{-1}(2\cos x-1)$ (c) $\sin^{-1}(2\cos x-1)$ (d) $\cos^{-1}(2\cos x-1)$

(45) $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx = \sqrt{2}\sin^{-1}($ _____ $)+c$

(a) $\sin x-\cos x$ (b) $\cos x-\sin x$ (c) $\sin\frac{x}{2}-\cos\frac{x}{2}$ (d) $\cos\frac{x}{2}-\sin\frac{x}{2}$

(46) $\int \frac{(x^5-x)^{1/5}dx}{x^6} =$ ________ $+c$

(a) $\frac{5}{24}\left(1-\frac{1}{x^4}\right)^{\frac{6}{5}}$ (b) $\frac{1}{24}\left(1-\frac{1}{x^4}\right)^{\frac{1}{5}}$ (c) $\frac{5}{24}\left(1-\frac{1}{x^4}\right)^{\frac{1}{5}}$ (d) $\frac{5}{24}\left(1-\frac{1}{x^4}\right)^{6}$

(47) $\int \frac{dx}{(x-1)^{\frac{3}{2}}(x-2)^{\frac{1}{2}}} =$ ________ $+c$

(a) $2\sqrt{\frac{x-1}{x-2}}$ (b) $\sqrt{\frac{x-1}{x+2}}$ (c) $2\sqrt{\frac{x-2}{x-1}}$ (d) $2\sqrt{\frac{x-1}{x+2}}$

(48) $\int \frac{x^2+1}{x^4-x^2+1}dx =$ ________ $+c$

(a) $\tan^{-1}\left(\frac{x^2+1}{x}\right)$ (b) $\tan^{-1}\left(\frac{x^2-1}{x}\right)$ (c) $\tan^{-1}(x+1)$ (d) $\tan^{-1}(x-1)$

(49) $\int \sqrt{\frac{\sin x-\sin^3 x}{1-\sin^3 x}}dx =$ _________ $+c$

(a) $\frac{2}{3}\sin^{-1}\left(\sin^{\frac{3}{2}}x\right)$ (b) $\frac{2}{3}\sin^{-1}\left(\cos^{3/2}x\right)$ (c) $\frac{-3}{2}\sin^{-1}\left(\sin^{\frac{3}{2}}x\right)$ (d) $\frac{3}{2}\sin^{-1}\left(\sin^{\frac{3}{2}}x\right)$

(50) $\int \cot^{-1}\sqrt{x}dx =$ __________ $+c$

(a) $(x+1)\cot^{-1}\sqrt{x}+\sqrt{x}$

(b) $(x+1)\cot^{-1}\sqrt{x}-\sqrt{x}$

(c) $x\cot^{-1}\sqrt{x}-\sqrt{x}$

(d) $\sqrt{x}\left(\cot^{-1}\sqrt{x}-x\right)$

(51) $\int \frac{\log x}{(1+\log x)^2}dx =$ _______ $+c$

(a) $\frac{x}{1+\log x}$ (b) $x(1+\log x)$ (c) $\frac{x}{\log x}$ (d) $x\log x+x^{-1}$

(52) $\int \frac{x^2dx}{(x^2+2)(x^3+3)} =$ ________ $+c$

(a) $\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}+\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}$

(b) $\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}-\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}$

(c) $\tan^{-1}\frac{x}{\sqrt{3}}+\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}$

(d) $\tan^{-1}\frac{x}{\sqrt{3}}-\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}$

(53) $\int \frac{1+x}{1+\sqrt[3]{x}}dx =$ _________ $+c$

(a) $\frac{3}{5}x^{5/3}-\frac{3}{4}x^{\frac{4}{3}}-x$

(b) $\frac{3}{5}x^{\frac{5}{3}}-\frac{3}{4}x^{\frac{4}{3}}+x$

(c) $\frac{3}{5}x^{\frac{5}{3}}+\frac{3}{4}x^{\frac{4}{3}}+x$

(d) $\frac{3}{5}x^{\frac{5}{3}}+\frac{3}{4}x^{\frac{4}{3}}-x$

(54) If $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}} = \frac{1}{\sqrt{2}}\cos^{-1}\left[f(x)\right]+c$ then $f(x) =$ _______

(a) $\sqrt{\frac{1-x^2}{1+x^2}}$ (b) $\sqrt{\frac{1+x^2}{1-x^2}}$ (c) $\sqrt{\frac{x^2-1}{x^2+1}}$ (d) $\sqrt{\frac{x^2+1}{x^2-1}}$

(55) $\int \frac{\cot x dx}{\sqrt{\cos^4 x+\sin^4 x}} =$ _______ $+c$

(a) $\frac{1}{2}\log\left|\cot^2 x+\sqrt{\cot^4+1}\right|$ (b) $-\frac{1}{2}\log\left|\cot^2 x+\sqrt{\cot^4+1}\right|$

(c) $\frac{1}{2}\log\left|\tan^2 x+\sqrt{\tan^4+1}\right|$ (d) $-\frac{1}{2}\log\left|\cot x+\sqrt{\cot^4+1}\right|$

(56) $\int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx =$ __________ $+c$

(a) $e^x\left(1+x^2\right)$ (b) $\frac{e^x}{1+x^2}$ (c) $e^x\left(\frac{1-x}{1+x^2}\right)$ (d) $e^x\left(1-x^2\right)$

(57) $\int \frac{dx}{\sqrt{\cos^3 x\sin(x+\alpha)}} =$ __________ $+c$

(a) $2\sec\alpha\sqrt{\sin\alpha+\cos\alpha\tan x}$ (b) $\sec\alpha\sqrt{\sin\alpha+\cos\alpha\tan x}$

(c) $\sqrt{\sin\alpha+\cos\alpha\tan x}$ (d) $2\sqrt{\sin\alpha+\cos\alpha\tan x}$

(58) If $\int \frac{dx}{1-\cos^4 x} = -\frac{1}{2}\cot x + A\tan^{-1}\left(f(x)\right)+c$ then $A=$ _____ and $f(x)=$ _______

(a) $-\frac{\sqrt{2}}{4}$ and $\sqrt{2}\cot x$ (b) $\sqrt{2}$ and $\sqrt{2}\tan x$

(c) $-\sqrt{2}$ and $\sqrt{2}\tan x$ (d) $\frac{1}{2\sqrt{2}}$ and $\sqrt{2}\tan x$

(59) $\int \frac{\sqrt{1-\sin x}}{1+\cos x}e^{\frac{-x}{2}}dx =$ __________ $+c$

(a) $e^{\frac{-x}{2}}\sec\frac{x}{2}$ (b) $-e^{\frac{-x}{2}}\sec\frac{x}{2}$ (c) $-2e^{\frac{-x}{2}}\sec\frac{x}{2}$ (d) $2e^{\frac{-x}{2}}\sec\frac{x}{4}$

(60) $\int \frac{dx}{(x+2)^{\frac{12}{13}}(x-5)^{\frac{14}{13}}} =$ __________ $+c$

(a) $\frac{-13}{7}\left(\frac{x+2}{x-5}\right)^{\frac{1}{13}}$ (b) $\frac{13}{7}\left(\frac{x+2}{x-5}\right)^{\frac{1}{13}}$ (c) $\frac{13}{7}\left(\frac{x-5}{x+2}\right)^{\frac{1}{13}}$ (d) $\frac{-13}{7}\left(\frac{x-5}{x-2}\right)^{\frac{1}{13}}$

(61) $\int \frac{x^2dx}{(x\sin x+\cos x)^2} =$ __________ $+c$

(a) $\frac{\sin x - x\cos x}{x\sin x+\cos x}$ (b) $\frac{\sin x + x\cos x}{x\sin x+\cos x}$ (c) $\frac{x\sin x-\cos x}{x\sin x+\cos x}$ (d) $\frac{x\sin x+\cos x}{x\sin x-\cos x}$

(62) $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx =$ __________ $+c$

(a) $(x+1)e^{x+\frac{1}{x}}$ (b) $(x-1)e^{x+\frac{1}{x}}$ (c) $-x\,e^{x+\frac{1}{x}}$ (d) $x\,e^{x+\frac{1}{x}}$

(63) If $\int \frac{5x+3}{\sqrt{x^2+4x+10}} = k_1\sqrt{x^2+4x+10} + k_2 \log\left|(x+2)+\sqrt{x^2+4x+10}\right| + c$

then $k_1 + k_2 =$ ________

(a) -1 (b) -2 (c) 1 (d) 2

(64) $\int (1-\cos x)\operatorname{cosec}^2 x dx =$ __________ $+c$

(a) $\tan\frac{x}{2}$ (b) $\cot\frac{x}{2}$ (c) $\frac{1}{2}\tan\frac{x}{2}$ (d) $2\tan\frac{x}{2}$

(65) $\int \frac{dx}{(2\sin x + 3\cos x)^2} =$ ____________ $+c$

(a) $-\frac{1}{2\tan x+3}$ (b) $\frac{1}{2\tan x+3}$ (c) $-\frac{1}{2(2\tan x+3)}$ (d) $\frac{1}{2(2\tan x+3)}$

(66) If $f(x) = \cos x - \cos^2 x + \cos^3 x - \cos^4 x + ...$ then $\int f(x)dx =$ ______ $+c$

(a) $\tan\frac{x}{2}$ (b) $x+\tan\frac{x}{2}$ (c) $x-\frac{1}{2}\tan\frac{x}{2}$ (d) $x-\tan\frac{x}{2}$

(67) $\int \frac{e^x dx}{(e^x+2012)(e^x+2013)} =$ __________ $+c$

(a) $\log\left(\frac{e^x+2012}{e^x+2013}\right)$ (b) $\log\left(\frac{e^x+2013}{e^x+2012}\right)$ (c) $\frac{e^x+2012}{e^x+2013}$ (d) $\frac{e^x+2013}{e^x+2012}$

(68) If $\int \frac{x^{2011}\tan^{-1}\left(x^{2012}\right)}{1+x^{4024}} dx = k\tan^{-1}\left(x^{2012}\right) + c$

(a) $\frac{1}{2012}$ (b) $-\frac{1}{2012}$ (c) $\frac{1}{4024}$ (d) $-\frac{1}{4024}$

(69) $\int \frac{dx}{\cos x - \sin x} =$ __________ $+c$

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|$ (b) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|$ (d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)\right|$

(70) If $\int \frac{\sin x\ dx}{\sin(x-\alpha)} = Ax + B\log\left|\sin(x-\alpha)\right| + c$ then $A^2 + B^2 =$ _______

(a) 1 (b) 0 (c) $\cos^2\alpha + 1$ (d) $sm^2\alpha + 1$

(71) If $\int \frac{5^x\,dx}{\sqrt{25^x-1}} = k\log\left|5^x+\sqrt{25^x-1}\right|+c$ then k = ___________

(a) $\log_e^{\frac{1}{5}}$ (b) $\log_e^5$ (c) $\log_e^{25}$ (d) $\log_e^{\frac{1}{25}}$

(72) If $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = f(x)-\log\left(1+x^2\right)+c$ then $f(x)=$ ________

(a) $x\tan^{-1}x$ (b) $-x\tan^{-1}x$ (c) $2x\tan^{-1}x$ (d) $-2x\tan^{-1}x$

(73) If $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx = k\cdot\frac{1}{2}\left[\sqrt{x-x^2}-(1-2x)\sin^{-1}\sqrt{x}\right]-x+c$ then k = ___

(a) $\frac{\pi}{4}$ (b) $\frac{4}{\pi}$ (c) $\frac{\pi}{2}$ (d) $\frac{2}{\pi}$

(74) If $\int \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx = A\sin^{-1}x+Bx\sqrt{1-x^2}+c$ then $A+B=$ ______

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $-\frac{1}{2}$

(75) If $\int \frac{\left(1+x^n\right)^{1/n}}{x^{n+2}}dx = a\left(1+\frac{1}{x^4}\right)^b+c$ then $a+b=$ _______

(a) $\frac{6}{5}$ (b) $\frac{11}{10}$ (c) $\frac{21}{10}$ (d) $\frac{16}{13}$

(76) If $\int 5^{5^{5^x}}5^{5^x}5^x dx = k5^{5^{5^x}}+c$ then $k=$ ________

(a) $\left(\log_e 5\right)^{-1}$ (b) $\left(\log_e 5\right)^{-2}$ (c) $\left(\log_e 5\right)^{-3}$ (d) $\left(\log_e 5\right)^{-4}$

(77) $\int \sqrt{1+\cos ecx}\,dx =$ __________ $+c$

(a) $2\sin^{-1}\left(\sqrt{\cos x}\right)$ (b) $2\cos^{-1}\left(\sqrt{\sin x}\right)$ (c) $2\sin^{-1}\left(\sqrt{\sin x}\right)$ (d) $2\cos^{-1}\left(\sqrt{\cos x}\right)$

(78) $\int \frac{dx}{\sqrt{1+\cos ec^2x}} =$ __________ $+c$

(a) $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right)$ (b) $\sin^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)$ (c) $\cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)$ (d) $\cos^{-1}\left(\frac{\sin x}{\sqrt{2}}\right)$

(79) $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}dx =$ ___________ $+c$

(a) $2^{\sqrt{x}}\log_2^e$ (b) $2^{\sqrt{x}}\log_e^2$ (c) $2^{\sqrt{x}+1}\log_2^e$ (d) $2^{\sqrt{x}+1}\log_e^2$

(80) $\int \operatorname{cosec}\left(x-\frac{\pi}{6}\right)\operatorname{cosec}\left(x-\frac{\pi}{3}\right)dx = k\left[\log\left|\sin\left(x-\frac{\pi}{6}\right)\right| - \log\left|\sin\left(x-\frac{\pi}{3}\right)\right|\right]+c$ then k = _____

(a) 2 (b) -2 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

(81) $\int \frac{dx}{\left(\sin^5 x \cos^7 x\right)^{1/6}} =$ __________ $+c$

(a) $4\left(\tan x\right)^{1/4}$ (b) $6\left(\tan x\right)^{\frac{1}{6}}$ (c) $4\left(\tan x\right)^{\frac{1}{6}}$ (d) $6\left(\cot x\right)^{\frac{1}{6}}$

(82) $\int e^x\left[\frac{x^3-x-2}{\left(x^2+1\right)^2}\right]dx =$ ___________ $+c$

(a) $e^x\left(\frac{2x-1}{x^2+1}\right)$ (b) $e^x\left(\frac{x+1}{x^2+1}\right)$ (c) $e^x\left(\frac{x-1}{x^2+1}\right)$ (d) $e^x\left(\frac{2x-2}{x^2+1}\right)$

(83) $\int \frac{\left(e^x-1\right)}{\left(e^x+1\right)}\frac{dx}{\sqrt{e^x+1+e^{-x}}} =$ _________ $+c$

(a) $\tan^{-1}\left(e^x+e^{-x}\right)$ (b) $\sec^{-1}\left(e^x+e^{-x}\right)$ (c) $2\tan^{-1}\left(e^{x/2}+e^{-x/2}\right)$ (d) $2\sec^{-1}\left(e^{x/2}+e^{-x/2}\right)$

(84) $\int \frac{dx}{x^{\frac{1}{5}}\sqrt{x^{\frac{8}{5}}-1}} =$ __________ $+c$

(a) $\frac{5}{4}\log\left|x^{\frac{4}{5}}+\sqrt{x^{\frac{8}{5}}-1}\right|$ (b) $\frac{-5}{4}\log\left|x^{\frac{4}{5}}+\sqrt{x^{\frac{8}{5}}-1}\right|$

(c) $\frac{4}{5}\log\left|x^{\frac{4}{5}}+\sqrt{x^{\frac{8}{5}}-1}\right|$ (d) $\frac{-4}{5}\log\left|x^{\frac{4}{5}}+\sqrt{x^{\frac{8}{5}}-1}\right|$

(85) If $\int\left(x^{30}+x^{20}+x^{10}\right)\left(2x^{20}+3x^{10}+6\right)^{\frac{1}{10}}dx = k\left(2x^{30}+3x^{20}+6x^{10}\right)^{\frac{11}{10}}+c$ then $k =$ ___

(a) $\frac{1}{60}$ (b) -$\frac{1}{60}$ (c) $\frac{1}{66}$ (d) -$\frac{1}{66}$

(86) $\int \frac{dx}{\sqrt{(x-4)(7-x)}} =$ _________ $+c$ $(4<x<7)$

(a) $2\sin^{-1}\sqrt{\frac{x-4}{3}}$ (b) $2\cos^{-1}\sqrt{\frac{x-4}{3}}$ (c) $\frac{1}{2}\sin^{-1}\sqrt{\frac{x-4}{3}}$ (d) -$\frac{1}{2}\sin^{-1}\sqrt{\frac{x-4}{3}}$

(87) If $\int \frac{2012x+2013}{2013x+2012}dx = \frac{2012}{2013}x + k\log|2013x+2012|+c$ *then* $k =$ _______

(a) $\frac{4025}{2013}$ (b) $\frac{4025}{(2013)^2}$ (c) $\frac{-4025}{2013}$ (d) $\frac{-4025}{(2013)^2}$

(88) If $\int \frac{2\sin x+\cos x}{7\sin x-5\cos x}dx = ax+b\log|7\sin x-5\cos x|+c$ then $a-b=$ ________

$(a)\frac{4}{37}$ $(b)-\frac{4}{37}$ $(c)\frac{8}{37}$ $(a)-\frac{8}{37}$

(89) If $\int \frac{\cos 9x+\cos 6x}{2\cos 5x-1}dx = k_1\sin 4x + k_2\sin x + c$ then $4k_1+k_2=$ ________

$(a)1$ $(b)2$ $(c)4$ $(d)5$

(90) $\int \frac{dx}{(x\tan x+1)^2} =$ ____________ $+c$

$(a)\frac{\tan x}{x\tan x+1}$ $(b)\frac{\cot x}{x\tan x+1}$ $(c)\frac{-\tan x}{x\tan x+1}$ $(d)-\frac{1}{x\tan x+1}$

(91) $\int \sqrt{1+\sin\frac{x}{4}}\,dx =$ ______________ $+c$

$(a)8\left(\sin\frac{x}{8}+\cos\frac{x}{8}\right)$ $(b)\sin\frac{x}{8}+\cos\frac{x}{8}$ $(c)\frac{1}{8}\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)$ $(d)8\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)$

(92) $\int \frac{(x+1)\,dx}{x(1+xe^x)^2} =$ ______________ $+c$

$(a)\log\left|\frac{xe^x}{1+xe^x}\right|-\frac{1}{1+xe^x}$ $(b)\log\left|\frac{xe^x+1}{xe^x}\right|+\frac{1}{1+xe^x}$

$(c)\log\left|\frac{xe^x}{1+xe^x}\right|+\frac{1}{1+xe^x}$ $(d)\log\left|\frac{1+xe^x}{xe^x}\right|-\frac{1}{1+xe^x}$

(93) $\int \frac{dx}{e^x+e^{-x}+2} =$ ______________ $+c$

$(a)-\frac{1}{e^x+1}$ $(b)\frac{1}{e^x+1}$ $(c)-\frac{2^x}{e^x+1}$ $(d)\frac{e^x}{e^x+1}$

(94) If $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,\frac{dx}{x} = k\log\left|\frac{1+\sqrt{1-x}}{\sqrt{x}}\right| - \cos^{-1}\sqrt{x}+c$

$(a)1$ $(b)2$ $(c)-1$ $(d)-2$

(95) $\int \frac{(x+2)^2}{(x+4)}e^x dx =$ ______________ $+c$

$(a)e^x\left(\frac{x}{x+4}\right)$ $(b)e^x\left(\frac{x+2}{x+4}\right)$ $(c)e^x\left(\frac{x-2}{x-4}\right)$ $(d)\frac{2xe^2}{x+4}$

(96) If $\int \frac{2e^{x}+3e^{-x}}{3e^{x}+4e^{-x}}dx = Ax + B\log\left|3e^{2x}+4\right| + c$ then $A+B=$ _______

(a) $\frac{11}{24}$ (b) $\frac{13}{24}$ (c) $\frac{15}{24}$ (d) $\frac{17}{24}$

(97) If $\int \frac{dx}{1+\tan^{4}x} = k\log\left|\frac{\sec^{2}x-\sqrt{2}\tan x}{\sec^{2}x+\sqrt{2}\tan x}\right| + \frac{x}{2} + c$ then $k=$ ________

(a) $\frac{1}{4\sqrt{2}}$ (b) $-\frac{1}{4\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $-\frac{1}{2\sqrt{2}}$

(98) If $\int \frac{3^{x}-1}{3^{x}+1}dx = k\log\left|3^{x/2}+3^{-x/2}\right| + c$ then $k=$ _________

(a) $\log_{3}^{e}$ (b) $\log_{e}^{3}$ (c) $2\log_{3}^{e}$ (d) $2\log_{e}^{3}$

(99) $\int \frac{\sin 2x}{\sin^{4}x+\cos^{4}x}dx =$ __________ $+c$

(a) $\tan^{-1}\left(\sqrt{\tan x}\right)$ (b) $\tan^{-1}\left(\frac{1}{2}\tan x\right)$ (c) $\tan^{-1}\left(\tan^{2}x\right)$ (d) $\tan^{-1}\left(2\tan x\right)$

(100) $\int e^{2x}\left(1+\tan x\right)^{2}dx =$ ________ $+c$

(a) $\tan e^{x}$ (b) $\tan xe^{2x}$ (c) $\tan\frac{x}{2}e^{x}$ (d) $\tan\frac{x}{2}e^{-x}$

(101) $\int \frac{2x^{12}+8x^{9}}{(x^{5}+x^{3}+1)^{2}}dx =$ _____________ $+c$

(a) $\frac{x^{10}+x^{5}}{(x^{5}+x^{3}+1)^{2}}$ (b) $\frac{x^{5}-x^{10}}{(x^{5}+x^{3}+1)^{2}}$ (c) $\frac{x^{10}}{2(x^{5}+x^{3}+1)^{2}}$ (d) $\frac{x^{5}}{2(x^{5}+x^{3}+1)^{2}}$

(102) $\int \frac{1}{\tan x+\cot x+\sec x+\operatorname{cosec} x}dx =$ _____________ $+c$

(a) $\frac{1}{2}\left(\cos x-\sin x\right)+\frac{x}{2}$ (b) $\frac{1}{2}\left(\sin x-\cos x\right)-\frac{x}{2}$

(c) $\frac{1}{2}\left(\sin x+\cos x\right)+\frac{x}{2}$ (d) $\frac{1}{2}\left(\sin x+\cos x\right)-\frac{x}{2}$

(103) $\int \frac{\sec^{\frac{3}{2}}\theta - \sec^{\frac{1}{2}}\theta}{2+\tan^2\theta}\tan\theta\, d\theta =$ _____________ $+ c$

$(a)\ \frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta - \sqrt{2\sec\theta}+1}{\sec\theta + \sqrt{2\sec\theta}+1}\right|$ $(b)\ \frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta + \sqrt{2\sec\theta}+1}{\sec\theta - \sqrt{2\sec\theta}+1}\right|$

$(c)\ \frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta - \sqrt{2\sec\theta}-1}{\sec\theta + \sqrt{2\sec\theta}-1}\right|$ $(d)\ \frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta + \sqrt{2\sec\theta}-1}{\sec\theta - \sqrt{2\sec\theta}-1}\right|$

(104) $\int \frac{\sec^2 x - 2009}{\sin^{2009} x}\,dx =$ _____________ $+ c$

$(a)\ \frac{\cot x}{\sin^{2009} x}$ $(b)\ \frac{-\cot x}{\sin^{2009} x}$ $(c)\ \frac{\tan x}{\sin^{2009} x}$ $(d)\ \frac{-\tan x}{\sin^{2009} x}$

(105) $\int x^{27}(1+x+x^2)^6(6x^2+5x+4)dx =$ _____________ $+ c$

$(a)\ \frac{(x^4+x^3+x^2)^7}{7}$ $(b)\ \frac{(x^4+x^5+x^6)^7}{7}$

$(c)\ \frac{(x+x^3+x^5)^7}{7}$ $(d)\ \frac{(x^5+x^6+x^7)^7}{7}$

(106) $\int \frac{1}{x^2(x^4+1)^{3/4}}dx =$ _____________ $+ c$

$(a)\left(1+\frac{1}{x^4}\right)^{1/4}$ $(b)(x^4+1)^{1/4}$ $(c)\left(1-\frac{1}{x^4}\right)^{1/4}$ $(d)-\left(1+\frac{1}{x^4}\right)^{1/4}$

(107) $\int \frac{dx}{x^4+x^3} = \frac{A}{x^2}+\frac{B}{x}+\log\left|\frac{x}{x+1}\right|+c$

$(a) A=\frac{1}{2}, B=1$ $(b) A=1, B=\frac{1}{2}$ $(c)\ A=-\frac{1}{2}, B=1$ $(d) A=-1, B=-\frac{1}{2}$

(108) $\int \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x \, dx =$ ____________ $+c$

$(a)\dfrac{\sin 16x}{1024}$ $(b)-\dfrac{\cos 32x}{1024}$ $(c)\dfrac{\cos 32x}{1096}$ $(d)-\dfrac{\cos 32x}{1096}$

(109) $\int \dfrac{\sin x + \sin^3 x}{\cos 2x} dx = A\cos x + B\log\left|f(x)\right| + c$

$(a) A = \dfrac{1}{4}, B = \dfrac{1}{\sqrt{2}}, f(x) = \dfrac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$ $(b) A = -\dfrac{1}{2}, B = \dfrac{-3}{4\sqrt{2}}, f(x) = \dfrac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

$(c)\ A = -\dfrac{1}{2}, B = \dfrac{3}{\sqrt{2}}, f(x) = \dfrac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$ $(d) A = \dfrac{1}{2}, B = \dfrac{-3}{4\sqrt{2}}, f(x) = \dfrac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

(110) $\int \dfrac{x^4 + 1}{x(x^2+1)^2} dx = A\log\left|x\right| + \dfrac{B}{1+x^2} + c.$ *then* $A =$ _____, $B =$ _____

$(a) A = 1; B = -1$ $(b) A = -1; B = 1$ $(c)\ A = 1; B = 1$ $(d) A = -1; B = -1$

Hints
(Indefinite Integration)

1. $\dfrac{1}{1+\tan x} = \dfrac{\cos x}{\sin x+\cos x} = \dfrac{1}{2}\left[\dfrac{\cos x+\sin x+\cos x-\sin x}{\cos x+\sin x}\right]$

2. $\dfrac{e^x+1}{e^x-1} = \dfrac{e^{\frac{x}{2}}+e^{-\frac{x}{2}}}{e^{\frac{x}{2}}-e^{\frac{-x}{2}}}$ and taking $e^{\frac{x}{2}}-e^{-\frac{x}{2}}=t$

3. $\dfrac{e^{5\log x}-e^{3\log x}}{e^{4\log x}-e^{2\log x}} = \dfrac{x^5-x^3}{x^4-x^2} = x$

4. $\dfrac{1}{x\left(x^n+1\right)} = \dfrac{x^{n-1}}{x^n\left(x^n+1\right)}$, taking $x^n = t$

5. Taking $\log(x+1) - \log x = t$

6. $e^{\cot^{-1}x}\left[1-\dfrac{x}{1+x^2}\right] = e^{\tan^{-1}x} - \dfrac{x}{1+x^2}e^{\tan^{-1}x}$ Integrate $e^{\cot^{-1}x}$ by parts

7. $\dfrac{\tan x}{\sqrt{\cos x}} = (\cos x)^{\frac{-3}{2}}\sin x$, taking $\cos x = t$

8. $e^{4\log x}\left(x^5+1\right)^{-1} = \dfrac{x^4}{x^5+1}$, taking $x^5+1=t$

9. $\operatorname{cosec}^3 x = \operatorname{cosec}^2 x\sqrt{1+\cot^2 x}$, taking $\cot x = t$

10. $\dfrac{2^{\frac{1}{x^2}}}{x^3}$, taking $2^{\frac{1}{x^2}} = t$

11. $(x-1)e^{-x} = x e^{-x} - e^{-x}$ Integrate $x e^{-x}$ by parts

12. $\sin(\log x) - \cos(\log x)$, $\log_e x = t \ \therefore\ x = e^t$

13. $(x+4)(x+3)^7 = [x+3+1][x+3]^7$

$= (x+3)^8 + (x+3)^7$

14. $\dfrac{1}{(x+3)\sqrt{x+2}}$, $x+2=t^2$

15. $\int \frac{1}{e^x + 2 + e^{-x}} = \frac{e^x}{\left(e^x + 1\right)^2}$ taking $e^x = t$

16. $\frac{\cos x}{\sqrt{\sin^2 x + 2\sin x + 1}}$, taking $\sin x = t$

17. $\frac{1}{e^x + 1} = \frac{e^{-x}}{1 + e^{-x}}$, taking $1 + e^{-x} = t$

18. $\sin^8 x - \cos^8 x = \left(1 - 2\sin^2 x \cos^2 x\right)\cos 2x$

19. Let $\log_c^x = t$ then $d\ (\log x) = dt$

20. $\frac{1 + \cos 8x}{\cot 2x - \tan 2x} = \frac{2\cos^2 4x}{\cos^2 2x - \sin^2 2x} \times \sin 2x \cos 2x = \frac{\sin 8x}{2}$

21. $9e^{2x} - 4 = t$

22. $\frac{1}{\sin^6 x + \cos^6 x} = \frac{1}{1 - 3\sin^2 x \cos^2 x} = \frac{4}{4 - 3\sin^2 2x} = \frac{4\sec^2 2x}{4 + \tan^2 2x}$, and taking $\tan 2x = t$

23. $1 - x^{\frac{3}{2}} = t^2$

24. $\frac{\sec x}{\sqrt{\sin(2x + \alpha) + \sin\alpha}} = \frac{\sec x}{\sqrt{2\sin(x + \alpha)\cos x}} = \frac{\sec^2 x}{\sqrt{2\tan x + \cos\alpha + \sin\alpha}}$

and taking $2\tan x \cos\alpha + \sin\alpha = t^2$

25. $\frac{x^4 + 1}{x^6 + 1} = \frac{x^4 - x^2 + 1 + x^2}{x^6 + 1} = \frac{1}{1 + x^2} + \frac{x^2}{x^6 + 1}$, taking $x^3 = t$

26. $\frac{\log_e x - 1}{\left(\log_e x\right)^2}$ taking $\log_e x = t$ $\quad \therefore x = e^t$

27. $\frac{e^x}{x}\log\left(e x^x\right) = \frac{e^x}{x}\left[\log_e e + x\log x\right] = e^x\left[\frac{1}{x} + \log x\right]$

28. $x\,\mathrm{cosec}^2 x$, Let $u = x, v = \mathrm{cosec}^2 x$, taking integration by parts

29. $x^6 \log_e x$, Let $u = \log x$, $v = x^6$, taking integration by parts

30. $\log(\log x) + \frac{1}{\log x}$, taking $\log_e x = t$ $\quad \therefore x = e^t$

31. $\left(\frac{x^2 + 1}{x^2}\right) e^{\frac{x^2 - 1}{x}} = \left(1 + \frac{1}{x^2}\right) e^{x - \frac{1}{x}}$ taking $x - \frac{1}{x} = t$

32. $\dfrac{x^2-1}{\left(x^4+3x^2+1\right)\tan^{-1}\left(\dfrac{x^2+1}{x}\right)} = \dfrac{1-\dfrac{1}{x^2}}{\left[\left(x+\dfrac{1}{x}\right)^2+1\right]\tan^{-1}\left(x+\dfrac{1}{x}\right)}$, taking $x+\dfrac{1}{x}=t$

33. $\cos x\, d\left(\sin x\right) = \cos x \cos x = \cos^2 x = \dfrac{1+\cos 2x}{2}$

34. taking $x e^x = t$

35. $\sin^3 x = \sin^2 x \sin x = \sin x - \sin x \cos^2 x$, taking $\cos x = t$

36. $\dfrac{1}{e^x + e^{-x}} = \dfrac{e^x}{e^{2x}+1}$ taking $e^x = t$

37. $e^{2x+\log x} = e^{2x}.\, x$ taking $u = x, v = e^{2x}$ (integration by parts)

38. $\dfrac{x-\sin x}{1-\cos x} = \dfrac{x - 2\sin\dfrac{x}{2}\cos\dfrac{x}{2}}{2\sin^2\dfrac{x}{2}} = x.\dfrac{1}{2}\operatorname{cosec}^2\dfrac{x}{2} - \cot\dfrac{x}{2}$, taking integration by parts

39. $\dfrac{5+\log x}{(6+\log x)^2}$, taking $\log_e x = t \therefore x = e^t$

40. $\dfrac{1}{5+4\cos x}$, taking $\tan\dfrac{x}{2} = t$

41. $\dfrac{\log x}{x^2}$, $\log_e x = t \Rightarrow x = e^t$, taking integration by parts

42. $\dfrac{\cos x - \sin x}{(\sin x + \cos x)\sqrt{\sin x \cos x + \sin^2 x \cos^2 x}}$

$= \dfrac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2 \sqrt{\left(\sin x \cos x + \dfrac{1}{2}\right)^2 - \dfrac{1}{4}}}$

$= \dfrac{2\cos 2x}{(1+\sin 2x)\sqrt{(1+\sin 2x)^2 - 1}}$, taking $1+\sin 2x = t$

43. $\dfrac{\cos x}{\sin^3 x + \cos^3 x} = \dfrac{\cos ec^2 x \cdot \cot x}{1 + \cot^3 x}$, taking $\cot x = t$

44. $\sqrt{1 + \sec x} = \sqrt{\dfrac{1 + \cos x}{\cos x}}$, taking cos α = y

45. $\sqrt{\tan x} + \sqrt{\cot x} = \dfrac{\sin x + \cos x}{\sqrt{\sin x \cos x}}$

$= \dfrac{(\sin x + \cos x)\sqrt{2}}{\sqrt{1 - (1 - 2\sin x \cos x)}} = \dfrac{\sqrt{2}\,(\sin x + \cos x)}{\sqrt{1 - (\sin x - \cos x)^2}}$

(taking sin x - cos x = t)

46. $\dfrac{(x^5 - x)^{\frac{1}{5}}}{x^6} = \dfrac{\left(1 - \dfrac{1}{x^4}\right)^{\frac{1}{5}}}{x^5}$, taking $1 - \dfrac{1}{x^4} = t$

47. $\dfrac{1}{(x-1)^{\frac{3}{2}}\,(x-2)^{\frac{1}{2}}} = \dfrac{1}{\left(\dfrac{x-1}{x-2}\right)^{\frac{3}{2}}(x-2)^2}$, taking $\dfrac{x-1}{x-2} = t$

48. $\dfrac{x^2 + 1}{x^4 - x^2 + 1} = \dfrac{1 + \dfrac{1}{x^2}}{\left(x - \dfrac{1}{x}\right)^2 + 1}$, taking $x - \dfrac{1}{x} = t$

49. $\sqrt{\dfrac{\sin x - \sin^3 x}{1 - \sin^3 x}} = \dfrac{\sqrt{\sin x}\,\cos x}{\sqrt{1 - \left(\sin^{\frac{3}{2}} x\right)^2}}$, taking $\sin^{\frac{3}{2}} x = t$

50. $\cot^{-1}\sqrt{x} = u$ and $v = 1$, taking integration by parts

51. $\dfrac{\log x}{(1 + \log x)^2}$, taking $\log_e x = t \Rightarrow x = e^t$

52. $\dfrac{x^2}{(x^2 + 2)(x^2 + 3)} = \dfrac{3(x^2 + 2) - 2(x^2 + 3)}{(x^2 + 2)(x^2 + 3)} = \dfrac{3}{x^2 + 3} - \dfrac{2}{x^2 + 2}$

53. $\dfrac{1+x}{1+\sqrt[3]{x}} = \dfrac{\left(1+\sqrt[3]{x}\right)\left(1-x^{\frac{1}{3}}+x^{\frac{2}{3}}\right)}{1+\sqrt[3]{x}} = 1-x^{\frac{1}{3}}+2^{\frac{2}{3}}$

54. $\dfrac{1}{\left(1+x^2\right)\sqrt{1-x^2}}$, taking $\dfrac{1-x^2}{1+x^2}=t^2 \Rightarrow x^2=\dfrac{1-t^2}{1+t^2}$, $2xdx=\dfrac{-2t\,dt}{\left(1+t^2\right)^2}$

55. $\dfrac{\cot x}{\sqrt{\cos^4 x+\sin^4 x}} = \dfrac{\cot x.\,\operatorname{cos}ec^2 x}{\sqrt{1+\cot^4 x}}$, taking $\cot^2 x = y$

56. $e^x\left[\dfrac{1-x}{1+x^2}\right]^2 = e^x\left[\dfrac{1}{1+x^2}-\dfrac{2x}{\left(1+x^2\right)^2}\right]$

57. $\dfrac{1}{\sqrt{\cos^3 x\sin\left(x+\alpha\right)}} = \dfrac{\sec^2 x}{\sqrt{\sin\alpha+\cos\alpha\;\tan x}}$, taking $\sin\alpha+\cos\alpha\;\tan x=t^2$

58. $\dfrac{1}{1-\cos^4 x} = \dfrac{1}{2}\left[\dfrac{1}{1-\cos^2 x}+\dfrac{1}{1+\cos^2 x}\right]$

59. $\dfrac{\sqrt{1-\sin x}}{1+\cos x}\,e^{-\frac{x}{2}}$, taking $-\dfrac{x}{2}=t \Rightarrow x=-2t$

60. $\dfrac{1}{(x+2)^{\frac{12}{13}}(x-5)^{\frac{14}{13}}} = \dfrac{1}{\left(\dfrac{x+2}{x-5}\right)^{\frac{12}{13}}(x-5)^2}$, taking $\dfrac{x+2}{x-5}=t$

61. $\dfrac{x^2}{(x\sin x+\cos x)^2} = \dfrac{x}{\cos x}\cdot\dfrac{x\cos x}{(x\sin x+\cos x)^2}$

$u=\dfrac{x}{\cos x}$, taking $v=\dfrac{x\cos x}{(x\sin x+\cos x)^2}$

62. $\left(1+x-\dfrac{1}{x}\right)e^{x+\frac{1}{x}} = e^{x+\frac{1}{x}} + x\left(1-\dfrac{1}{x^2}\right)e^{x+\frac{1}{x}}$

taking $u=x,\ v=e^{x+\frac{1}{x}}\left(1-\dfrac{1}{x^2}\right)$

63. $\dfrac{5x+3}{\sqrt{x^2+4x+10}} = \dfrac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}$

64. $(1-\cos x)\,\cos ec^2 x = \cos ec^2 x - \cos ec\ x \,.\cot x$

65. $\dfrac{1}{(2\sin x + 3\cos x)^2} = \dfrac{\sec^2 x}{(2\tan x + 3)^2}$, taking $\tan x = t$

66. $f(x) = \dfrac{\cos x}{1+\cos x}$ $\quad \therefore \underset{n\to\infty}{lin} Sn = \dfrac{a}{1-r}$ $\quad a = \cos x,\ r = -\cos x$

67. $\dfrac{e^x}{(e^x+2012)(e^x+2013)}$, taking $e^x = t$

68. $\dfrac{x^{2011}\tan^{-1}x^{2012}}{1+x^{4024}}$, taking $\tan^{-1} x^{2012} = \text{t}$

69. $\dfrac{1}{\cos x - \sin x}$

$$= \frac{1}{\sqrt{2}\sin\left(x+\frac{3\pi}{4}\right)} = \frac{1}{\sqrt{2}}\cos ec\left(x+\frac{3\pi}{4}\right)$$

70. $\dfrac{\sin x}{\sin(x-\alpha)} = \dfrac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)} = \cos\alpha + \sin\alpha\,.\cot(x-\alpha)$

71. $\dfrac{5^x}{\sqrt{(5^x)^2-1}}$ taking $5^x = t$

72. $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$, taking $x = \tan\theta$

73. $\dfrac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} = \dfrac{4}{\pi}\sin^{-1}\sqrt{x} - 1$, taking $\sqrt{x} = \sin\theta$

74. $\sin^{-1}\left(2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right)$, taking $x = \cos 2\theta$

75. $\dfrac{\left(1+x^{n}\right)^{\frac{1}{n}}}{x^{n+2}}=\dfrac{\left(\dfrac{1}{x^{n}}+1\right)^{\frac{1}{n}}}{x^{n+1}}$, taking $x^{-n}+1=t$

76. $5^{5^{5^{x}}}5^{5^{x}}5^{x}$, *taking* $5^{5^{5^{x}}}=t$

77. $\sqrt{1+\operatorname{cos}ecx}=\sqrt{\dfrac{1+\sin x}{\sin x}}$, taking $\sin x=t^{2}$

78. $\dfrac{1}{\sqrt{1+\operatorname{cos}ec^{2}x}}=\dfrac{\sin x}{\sqrt{2-\cos^{2}x}}$, taking $\cos x=t$

79. $\dfrac{2^{\sqrt{x}}}{\sqrt{x}}$, taking $x=t^{2}$

80. $\operatorname{cos}ec\left(x-\dfrac{\pi}{6}\right)\operatorname{cos}ec\left(x-\dfrac{\pi}{3}\right)=2\left[\cot\left(x-\dfrac{\pi}{3}\right)-\cot\left(x-\dfrac{\pi}{6}\right)\right]$

81. $\dfrac{1}{\left(\sin^{5}x\cot^{7}x\right)^{\frac{1}{6}}}=\dfrac{\sec^{2}x}{(\tan x)^{\frac{5}{6}}}$ taking $\tan x=t$

82. $e^{x}\left[\dfrac{x^{3}-x-2}{\left(x^{2}+1\right)^{2}}\right]=e^{x}\left[\dfrac{x+1}{x^{2}+1}+\dfrac{1-2x-x^{2}}{\left(x^{2}+1\right)^{2}}\right]$,

$f(x)=\dfrac{x+1}{x^{2}+1}$ $\qquad f'(x)=\dfrac{1-2x-x^{2}}{\left(x^{2}+1\right)^{2}}$

83. $\dfrac{\left(e^{x}-1\right)}{\left(e^{x}+1\right)\sqrt{e^{x}+1+e^{-x}}}=\dfrac{e^{\frac{x}{2}}-e^{\frac{-x}{2}}}{\left(e^{\frac{x}{2}}+e^{\frac{-x}{2}}\right)\sqrt{\left(e^{\frac{x}{2}}+e^{\frac{-x}{2}}\right)^{2}-1}}$, taking $e^{\frac{x}{2}}+e^{\frac{-x}{2}}=t$

84. $\dfrac{1}{x^{\frac{1}{5}}\sqrt{5^{\frac{8}{5}}-1}}$, taking $x^{\frac{4}{5}}=t$

85. $\left(x^{30}+x^{20}+x^{10}\right)\left(2x^{20}+3x^{10}+6\right)^{\frac{1}{10}}$

$$= \left(x^{30} + x^{20} + x^{10}\right)\left(2x^{30} + 2x^{20} + 6x^{10}\right)^{\frac{1}{10}}$$

taking $2x^{30} + 3x^{20} + 6x^{10} = t$

86. $\dfrac{1}{\sqrt{(x-4)(7-x)}}$, taking $x - 4 = t^2$

87. $\dfrac{2012\,x + 2013}{2013x + 2012}$, Nr = A (Dr)+ B

88. $\dfrac{2\sin x + \cos x}{7\sin x - 5\cos x}$; Nr = A + B (Dr)

89. $$\frac{\cos 9x + \cos 6x}{2\cos 5x - 1} = \frac{2\cos\frac{15x}{2}\cos\frac{3x}{2}}{4\cos^2\frac{5x}{2} - 3} = \frac{2\left[4\cos^3\frac{5x}{2} - \cos\frac{5x}{2}\right]\cos\frac{3x}{2}}{4\cos^2\frac{5x}{2} - 3} = 2\cos\frac{5x}{2}\cos\frac{3x}{2}$$

90. $$\frac{1}{(x\ \tan x + 1)^2} = \frac{\cos^2 x}{(x\sin x + \cos x)^2} = \frac{\cos x}{x}\cdot\frac{x\cos x}{(x\sin x + \cos x)^2}$$

taking $u = \dfrac{\cos x}{x}$, taking $v = \dfrac{x\cos x}{(x\sin x + \cos x)^2}$

91. $$\sqrt{1 + \sin\frac{x}{4}} = \sqrt{\left(\sin\frac{x}{8} + \cos\frac{x}{8}\right)^2} = \sin\frac{x}{8} + \cos\frac{x}{8}$$

92. $\dfrac{x+1}{x\left(1 + xe^x\right)^2} = \dfrac{(x+1)\,e^x}{x e^x\left(1 + x\,e^x\right)^2}$, taking $x e^x = t$

93. $\dfrac{1}{e^x + e^{-x} + 2} = \dfrac{e^x}{\left(e^x + 1\right)^2}$, taking $e^x = t$

94. $\sqrt{\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}} \times \dfrac{1}{x}$, taking $x = \cos^2\theta$

95. $$\frac{(x+2)^2}{(x+4)^2}\,e^x = \left(\frac{x\,(x+4)}{(x+4)^2} + \frac{4}{(x+4)^2}\right)e^x$$

$$= \left(\frac{x}{x+4} + \frac{4}{(x+4)^2} \right) e^x, \; f(x) = \frac{x}{x+4} \; \text{and} \; f(x) = \frac{4}{(x+4)^2}$$

96. $\dfrac{2e^x + 3e^{-x}}{3e^x + 4e^{-x}} = \dfrac{2e^{3x}+3}{3e^{2x}+4}$, then taking $Nr = A(Dr) + B$

97. $\dfrac{1}{1+\tan^4 x} = \dfrac{\sec^2 x}{(1+\tan^2 x)(1+\tan^4 x)}$, taking $\tan x = t$

98. $\dfrac{3^x - 1}{3^x + 1} = \dfrac{3^{\frac{x}{2}} - 3^{\frac{-x}{2}}}{3^{\frac{x}{2}} + 3^{\frac{-x}{2}}}$, taking $3^{\frac{x}{2}} + 3^{\frac{-x}{2}} = t$

99. $\dfrac{\sin 2x}{\sin^4 x + \cos^4 x} = \dfrac{2\sin x \cos x}{\sin^4 x + \cos^4 x} = \dfrac{2\tan x \cdot \sec^2 x}{1 + \tan^4 x}$, taking $\tan^2 x = t$

100. $e^{2x}(1 + \tan x)^2 = e^{2x}(\tan x + \sec^2 x)$, taking $2x = t$

101 to **110** Try yourself

Answer Key							
1	d	30	d	59	b	88	b
2	a	31	a	60	a	89	b
3	d	32	a	61	a	90	a
4	b	33	c	62	d	91	a
5	c	34	c	63	b	92	c
6	c	35	a	64	a	93	a
7	a	36	b	65	c	94	d
8	d	37	a	66	c	95	a
9	a	38	b	67	a	96	d
10	a	39	b	68	c	97	b
11	b	40	d	69	b	98	c
12	c	41	a	70	a	99	c
13	b	42	a	71	b	100	b
14	a	43	b	72	c	101	c
15	b	44	b	73	b	102	b
16	a	45	a	74	c	103	a
17	a	46	a	75	c	104	c
18	d	47	c	76	c	105	b
19	b	48	b	77	c	106	d
20	c	49	a	78	c	107	c
21	c	50	a	79	c	108	b
22	c	51	a	80	b	109	d
23	c	52	b	81	b	110	c
24	b	53	b	82	c		
25	b	54	a	83	d		
26	c	55	b	84	a		
27	b	56	b	85	c		
28	c	57	a	86	a		
29	a	58	a	87	b		

QUESTION BANK

(Definite Integration)

(1) $\int_{-K}^{K} |x| dx = \frac{1}{K}$; where K $\in$ N then K is ....

(a) 0 (b) 1 (c) 2 (d) not possible

(2) If $\int_{-1}^{n} x|x|dx = \frac{7}{3}, n \in N$ then n is ....

(a) 1 (b) 2 (c) 0 (d) 3

(3) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} [\cot x] dx$ is equal to ....

(a) 1 (b) 0 (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

(4) $\int_{0}^{\frac{3}{2}} [x^2] dx$ is equal to ....

(a) $\frac{3}{4}$ (b) 3 (c) $2+\sqrt{2}$ (d) $2-\sqrt{2}$

(5) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1-\sin 2x}\, dx$ is equal to ....

(a) $\sqrt{2}+1$ (b) $\sqrt{2}-1$ (c) $1-\sqrt{2}$ (d) 0

(6) The value of the integral $\int_{0}^{1} 2^{2x} \cdot 3^{-x} dx$ is ....

(a) $\log_e \frac{64}{27}$ (b) $\log_e \frac{27}{64}$ (c) $\log_{\frac{3}{4}} e$ (d) $\log_{\frac{64}{27}} e$

(7) The value of the integral $\int_{-5}^{5} (x-[x]) dx$ is ....

(a) 0 (b) 5 (c) 10 (d) 15

(8) $\int_{0}^{\frac{\pi}{2}} e^{\sin^{-1} x} \cdot e^{\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)} dx$ is equal to ....

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2} e^{\frac{\pi}{2}}$ (c) $\frac{\pi}{4} e^{\frac{\pi}{2}}$ (d) $e^{\frac{\pi}{2}}$

(9) The value of the integral $\int_0^{\frac{\pi}{2}}\left[\tan^{-1}(\cot x)+\cot^{-1}(\tan x)\right]dx$ is ....

(a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{2}$

(10) The value of the integral $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+3^x}dx$ is ....

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

(11) The value of the integral $\int_{-1}^{1}\log\left(\frac{1}{x+\sqrt{x^2+1}}\right)dx$ is ....

(a) log 2 (b) 0 (c) log 3 (d) not possible

(12) The value of the integral $\int_0^{e}\frac{x}{\left(x+\sqrt{e^2-x^2}\right)\sqrt{e^2-x^2}}dx$ is ....

(a) 0 (b) $\frac{e}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

(13) $\int_0^{2\pi}\left(\sin x+\left|\sin x\right|\right)dx$ is equal to ....

(a) 0 (b) 2 (c) -2 (d) 4

(14) $\int_0^{\frac{\pi}{9}}\left(\tan x+\tan 2x+\tan 3x+\tan x\cdot\tan 2x\cdot\tan 3x\right)dx$ is equal to ....

(a) $\frac{1}{3}\log 2$ (b) $\log\sqrt[3]{4}$ (c) $3\log 2$ (d) $4\log\sqrt{3}$

(15) $\int_1^{e}\left(x^x+\log x^{x^x}\right)dx$ is equal to ....

(a) $\frac{e-1}{2}$ (b) e^e-1 (c) e^e+1 (d) e^e

(16) $I=\int_{-1}^{1}\left(x^7+\cos^{-1}x\right)dx$ then $\cos I$ is equal to ....

(a) 1 (b) 0 (c) -1 (d) $\frac{1}{2}$

(17) $\int_{-\pi/2}^{\pi/2}\sqrt{\cos x-\cos^3 x}\,dx$ is equal to ....

(a) $-\frac{1}{3}$ (b) $-\frac{1}{4}$ (c) $-\frac{2}{3}$ (d) $-\frac{4}{3}$

(18) $\int_{-a}^{a}\left(\frac{|x+a|}{x+a}+\frac{|x-a|}{x-a}\right)dx$ is equal to .... (where a > 0)

(a) 0 (b) a (c) 2a (d) 4a

(19) The value of the integral $\int_{1}^{e}(\log x)^{8}\,dx+8\int_{1}^{e}(\log x)^{7}\,dx$ is ....

(a) e-1 (b) $\frac{e-1}{2}$ (c) 0 (d) e

(20) If $\int_{\sqrt{2}}^{2}\frac{Kdx}{\sqrt{x^4-x^2}}dx=\frac{\pi}{4}$ then K is equal to ....

(a) 1 (b) 2 (c) 3 (d) 4

(21) $\int_{\log\frac{1}{3}}^{\log 3}2^{x^2}\cdot x^3dx$ is equal to ....

(a) 0 (b) log 3 (c) -log3 (d) log2

(22) If f is an even function and $\int_{0}^{2}f(x)dx=K$

then $\int_{-1}^{1}\left(\frac{x^2-1}{x^2}\right)f\left(x+\frac{1}{x}\right)dx$ is equal to ....

(a) 0 (b) 2K (c) K (d) 4K

(23) The value of $\int_{\pi/6}^{\pi/3}\cos ec2\theta\log\tan^2\theta d\theta$ is ....

(a) 0 (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) 1

(24) The value of $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}}\frac{dx}{1+\sqrt[n]{\tan x}}=\alpha$ then $\tan\alpha$ is equal to ....

(a) $\sqrt{3}$ (b) 1 (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(25) The value of $\int_{0}^{\pi/4}\frac{8\tan^2 x+8\tan x+8}{\tan^2 x+2\tan x+1}dx$ is ....

(a) 0 (b) π (c) $\pi+2$ (d) π-2

(26) The value of $\int_{0}^{\pi}\frac{\cos 3\theta}{\cos\theta+\sin\theta}d\theta$ is ....

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

(27) $\int_0^{\frac{\pi}{2}} \log\left(\tan\frac{x}{2}+\cot\frac{x}{2}\right)dx$ is equal to ....

(a) $\frac{\pi}{2}\log 2$ (b) $-\frac{\pi}{2}\log 2$ (c) $\pi\log 2$ (d) $-\pi\log 2$

(28) The value of integral $\int_0^{\pi} \frac{\sin(2n+1)\frac{x}{2}}{\sin\frac{x}{2}}dx$ is ....

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

(29) The value of the integral $\int_0^{100\pi} \sqrt{1-\cos 2x}\,dx$ is ....

(a) 50π (b) 100π (c) $100\sqrt{2}$ (d) $200\sqrt{2}$

(30) The value of $\int_e^{e^2} \frac{dx}{\log x} - \int_1^2 \frac{e^x}{x}dx$ is ....

(a) e^2 (b) e (c) $\frac{1}{e}$ (d) 0

(31) If $f(x)$ is an odd periodic function with period P then $\int_{2p-a}^{2p+a} f(x)dx$ is equal to ....

(a) p (b) 2p (c) 4p (d) 0

(32) If $I_n = \int_0^1 x^n \cdot e^x dx$ for $n \in N$ then $I_{100} + 100I_{99}$ is equal to ....

(a) 0 (b) 1 (c) e (d) e^{-1}

(33) $\int_0^{\pi/2} \frac{x\sin x\cos x}{\cos^4 x+\sin^4 x}dx$ is equal to ....

(a) $\frac{\pi}{4}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi}{16}$

(34) $\int_0^{\pi/2} \log\left(\frac{a+b\sin x}{a+b\cos x}\right)dx$ is equal to ....

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) πab

(35) The value of intergral $\int_1^2 \frac{dx}{x+x^7}$ is ....

(a) $\frac{1}{6}\log\frac{64}{65}$ (b) $\frac{1}{6}\log\frac{128}{65}$ (c) $\frac{1}{6}\log\frac{32}{65}$ (d) $6\log\frac{64}{65}$

(36) If $I_n = \int_0^{\pi/4} \tan^n x dx$ then $\sum_{r=1}^{5}\frac{1}{I_r + I_{r+2}}$ is equal to ....

(a) 5 (b) 10 (c) 15 (d) 20

(37) If $\int_{3+\pi}^{4+\pi} f(x-\pi)dx = \int_a^b f(x)dx$ then $a+b$ is equal to ....

(a) $2\pi+7$ (b) $\pi+\frac{7}{2}$ (c) $\frac{1}{2}$ (d) 7

(38) $\int_0^1 \sqrt[3]{x^3-x^4}\ dx$ is equal to ....

(a) $\frac{1}{2}$ (b) $\frac{3}{7}$ (c) $\frac{9}{28}$ (d) $\frac{29}{28}$

(39) The value of the integral $\int_0^1 \left(x^5+6x^4+5x^3+4x^2+3x+1\right)e^{x-1}dx$ is equal to ....

(a) 5 (b) $5e$ (c) $5e^2$ (d) $5e^4$

(40) $\int_0^2 x^{[x]}dx$ is equal to ....

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

(41) If $f(x) = f(\pi+e-x)$ and $\int_e^{\pi} f(x)dx = \frac{2}{e+\pi}$ then $\int_e^{\pi} xf(x)dx$ is equal to ....

(a) $\frac{\pi+e}{2}$ (b) $\frac{\pi-e}{2}$ (c) 1 (d) -1

(42) The value of integral $\int_0^1 \frac{1}{1-x+\sqrt{2x-x^2}}dx$ is ....

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(43) If $\int_{\frac{\pi}{4}}^{\pi/2} \frac{\cos^n x}{\sin^{n+2} x}dx = \frac{1}{K-1}$ then K is equal to ....

(a) n (b) $n+1$ (c) $n+2$ (d) $n+3$

(44) $\int_0^{\pi/4} \log(\cot 2x)^{\sin 4x} dx$ is equal to ....

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{2}$

(45) If $\int_n^{n+1} f(x)dx = n$ where $n = 0, 1, 2......,$ and $\int_0^{100} f(x)dx = \frac{k^2 - k}{2}$ then k is

(a) 50 (b) 49 (c) 99 (d) 100

(46) $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is equal to ....

(a) 0 (b) π (c) $\frac{\pi^2}{2}$ (d) π^2

(47) The value of integral $\int_a^{a+1} |a - x| dx$ is $(a \in R^+) =$ ....

(a) a (b) $\frac{a}{2}$ (c) 1 (d) $\frac{1}{2}$

(48) The value of $\int_0^{\frac{\pi}{2}} \sin\theta\sqrt{\sin 2\theta}\, d\theta$ is ....

(a) 1 (b) 0 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

(49) $\int_{e^{-1}}^{1} \left|\log x^{\frac{1}{x}}\right| dx$ is equal to

(a) $\frac{1+e}{2}$ (b) $\frac{e-1}{2}$ (c) 1 (d) $\frac{1}{2}$

(50) If $a < 0 < b$ then the value of $\int_a^b \frac{|x|}{x} dx$ is ....

(a) $a + b$ (b) $b - a$ (c) $a - b$ (d) $\frac{b-a}{2}$

(51) $\int_0^{\pi/2} \sqrt{\sec x + 1}\, dx$ is equal to ....

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

(52) The value of the integral $\int_{-\pi/4}^{\pi/4} \log(\sec\theta - \tan\theta)\, d\theta$ is ....

(a) $\pi/4$ (b) $\pi/2$ (c) π (d) 0

(53) The value of the integral $\int_{0}^{\pi} \sqrt{\sin x} \cdot \cos\frac{x}{2}\, dx$ is ....

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

(54) $\int_{0}^{1} \sqrt{x}\sqrt{1-\sqrt{x}}\, dx$ is equal to ....

(a) $\frac{4}{105}$ (b) $\frac{8}{105}$ (c) $\frac{16}{105}$ (d) $\frac{32}{105}$

(55) $\int_{0}^{\pi/4} \frac{\sin 2\theta}{\cos^4\theta + \sin^4\theta}\, d\theta$ is equal to ....

(a) 0 (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(56) $\int_{0}^{\pi} \frac{\sin 100x}{\sin x}\, dx$ is equal to ....

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) 2π

(57) $\int_{-\pi/2}^{\pi/2} \sin x f(\cos x)\, dx$ is equal to ....

(a) 1 (b) -1 (c) 0 (d) $\frac{\pi}{2}$

(58) The value of the integral $\int_{-1}^{1} (x^2 + x)|x|\, dx$ is ....

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

(59) The value of $\int_{0}^{\pi} [\cot x]\, dx$ is equal to .... (where [ ] denotes the greatest integer function)

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{-\pi}{2}$ (d) -1

(60) If $f(x)=\int_0^x log\left(\frac{1-t}{1+t}\right)dt$ then $f\left(\frac{1}{2}\right)-f\left(\frac{-1}{2}\right)$ is equals to ....

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) 1

(61) The value of $c\int_{1+c}^{a+c}\left[f(cx)+1\right]dx-\int_c^{ac} f\left(c^2+x\right)dx, c\neq 0,$ is equal to ....

(a) 0 (b) $c(a-1)$ (c) ac (d) $a(c+1)$

(62) $f: R\to R$ and satisfies $f(2)=-1, f'(2)=4$ If $\int_2^3 (3\text{-x})\, f''(x)dx=7,$

then $f(3)$ is equal to ....

(a) 2 (b) 4 (c) 8 (d) 10

(63) $\int_1^{\frac{1}{e}}\frac{log\, t}{1+t}dt+\int_1^{e}\frac{log\, t}{1+t}dt$ is equal to ....

(a) e (b) $\frac{1}{e}$ (c) 2 (d) $\frac{1}{2}$

(64) If $\int_0^{\pi} f(sin\, x)\,dx=2$ then the value of $\int_0^{\pi} xf(sin\, x)\,dx$ is ....

(a) 0 (b) 4 (c) $\frac{\pi}{2}$ (d) π

(65) $\int_{-3\pi/2}^{-\pi/2}\left[(x+\pi)^3+cos^2(x+3\pi)\right]dx$ is equal to ....

(a) $\frac{\pi^3}{8}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$-1 (d) $\frac{\pi}{4}$+1

(66) If $f(x)=1-\frac{1}{x}$ then $\int_{1/3}^{2/3} fof(x)\,dx$ is equal to ....

(a) 1 (b) $\frac{1}{2}$ (c) -log2 (d) -log$\frac{1}{2}$

(67) The value of the integral $\int_0^1\frac{dx}{x^{3/2}+x^{1/2}}$ is ....

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) π

(68) The value of the integral $\int_0^{1/2}\frac{dx}{(1-x)^{3/2}\sqrt{1+x}}$ is ....

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

(69) If $h(x)=\left[f(x)+g(x)\right]\left[g(x)-f(x)\right]$ where f is an odd and g is an even function the $\int_{-\pi/2}^{\pi/2} h(x)\,dx$ is equal to ....

(a) 0 (b) $\frac{\pi}{2}$ (c) $\int_{0}^{\pi/2} h(x)\,dx$ (d) $2\int_{0}^{\pi/2} h(x)\,dx$

(70) If $\int_{0}^{100} f(x)dx=10$ then $\sum_{K=1}^{100}\int_{0}^{1} f(x+K-1)\,dx$ is equal to ....

(a) 0 (b) 10 (c) 100 (d) 1000

(71) If f is an odd function the value of integral $\int_{1/e}^{e} \frac{1}{x} f\left(x-\frac{1}{x}\right)dx$ is equal to ....

(a) e (b) $\frac{e^2+1}{e}$ (c) $\frac{e^2-1}{2e}$ (d) 0

(72) The value of the integral $\int_{0}^{\pi/2} \sin\theta\cdot\log\sin\theta\cdot d\theta$ is ....

(a) $\log\frac{2}{e}$ (b) $\log 2e$ (c) $\log 2$ (d) $\log\frac{e}{2}$

(73) The value of the integral $\int_{0}^{1} \log\left(\frac{1}{x}-1\right)dx$ is ....

(a) 1 (b) $\frac{1}{2}$ (c) 0 (d) 2

(74) The value of integral $\int_{0}^{\frac{\pi}{4}} \frac{2}{\sec x+\operatorname{cosec}x+\tan x+\cot x}dx$ is ....

(a) 0 (b) $1-\frac{\pi}{4}$ (c) $\frac{\pi}{4}+1$ (d) $\frac{\pi}{2}+1$

(75) The value of the integral $\int_{\frac{\sqrt{5}+1}{2}}^{\sqrt{3}} \frac{(x^2+1)\,dx}{(x^2-1)\sqrt{x^4-3x^2+1}}$ is ....

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{4}$

(76) The value of the integral $\int_{0}^{1} \log\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$ is ....

(a) $\frac{1}{2}\left[log(2)-\frac{1}{2}+\frac{\pi}{4}\right]$ (b) $\frac{1}{2}\left(log\,2-1+\frac{\pi}{2}\right)$

(c) $\frac{1}{3}\left(log\,4-1+\frac{\pi}{4}\right)$ (d) $\frac{1}{4}\left(log\,3-1+\frac{\pi}{2}\right)$

(77) The area enclosed by the parabola $x^2 = 4by$ and its latusrectum is $\frac{8}{3}$ then $b > 0$ is equal to ....

(a) 2 (b) $\sqrt{2}$ (c) 1 (d) 4

(78) The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded the lines $x = 4, y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numered from top to bottom then $S_1 : S_2 : S_3$ is ....

(a) 1:2:3 (b) 2:1:2 (c) 3:2:3 (d) 1:1:1

(79) The area enclosed between the curves $y = log_e(x+e)$ and the coordinate axes is ....

(a) 1 (b) 4 (c) 2 (d) 3

(80) Ratio of the area cut off by a parabola $y^2 = 32x$ and line $x = 8$ corresponding rectangle contained the area formed by above curves region is ....

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) 3

(81) The area bounded by $|x|-|y| = 2$ is ....

(a) 2 Sq. unit (b) 4 Sq. unit (c) 8 Sq. unit (d) 16 Sq. unit

(82) The area bounded by the curves $x^2 = y$ and $2x + y - 8 = 0$ and $y - axis$ in the second quadrant is ....

(a) 9 Sq. unit (b) 18 Sq. unit (c) $\frac{80}{3}$ Sq. unit (d) 36 Sq. unit

(83) The area of common region of the circle $x^2 + y^2 = 4$ and $x^2 + (y-2)^2 = 4$ is ....

(a) $\frac{1}{3}\left(4\pi\text{-}2\sqrt{3}\right)$ (b) $\frac{4}{3}\left(2\pi\text{-}\sqrt{3}\right)$ (c) $\frac{4}{3}\left(\sqrt{3}\text{-}2\pi\right)$ (d) $\frac{2}{3}\left(4\pi\text{-}3\sqrt{3}\right)$

(84) The area enclosed between the curves $y = kx^2$ and $x^2 = ky^2\ (k > 0)$ is 12 Sq. unit Then the value of $'k'$ is ....

(a) 6 (b) $\frac{1}{6}$ (c) 12 (d) $\frac{1}{12}$

(85) The area enclosed by $y^2 = 32x$ and $y = mx(m > 0)$ is $\frac{8}{3}$ then m is ....

(a) 1 (b) 2 (c) 4 (d) $\frac{1}{4}$

(86) The area of the region bounded by the circle $x^2 + y^2 = 12$ and parabola $x^2 = y$ is ....

(a) $(2\pi - \sqrt{3})$ Sq.unit (b) $4\pi + \sqrt{3}$ Sq.unit

(c) $2\pi + \sqrt{3}$ Sq.unit (d) $\pi + \frac{\sqrt{3}}{2}$ Sq.unit

(87) The area bounded by the curves $|x| + |y| \geq 2$ and $x^2 + y^2 \leq 4$ is ....

(a) 4π-4 (b) 4π-2 (c) $4(\pi\text{-}2)$ (d) $4(\pi\text{-}1)$

(88) The area bounded by the curves $y = x^2$ and $y = |x|$ is ....

(a) 1 Sq.unit (b) 2 Sq.unit (c) $\frac{1}{3}$ Sq.unit (d) $\frac{2}{3}$ Sq.unit

(89) The area of the region bounded by curves $f(x) = \sin x, g(x) = \cos x, x = \frac{\pi}{4}, x = \frac{5\pi}{4}$ is ....

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

(90) The area enclosed by the curves $x^2 = y, y = x + 2$ and $x-$axis is ....

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{5}{6}$ (d) $\frac{7}{6}$

(91) The area bounded by ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and its auxiallary circle is ....

(a) 2π (b) 3π (c) 6π (d) 9π

(92) The area of the region bounded by curves $x^2 + y^2 = 4, x = 1 \& x = \sqrt{3}$ is..........................

(a) $\frac{\pi}{3}$ sq.unit (b) $\frac{2\pi}{3}$ sq.unit (c) $\frac{s\pi}{6}$ sq.unit (d) $\frac{4\pi}{3}$ sq.unit

(93) The area of the region bounded by the lines $y = mx, x = 1, x = 2$ and $x-$axis is 6 Sq.unit then m is ....

(a) 1 (b) 2 (c) 3 (d) 4

Hints
(Definite Integration)

1. $|x|$ is an even function

$\therefore \int_{-k}^{k} |x| \, dx = 2\int_{0}^{k} x \, dx = k^2$

$\therefore k^2 = \frac{1}{k}$

2. Here $\int_{-1}^{1} x|x| dx + \int_{1}^{n} x|x| dx$

$\frac{7}{3} = 0 + \int_{1}^{n} x^2 \, dx$ $\quad (\therefore x|x|)$ is an odd function)

$\frac{7}{3} = \frac{4^3 - 1}{3}$ $\quad (\therefore x > 0)$

3. $I = \int_{\pi/3}^{\pi/2} 0 \, dx$ $\quad \left[\begin{array}{l} \because \frac{\pi}{3} \le x \le \frac{\pi}{2} \\ \therefore \frac{1}{\sqrt{3}} > \cot x > 0 \end{array}\right]$

$= 0$

4. $\int_{0}^{3/2} [x^2] \, dx = \int_{0}^{1} [x^2] \, dx + \int_{1}^{\sqrt{2}} [x^2] \, dx + \int_{\sqrt{2}}^{3/2} [x^2] \, dx$

$= 0 + \int_{1}^{\sqrt{2}} 1. \, dx + \int_{\sqrt{2}}^{3/2} 2 \, dx$

5. Here $\frac{\pi}{4} < x < \frac{\pi}{2}$

$\therefore \cos x < \sin x$

$\therefore I = \int_{\pi/4}^{\pi/2} (\sin x - \cos x) \, dx$

6. $$\int_0^1 \left(\frac{4}{3}\right)^x dx = \left[\frac{\left(4/3\right)^x}{\log_e 4/3}\right]_0^1$$

$$= \frac{1}{\log_e 4/3}\left[\frac{4}{3}-1\right]$$

7. $$\int_{-5}^{5}(x-[x])\,dx$$

$$= \int_{-5}^{5} x\,dx - \left[\int_{-5}^{-4}[x]\,dx + \int_{-4}^{-3}[x]dx + \ldots\ldots\ldots + \int_{4}^{5}[x]\,dx\right]$$

$$= 0 - [-5 \ -4 - + \ldots\ldots\ldots\ldots 3 + 4]$$

$$= 5$$

8. $$\int_0^{\frac{\pi}{2}}\left(e^{\sin^{-1}x} + e^{\cos^{-1}x}\right)dx = e^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} dx$$

9. $$I = \int_0^{\frac{\pi}{2}}\left[\tan^{-1}\left(\cot\left(\frac{\pi}{2}-x\right)\right)+\cot^{-1}\left(\tan\left(\frac{\pi}{2}-x\right)\right)\right]dx$$

$$= \int_0^{\frac{\pi}{2}} (x + x)\,dx$$

10. $$I = \int_{-\pi}^{\pi}\frac{\cos^2 x}{1+3^x} \qquad \ldots.(I)$$

$$I = \int_{-\pi}^{\pi}\frac{\cos^2(o-x)}{1+3^{o-x}}\,dx = \int_{-\pi}^{\pi}\frac{\cos^2 x}{1+\frac{1}{3^x}}\,dx \qquad \ldots.(II)$$

$$2I = \int_{-\pi}^{\pi}\cos^2 x\,dx$$

11. $f(x) = \log\left(\frac{1}{x+\sqrt{x^2+1}}\right)$

$f(-x) = \log\left(\frac{1}{-x+\sqrt{x^2+1}}\right)$

$= \log\left(x + \sqrt{x^2+1}\right)$

$= -f(x)$

$\therefore \int_{-1}^{1} f(x)dx = 0$

12. $\int_0^e \frac{xdx}{\left(x+\sqrt{e^2-x^2}\right)\sqrt{e^2-x^2}}$

(*Take* x = esin θ ∴ dx = ecosθdθ)

$= \int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta+\cos\theta}\, d\theta = \frac{\pi}{4}$

13. $I = \int_0^{\pi}\left(\sin x + |\sin x|\right)dx + \int_{\pi}^{2\pi}\left(\sin x + |\sin x|\right)dx$

$= 2\int_0^{\pi} \sin x\ dx + 0$ $\left(\because\ \pi < x < 2\pi \Rightarrow \sin x < 0\ \&\ 0 < x < \pi \Rightarrow \sin x > 0\right)$

14. $\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan x \cdot \tan 2x}$

∴ $\tan x + \tan 2x + \tan 3x + \tan x \,.\, \tan 2x \,.\, \tan 3x = 2 \tan 3x$

$\therefore\ I = 2\int_0^{\frac{\pi}{9}} \tan 3x\, dx$

15. $I = \int_1^{e^e} .dt$

Put $x^x = t$ $\therefore\ x^x(\log x + 1)\, dx = dt$

16. $I = \int_{-1}^{1} x^7\, dx + \int_{-1}^{1} \cos^{-1} x\, dx$

$$= 0 + \int_{-1}^{1} \cos^{-1}\left(1 + (-1) - x\right) dx$$

$$= \int_{-1}^{1} \left(\pi - \cos^{-1} x\right) dx = \int_{-1}^{1} \pi\, dx - I$$

$$2I = \int_{-1}^{1} \pi\, dx$$

17. $I = 2\int_{0}^{\frac{\pi}{2}} \sqrt{\cos x} \cdot |\sin x|\, dx$ (even function)

$$= 2\int_{0}^{\frac{\pi}{2}} \sqrt{\cos x} \cdot \sin x\, dx \qquad (\because \sin x > 0)$$

18. $-a < x < a$

$$0 < x + a < 2a \ \& \ -2a < x - a < 0$$

$$\therefore I = \int_{-a}^{a} \left(\frac{x+a}{x+a} + \frac{a-x}{x-a} \right) dx = 0$$

19. $\int_{1}^{e} (\log x)^8\, dx = \left[x\, (\log x)^8\right]_1^e - 8\int_{1}^{e} x\, (\log x)^7 \cdot \frac{1}{x}\, dx$

$$\therefore \int_{1}^{e} (\log x)^8\, dx + 8\int_{1}^{e} (\log x)^7\, dx = \left[x\, (\log x)^8\right]_1^e$$

20. $\int_{\sqrt{2}}^{2} \frac{k\,dx}{x\sqrt{x^2 - 1}} = \frac{\pi}{4}$

$$k\left[\sec^{-1} x\right]_{\sqrt{2}}^{2} = \frac{\pi}{4}$$

21. $I = \int_{-\log 3}^{\log 3} 2^{x^2} \cdot x^3\, dx$ ($\because f$ is an odd function)

$$= 0$$

22. $x + \frac{1}{x} = t$

$$\left(1 - \frac{1}{x^2}\right) dx = dt$$

$$\therefore\ I = \int_{-2}^{2} f(t)\,dt = 2\int_{0}^{2} f(t)\,dt \quad (\because f \text{ is an odd function})$$

23. $\log\tan\theta = t$

$$\frac{1}{\tan\theta} \cdot \sec^2\theta \cdot d\theta = dt$$

$$I = \frac{1}{2}\int_{-\log\sqrt{3}}^{\log\sqrt{3}} t\,dt = 0$$

24. $$I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt[n]{\cos x}}{\sqrt[n]{\cos x} + \sqrt[n]{\cos x}}\,dx$$

$$I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt[n]{\cos x}}{\sqrt[n]{\cos x} + \sqrt[n]{\sin x}}\,dx$$

$$2I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} 1.dx = \frac{\pi}{3}$$

25. $$I = 4\int_{0}^{\frac{\pi}{4}} \frac{2\tan^2 x + 2\tan x + 2}{\tan^2 x + 2\tan x + 1}\,dx$$

$$= 4\int_{0}^{\frac{\pi}{4}} 1.dx + 4\int_{0}^{\frac{\pi}{4}} \frac{1 + \tan^2 x}{(\tan x + 1)^2} \qquad \begin{pmatrix} \because\ 1 + \tan x = t \\ \sec^2 x\ dx = dt \end{pmatrix}$$

$$= \pi + 4\int_{1}^{2} \frac{1}{t^2}.dt$$

26. $$I = \int_{0}^{\pi} \frac{\cos 3\theta}{\cos\theta + \sin\theta}\,d\theta \qquad \text{...(i)}$$

$$I = \int_{0}^{\pi} \frac{\cos(3(\pi - \theta))}{\cos(\pi - \theta) + \sin(\pi - \theta)}\,d\theta \qquad \text{...(ii)}$$

$$(i) + (ii) \Rightarrow\ 2I = \int_{0}^{\pi} \frac{2\cos 3\theta.\cos\theta}{\cos 2\theta}\,d\theta$$

$$= \int_0^{\pi} \frac{\cos 4\theta + \cos 2\theta}{\cos 2\theta}\, d\theta$$

27. $$I = \int_0^{\frac{\pi}{2}} \log\left(2\frac{\left(1+\tan^2\frac{x}{2}\right)}{2\tan\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \log 2\, dx - \int_0^{\frac{\pi}{2}} \log \sin x\, dx$$

$$= \frac{\pi}{2}\log 2 - \left(-\frac{\pi}{2}\log 2\right)$$

28. $$\sin(2n+1)\frac{x}{2} = \sin(2n+1)\frac{x}{2} - \sin(2n-1)\frac{x}{2} + \sin(2n-1)\frac{x}{2}$$

$$-\sin(2n-3)\frac{x}{2} + \dots\dots\dots + \sin\frac{3x}{2} - \sin\frac{x}{2} + \sin\frac{x}{2}$$

$$= 2\cos nx.\sin\frac{x}{2} + 2\cos(n-1)x.\sin\frac{x}{2} + \dots\dots\dots + 2\cos x.$$

$$\sin\frac{x}{2} + \sin\frac{x}{2}$$

$$I = 2\int_0^{\pi}\left(\cos nx + \cos(n-1)x + \dots\dots\dots + \cos x + \frac{1}{2}\right)dx$$

29. $$I = \int_0^{100\pi} \sqrt{2}\,|\sin x|\, dx$$

$$= \sqrt{2}\left[\int_0^{\pi}\sin x\, dx - \int_{\pi}^{2\pi}\sin x\, dx + \int_{2\pi}^{3\pi}\sin x dx + \dots\dots\dots + \int_{98\pi}^{99\pi}\sin x\, dx - \int_{99\pi}^{100\pi}\sin x\, dx\right]$$

30. $$\int_e^{e^2}\frac{dx}{\log x} = \int_1^2 \frac{e^t}{t}\, dt$$ [put $\because$ $\log x = t$ $\therefore$ $x = e^t$ $\therefore$ $dx = e^t dt$]

$$= \int_1^2 \frac{e^x}{x} dx$$

$$\therefore \int_e^{e^2}\frac{dx}{\log x} - \int_e^{2}\frac{e^x}{x}\, dx = 0$$

31. $\int_{2P-a}^{2P+a} f(x)\,dx = \int_{2P-a}^{2P+a} f\,(4P-x)\,dx$

$= -\int_{2p-a}^{2p+a} f\,(x + (-4P))\,dx$ [$\because$ $f(-x) = -f(x)$][$-4p$ is period of f]

$= -\int_{2P-a}^{2P+a} f(x)\,dx$

$= -\,I$

32. $I_{100} = \int_0^1 x^{100} e^x\,dx$

$= \left[x^{100}e^x\right]_0^1 - \int_0^1 100x^{99}e^x dx$

$= e - 100\,I_{99}$

33. $I = \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$

$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right)\cos x \cdot \sin x}{\cos^4 x + \sin^4 x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\pi}{2}\,\frac{\cos x \sin x}{\cos^4 x + \sin^4 x} - I$

34. $I = \int_0^{\frac{\pi}{2}} \log\left(\frac{a + b\sin x}{a + b\cos x}\right) dx$

$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{a + b\cos x}{a + b\sin x}\right) dx$

$2I = \int_0^{\frac{\pi}{2}} \log 1\,dx = 0$

35. $\int_1^2 \frac{dx}{x\left(1 + x^6\right)}$ [Put $t = x^6$, $dt = 6x^5\,dx$]

$= \int_1^{64} \frac{dt}{6t\,(t+1)}$

36. $I_K + I_{K+2} = \int_0^{\frac{\pi}{4}} \tan^k x \left(1 + \tan^2 x\right) dx$

$$= \left[\frac{\tan^{k+1} x}{k+1}\right]_0^{\frac{\pi}{4}} = \frac{1}{k+1}$$

$$\sum_{r=1}^{5} \frac{1}{I_r + I_{r+2}} = \frac{1}{I_1 + I_3} + \ldots\ldots\ldots + \frac{1}{I_5 + I_7} = 2 + 3 + \ldots\ldots\ldots + 6$$

$= 20$

37. $\int_{3+\pi}^{4+\pi} f(x - \pi) dx$

$= \int_3^4 f(t)\, dt$ [Put $x - \pi = t$ $dx = dt$]

$a = 3,\ b = 4$ $a + b = 7$

38. $\int_0^1 \sqrt[3]{x^3 - x^4} = \int_0^1 x \sqrt[3]{1 - x}$

$= \int_0^1 (1 - x) \sqrt[3]{x}\ dx$

39. $\int_0^1 (x^5 + 5x^4 + x^4 + 4x^3 + x^3 + 3x^2 + x^2 + 2x + x + 1) \frac{e^x}{e}\ dx$

$$= \frac{1}{e}\left[x^5 + x^4 + x^3 + x^2 + x\ e^x\right]_0^1$$

40. $I = \int_0^1 x^{[x]}\ dx + \int_1^2 x^{[x]}\ dx$

$= \int_0^1 dx + \int_1^2 x\ dx$

41. $I = \int_e^{\pi} (e + \pi - x)\ f\ (e + \pi - x)\ dx$

$$= \int_{e}^{\pi} (e + \pi) f(x)\, dx - I \quad (\because f(e + \pi - x) = f(x))$$

$$I = \frac{e+\pi}{2} \cdot \frac{2}{e+\pi} = 1$$

42. $$I = \int_0^1 \frac{dx}{1-(1-x)+\sqrt{2(1-x)-(1-x)^2}}$$

$$= \int_0^1 \frac{1}{x+\sqrt{1-x^2}}$$

$$= \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta+\cos\theta}\, dQ \qquad [\because x = \sin\theta,\ dx = \cos\theta\ .\ d\theta]$$

43. $$\frac{1}{k-1} = \int_{\pi/4}^{\pi/2} cot^n x\,.\,cosec^2 \times dx$$

$$= -\left[\frac{\cot^{n+1} x}{n+1}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{n+1}$$

44. $$I = \int_0^{\pi/4} \sin 4x \log \cot 2x\, dx$$

$$= \int_0^{\pi/4} \sin\left[\frac{4\pi}{4} - 4x\right] \log \cot\left[\frac{\pi}{2} - 2x\right] dx$$

$$I = \int_0^{\pi/4} \sin 4x\ .\ \log \tan 2x\, dx$$

$$2I = 0$$

45. $$\int_0^{100} f(x)\, dx = \int_0^1 f(x)\, dx + \int_0^2 f(x)\, dx + \ldots + \int_{99}^{100} f(x)\, dx$$

$$\frac{k(k-1)}{2} = 0 + 1 + 2 + \ldots\ldots.. + 99$$

46. $I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\, dx$

$= \int_{-\pi}^{\pi} \frac{2x\,dx}{1+cos^2 x} + \int_{-\pi}^{\pi} \frac{2x \sin x}{1+\cos^2 x}\, dx$

$= 0 + 2\int_{0}^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$

47. $\int_{a}^{a+1} | a - x |\, dx$

$= \int_{a}^{a+1} (x - a)\, dx$

$(a < x < a + 1;\ 0 < x - a < 1)$

48. $I = \int_{0}^{\pi/2} \sqrt{\sin 2\theta}\, \sin\theta\, d\theta$ .... (i)

$I = \int_{0}^{\pi/2} \sqrt{\sin(\pi - 2\theta)}\ .\ \sin\left(\frac{\pi}{2} - 0\right) d\theta$

$= \int_{0}^{\pi/2} \sqrt{sin\, 2\theta}\ .\cos\theta.d\theta$ ..... (ii)

$2I = \int_{0}^{\pi/2} \sqrt{sin\, \alpha\theta}\ (\sin\theta + \cos\theta)\, d\theta$ ($\because$ (i) + (ii))

$= \int_{0}^{\pi/2} \sqrt{1 - (\sin\theta - \cos\theta)^2}\ .\ (\sin\theta + coos\theta)\, d\theta$

put $\sin\theta - \cos\theta = t$

$(\cos\theta + \sin\theta)\, d\theta = dt$

49. $\int_{1/e}^{1} \left| \log x^{\frac{1}{x}} \right| dx$

$= -\int_{1/e}^{1} \frac{1}{x} \log x\, dx$ $\left[\because \frac{1}{x} > 0\ \&\ ;\log x < 0\right]$

$$= -\left[\frac{(\log x)^2}{2}\right]_{\frac{1}{e}}^{1}$$

50. $a < 0 < b$

$$\therefore \int_a^b f(x)\, dx = \int_a^0 \frac{|x|}{x}\, dx + \int_a^b \frac{|x|}{x}\, dx$$

$$= -\int_a^0 1 \,.\, dx + \int_a^b 1 \,.\, dx$$

51. $$I = \int_0^{\pi/2} \sqrt{\frac{1 + \cos x}{\cos x}}\, dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{2}\cos\frac{x}{2}}{\sqrt{1 - 2\sin^2\frac{x}{2}}}\, dx$$

$$t = \sin\frac{x}{2}$$

$$dt = \frac{1}{2}\cos\frac{x}{2}\, dx$$

$$\therefore I = \sqrt{2}\int_0^{\frac{1}{\sqrt{2}}} \frac{2dt}{\sqrt{1 - 2t^2}}$$

52. $$I = \int_{-\pi/4}^{\pi/4} \log(\sec\theta - \tan\theta)\, d\theta$$

$$I = \int_{-\pi/4}^{\pi/4} \log(\sec\theta + \tan\theta)\, d\theta$$

$$2I = 0$$

53. $$I = \int_0^{\pi} \sqrt{\sin x} \,.\, \cos\frac{x}{2}\, dx$$

$$I = \int_0^{\pi/2} \sqrt{\sin 2\theta}\,.\cos\theta\, d\theta ... (i) \qquad \left[\because \frac{x}{2} = \theta,\ dx = 2d\theta\right]$$

$$I = 2\int_0^{\pi/2} \sqrt{\sin 2\theta}\,.\sin\theta\, d\theta ... (ii)$$

$$(i) + (ii) \Rightarrow 2I = 2\int_0^{\pi/2} \sqrt{\sin 2\theta}\,(\cos\theta + \sin\theta)\, d\theta$$

take $\sin\theta - \cos\theta = t$

$(\cos\theta + \sin\theta)\, d\theta = dt$

54. $I = \int_0^1 \sqrt{x}\,\sqrt{1-\sqrt{x}}$

$\sqrt{x} = t$

$dx = 2tdt$

$$I = 2\int_0^1 t^2 \left(\sqrt{1-t}\right) dt$$

$$= 2\int_0^1 (1-t)^2 \sqrt{t}\, dt$$

55. $I = \int_0^{\frac{\pi}{4}} \frac{\sin 2\theta}{1-\frac{1}{2}(\sin 2\theta)} d\theta$

$\cos 2\theta = t$

$-2\sin 2\theta d\theta = dt$

$$I = \int_0^1 \frac{2dt}{1+t^2}$$

56. $I = \int_0^{\pi} \frac{\sin 100x}{\sin x}\, dx$

$$= \int_0^{\pi} \frac{\sin 100(\pi - x)}{\sin(\pi - x)}\, dx = -I$$

57. $I = \int_{-\pi/2}^{\pi/2} \sin x \, f(\cos x) \, dx$

$$= \int_{-\pi/2}^{\pi/2} \sin\left(-\frac{\pi}{2}+\frac{\pi}{2}-x\right) f\left(\cos\left(-\frac{\pi}{2}+\frac{\pi}{2}-x\right)\right) dx$$

$= -\,I$

58. $I = \int_{-1}^{1} (x^2 + x)\,|\,x\,|\,dx = \int_{-1}^{1} x^2\,|\,x\,|\,dx + \int_{-1}^{1} x\,|\,x\,|\,dx$

$= 2\int_{0}^{1} x^2 \,.\, x\,dx + 0$ ($\because$ $x\,|x|$ is an odd function)

59. $I = \int_{0}^{\pi} [\cot x]\,dx = \int_{0}^{\pi} [\cot(\pi - x)]dx = \int_{0}^{\pi} [-\cot x]\,dx$

$2I = \int_{0}^{\pi} ([\cot x] + [-\cot x])\,dx$

$= \int_{0}^{\pi} (-1)\,\mathrm{dx}$

[$\because$ $x \in$ R if x is an integer then $[x] + [-x] = 0$ and if x is not an integrer then $[x] + [-x] = -1$]

60. $f\left(\frac{1}{2}\right) - f\left(-\frac{1}{2}\right) = \int_{0}^{\frac{1}{2}} \log\left(\frac{1-t}{1+t}\right) dt - \int_{0}^{-\frac{1}{2}} \log\left(\frac{1-t}{1+t}\right) dt$

$$= \int_{0}^{\frac{1}{2}} \log\left(\frac{1-t}{1+t}\right) dt + \int_{-\frac{1}{2}}^{0} \log\left(\frac{1-t}{1+t}\right) dt$$

$$= \;= \int_{\frac{-1}{2}}^{\frac{1}{2}} \log\left(\frac{1-t}{1+t}\right) dt$$

$= 0$ ($\because$ $\log\left(\frac{1-t}{1+t}\right)$ is an odd function t)

61. cx = c^2 + t

$$I = \int_{1+c}^{a+c} 1\ dx = c\ (a - 1)$$

62. $7 = [(3-x)f'(x)]_2^3 - \int_2^3 (0-1)f'(x)dx$

$7 = 0 - f'(2) + f(3) - f(2)$

63. $\int_1^{\frac{1}{e}} \frac{\log t}{1+t}$ dt

$$= -\int_1^e \frac{\log\frac{1}{u}}{u(u+1)}dy \qquad \left[\because t = \frac{1}{u}dt = -\frac{1}{u}dy\right]$$

$$= \int_1^e \frac{\log u}{u(u+1)}$$

$$\int_1^e \frac{\log t}{1+t}dt + \int_1^{\frac{1}{e}} \frac{\log t}{1+t}dt$$

$$= \int_1^e \frac{1}{t} \log t\ dt$$

64. $I = \int_0^{\pi} x\ f\ (\sin x)\ dx$

$$= \int_0^{\pi} (\pi - x)\ f\ (\sin\ (\pi - x)\ dx$$

$$= \pi \int_0^{\pi} f\ (\sin x)\ dx - I$$

65. $I = \int_{-\frac{3\pi}{2}}^{-\pi/2} [(x + \pi)^3 + \cos^2\ (x + 3\pi)]\ dx$

$$= \int_{-\frac{3\pi}{2}}^{-\pi/2} \left[\left(-\frac{\pi}{2} - \frac{3\pi}{2} - x + \pi\right)^3 + \cos^2\left(-\frac{3\pi}{2} - \frac{\pi}{2} - x + 3\pi\right)\right] dx$$

$$= -\,I + \int_{-\frac{3\pi}{2}}^{-\pi/2} (1 + \cos 2x)\, dx$$

66. $\int_{1/3}^{2/3} f\text{o}\, f(x)\, dx = \int_{1/3}^{2/3} \frac{1}{1-x}\, dx$

67. $\int_0^1 \frac{dx}{\sqrt{x}\,(x+1)}$ $\qquad \sqrt{x} = t$

$= \int_0^1 \frac{2dt}{1+t^2}$ $\qquad \frac{1}{2\sqrt{x}}\, dx = dt$

68. $\int_0^{\frac{1}{2}} \frac{dx}{(1-x)^2 \sqrt{\frac{1+x}{1-x}}}$

$= \int_0^{\sqrt{3}} dt$ $\qquad \left[\frac{1+x}{1-x} = t^2, \quad \frac{2}{(1-x)^2}\, dx = 2t\, dt\right]$

69. $h(-x) = (f(-x) + g(-x))\,(g(-x) - f(-x))$

$= (-f(x) + g(x))\,(g(x) + f(x))$

$= h(x)$

$\therefore \int_{-\pi/2}^{\pi/2} h(x)\, dx = 2\int_0^{\pi/2} h(x)\, dx$

70. $\sum_{k=1}^{100} \int_0^1 f(x + k - 1)\, dx$

$= \sum_{k=1}^{100} \int_{k-1}^{k} f(t)\, dt$ $\qquad$ [Putting x+k-1=t, dx=dt]

$= \int_0^1 f(t)\, dt + \int_1^2 f(t)\, dt + \ldots\ldots + \int_{99}^{100} f(t)\, dt$

$= \int_0^{100} f(t)\, dt$

71. $\int_{1/e}^{e} \frac{1}{x} f\left(x - \frac{1}{x}\right) dx$ $\qquad \because \frac{1}{x} = t \Rightarrow -\frac{1}{x^2}\, dx = dt$

$= \; = \int_{e}^{\frac{1}{e}} \frac{1}{t} f\left(\frac{1}{t} - t\right) dt$

$= \; = \int_{\frac{1}{e}}^{e} \frac{1}{t}\left[-f\left(t - \frac{1}{t}\right)\right] dt$

$= -\mathrm{I}$

72. Putting t = cosθ

$\int_0^{\pi/2} \sin\theta \log\left(1 - \cos^2\theta\right)^{\frac{1}{2}} d\theta = -\frac{1}{2}\int_1^0 [\log\,(1 \,\square\, t) + \log\,(1 + t)]\, dt$

73. $\int_1^0 \log\left(\frac{1-x}{x}\right) dx$

$= \int_0^1 log\ (1 - x)\ dx - \int_0^1 log\ x\ dx$

$= \int_0^1 log\ (1 - (1 - x))\ dx - \int_0^1 log\ x\ dx$

$= 0$

74. $\mathrm{I} = \int_0^{\frac{\pi}{4}} \frac{2\sin x \cos x}{(\sin x + \cos x + 1)}\, \mathrm{d}x$

$= \int_0^{\frac{\pi}{4}} \frac{2\sin x}{(\tan x + 1 + \sec x)}\left(\frac{\tan x + 1 - \sec x}{\tan x + 1 - \sec x}\right) \mathrm{d}x$

$= \int_0^{\frac{\pi}{4}} \left(\sin x + \cos x - 1\right) \mathrm{d}x$

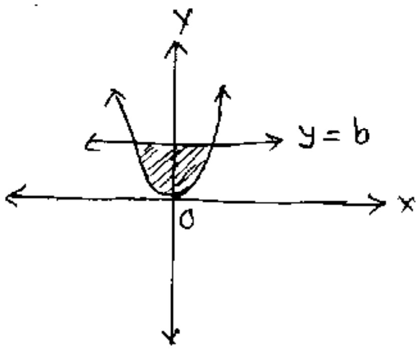
75. Take $x - \frac{1}{x} = t$

$\left(1 + \frac{1}{x^2}\right) dx = dt$

76. Applying integration by parts

77. $I = \int_0^b xdy$

$\frac{4}{3} = \int_0^b 2\sqrt{b}\,\sqrt{y}\,dy$



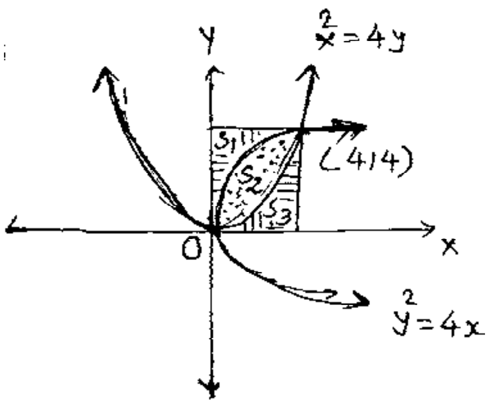
78. $S_1 = S_3$ ..... (i)

$\& S_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \frac{16}{3}$

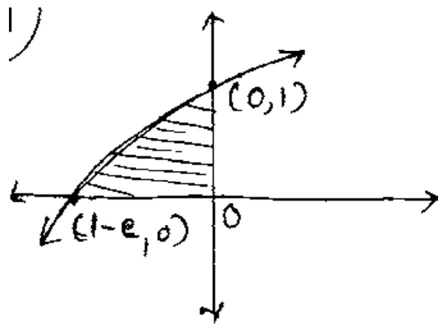
$S_1 + S_2 + S_3 = 4 \times 4$

$2S_1 = 16 - \frac{16}{3}$

$S_1 = \frac{16}{3}$ Sq. unit

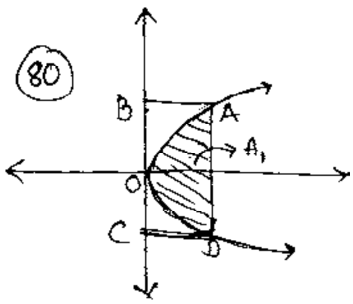


79. $A = \int_{1-e}^{0} \log_e (x + e)\, dx$



80. $A_1 = 2 \mid I \mid$

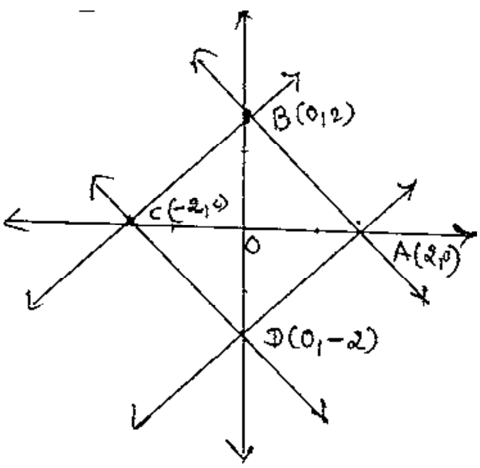
$I = \int_0^8 4\sqrt{2}\,\sqrt{x}\,dx$



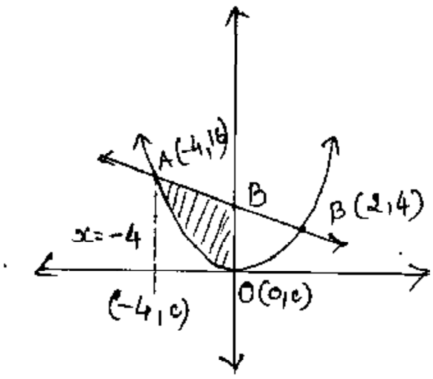
81. $A = 4\,|\,I\,|$

$= 4\int_0^2 (2 - x)\,dx$

OR $A = 4\cdot\frac{1}{2}\,(2)\,(2)$



82. $I = \int_{-4}^{0} (8 - 2x - x^2)\,dx$



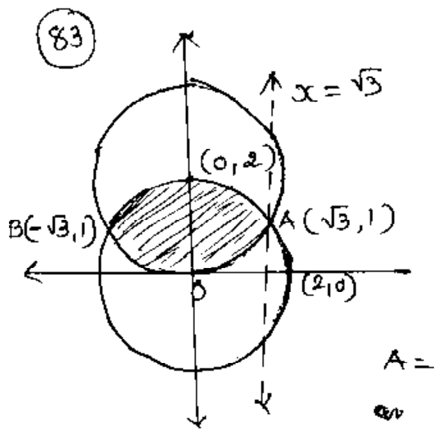
83. $A = 2\,|\,I\,|$

$I\int_0^{\sqrt{3}}\left(\sqrt{2^2 - x^2} - 2 + \sqrt{2^2 - x^2}\right)dx$

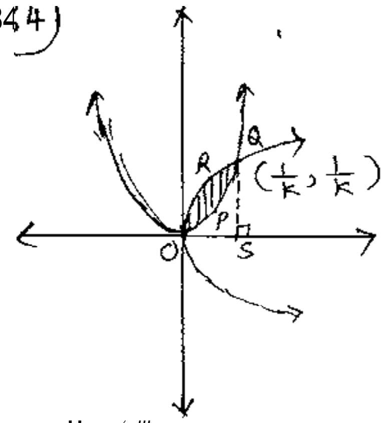
OR

$A = 4\,|\,I\,|$

$I = \int_0^1 \sqrt{4 - (y - 2)^2}\;dy$



84



$$12 \int_0^{\frac{1}{k}} \left[\sqrt{\frac{x}{k}} - kx^2 \right] dx$$

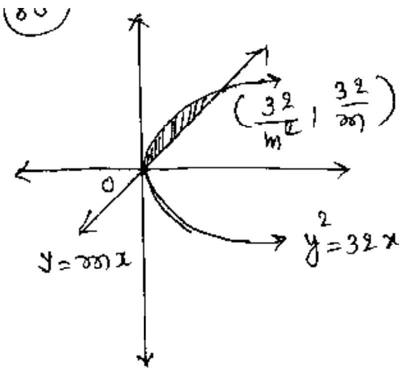
$$3k^2 = \frac{1}{12}$$

$$k = \frac{1}{6} \quad (\because\ k > 0)$$

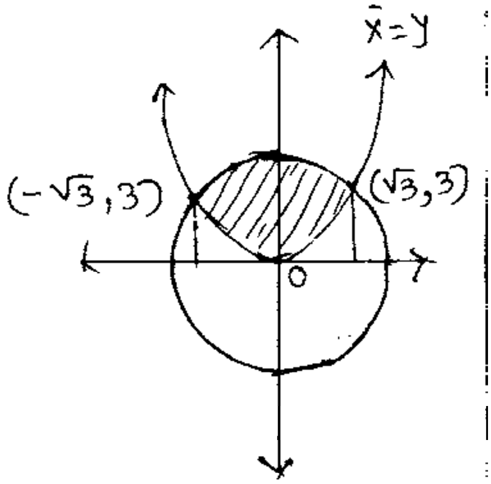
85. $\frac{8}{3} = \int_0^{} 4\sqrt{2}\sqrt{x} - mx \quad dx$

$$\frac{8}{3} = \frac{512}{3m^3}$$

$$m = 4$$

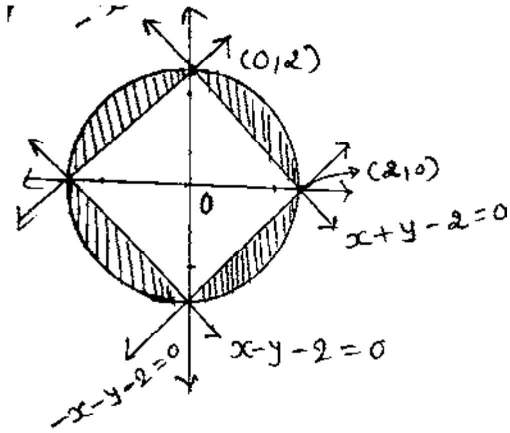


86. $A = 2\int_0^{\sqrt{3}} \sqrt{12 - x^2} - x^2 \quad dx$

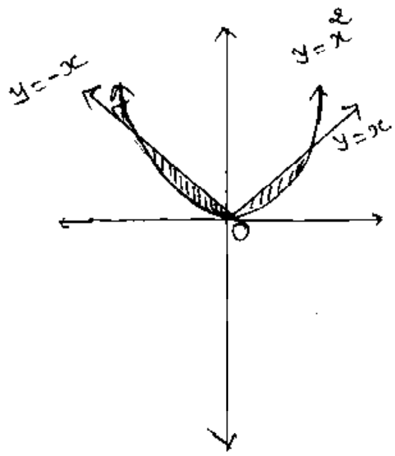


87. $A = \pi\,(2)^2 - \left(2\sqrt{2}\right)^2$

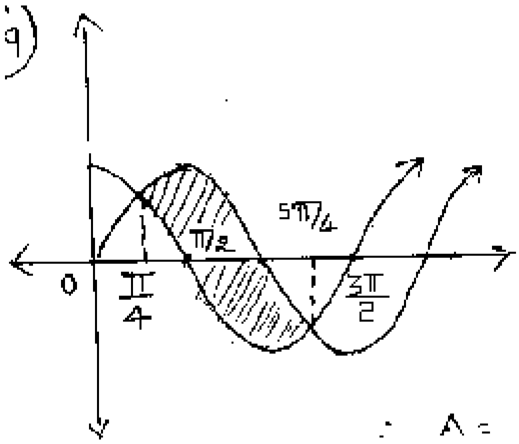
$= 4\pi - 8$



88. $A = 2\int_0^1 (x - x^2)\,dx$



89.



$\frac{\pi}{4} \le x \le \frac{5\pi}{4}$

$\Rightarrow \sin x \le \cos x$

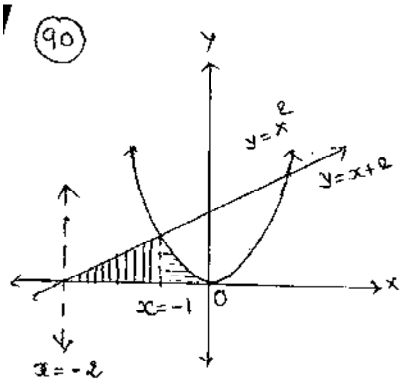
$\therefore\ A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$

$I_2 = \int_{-1}^{0} x^2 dx$

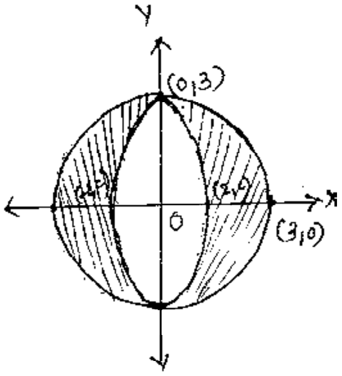
90. $A=|I_1|+|I_2|$

$$I_1=\int_{-2}^{-1}(x+2)\,dx$$

$$I_2=\int_{-1}^{0}x^2\,dx$$



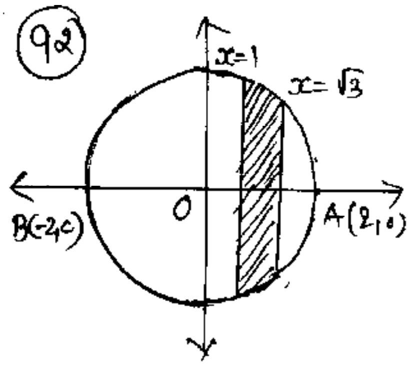
91.



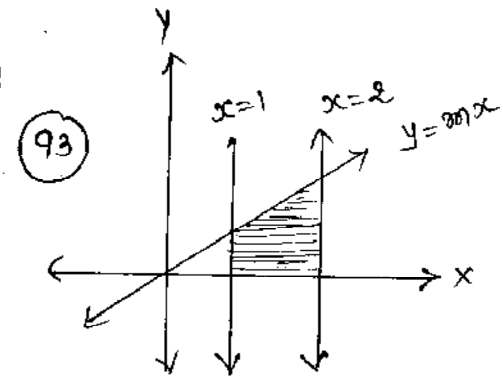
$$=4\int_0^3\left(\sqrt{9-y^2}-\frac{2}{3}\sqrt{9-y^2}\right)dy$$

$$=\frac{4}{3}\int_0^3\sqrt{(3)^2-y^2}\;dy$$

92.



$$A=2\int_1^{\sqrt{3}}\sqrt{4-x^2}\,dx$$



$$6=\int_1^2 mx\,dx$$

$$6=m\left[\frac{x^2}{2}\right]_1^2$$

Answer

(1) b	(26) b	(51) d	(76) b
(2) b	(27) c	(52) d	(77) c
(3) b	(28) c	(53) c	(78) d
(4) d	(29) d	(54) d	(79) a
(5) b	(30) d	(55) c	(80) b
(6) d	(31) d	(56) a	(81) c
(7) b	(32) c	(57) c	(82) c
(8) b	(33) b	(58) b	(83) d
(9) c	(34) a	(59) c	(84) b
(10) c	(35) b	(60) a	(85) c
(11) b	(36) d	(61) b	(86) c
(12) d	(37) d	(62) d	(87) c
(13) d	(38) c	(63) d	(88) c
(14) b	(39) a	(64) d	(89) d
(15) b	(40) c	(65) b	(90) c
(16) c	(41) c	(66) d	(91) b
(17) d	(42) c	(67) c	(92) b
(18) a	(43) c	(68) c	(93) d
(19) d	(44) a	(69) d	
(20) c	(45) d	(70) b	
(21) a	(46) d	(71) d	
(22) b	(47) d	(72) a	
(23) a	(48) d	(73) c	
(24) c	(49) d	(74) b	
(25) c	(50) a	(75) a	

Gujarat Secondary and Higher Secondary Education Board

Sector 10-B, Near Old Sachivalaya, Gandhinagar-382 010.