AMTI (NMTC) - 2004

BHASKARA CONTEST - JUNIOR LEVEL

1. Given the sequence a, ab, aab, aaabb, aaabbb, ... upto 2004 terms, the total number of times a's and b's are used from 1 to 2004 terms are
   (A) 2004 a's and 2003 b's (B) 4008 a's and b's
   (C) 1002 x 1003 a's and $(1002)^2$ b's (D) 1003^2 a's and 1002 x 1003 b's
2. The number of two digit numbers divisible by the product of the digits is
   (A) 5 (B) 8 (C) 14 (D) 33
3. Given that $(a - 5)^2 + (b - c)^2 + (c - d)^2 + (b + c + d - 9)^2 = 0$ then $(a + b + c)(b + c + d)$ is
   (A) 0 (B) 11 (C) 20 (D) 99
4. $x^4 - y^4 = 15$, x and y are positive integers. Then $x^4 + y^4$ is
   (A) 17 (B) 31 (C) 32 (D) 113
5. In this addition each letter represents a different digit. Which is the absent digit ?

$$\begin{array}{r} ABCD \\ +\ BCD \\ \hline GHIJK \\ \hline \end{array}$$

   (A) 1 (B) 3 (C) 4 (D) 5
6. Five children each owned a different number of rupees. The ratio of any one's fortune to the fortune of every child poorer than himself was an integer. The combined fortune of the children was 847 rupees. The least number of rupees that a child had was
   (A) 12 Rs. (B) 10 Rs. (C) 7 Rs. (D) 5 Rs.
7. A number with 8 digits is multiple of 73 and also a multiple of 137. The second digit from the left equals 7. Then the 6^{th} digit from the left equals
   (A) 1 (B) 7 (C) 9 (D) can be any digit
8. Let n be the least positive integer such that 1260 n is the cube of a natural number. Then n satisfies
   (A) $1 < n < 50$ (B) $50 < n < 100$
   (C) $100 < n < 1000$ (D) $1000 < n < 10000$
9. If $(43)_x$ in base x number system is equal to $(34)_y$ in base y number system the possible value for x + y is
   (A) 16 (B) 14 (C) 12 (D) 10
10. In each of the following 2003 fractions the sum of the numerator and denominator equals 2004 : $\left[\frac{1}{2003}, \frac{2}{2002}, \frac{3}{2002}, ..., \frac{2003}{1}\right]$. The number of fractions < 1 which are irreducible (no common factor between numerator and denominator) is
   (A) 664 (B) 332 (C) 1002 (D) 1001
11. For how many integers n is $\sqrt{9-(n+2)^2}$ a real number ?
   (A) 3 (B) 5 (C) 7 (D) infinitely many
12. During holidays, five people A, B, C, D and E went swimming regularly. Each time they went, exactly one of them was missing. A went the least number of times (5 times) and E most often (8 times). What can we say about the number of times B, C and D went ?
   (A) each went six times
   (B) each went seven times
   (C) 2 went 6 times and one went 7 times
   (D) 2 went 7 times and 1 went 6 times

13. Let A = {a, b, c} and B = {a, b, d, e, f}. How many sets C consisting of characters from the English alphabet can be constructed so that $C \subseteq B$ and such that $A \cap C$ has one element and $C \not\subseteq A$.

(A) 16 (B) 14 (C) 8 (D) 6

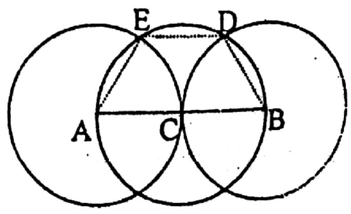
14. The sum of all angles except one of a convex polygon is 2190^0. (where the angles are less than 180^0). Then the possible number of sides of the polygon is

(A) 13 (B) 15 (C) 17 (D) 19

15. In a right angled let triangle with legs 4 and 8, the area of the largest square that can be inscribed in the triangle is

(A) $\frac{8}{3}$ (B) $\frac{4}{3}$ (C) $\frac{16}{9}$ (D) $\frac{16}{9}$

16. Two circles with centres A and B and radius 2 touch each other externally at C. A third circle with center C and radius 2 meets the other two at D, E (see the figure). Then area ABDE is



(A) $3\sqrt{2}$ (B) $6\sqrt{2}$ (C) $3\sqrt{3}$ (D) $6\sqrt{3}$

17. In ΔABC, $\angle A = 90^0$ and I is the incentre. The perpendicular distance of I from BC is $\sqrt{8}$. Then AI is equal to

(A) $\sqrt{8}$ (B) 3 (C) $\sqrt{12}$ (D) 4

18. In an isosceles triangle, the centriod, the orthocentre, the incentre and the circumcentre are

(A) conincident (B) collinear
(C) in the interior of the circumcircle (D) in the interior of the incircle

19. If a, b are positive real numbers and $\sqrt{a\frac{a}{b}} = a \cdot \sqrt{\frac{a}{b}}$ where $a\frac{a}{b}$ is a mixed fraction, which of the following is true ?

(A) $b = a^2 + 1$ (B) $a = b^2 - 1$ (C) $a = b^2 + 1$ (D) $b = a^2 - 1$

20. If $\frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1$ and $\frac{a}{p}+\frac{b}{q}+\frac{c}{r}=0$ then the value of $\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}$ is

(A) 0 (B) –11 (C) 9 (D) 1

21. Let [x] denote the greatest integer less than or equal to x, what is the value of

$[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+....+[\sqrt{2004}]$?

(A) 58850 (B) 59730 (C) 59950 (D) 56718

22. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3 then

(A) $a < 2$ (B) $2 \le a \le 3$ (C) $3 < a \le 4$ (D) $a > 4$

23. If a function f(x) is defined such that $10^{f(x)} = \frac{1-x}{1+x}$ where $x < 1$, then f(a) + f(b) is equal o

(A) $f\left(\frac{a+b}{1+ab}\right)$ (B) $f\left(\frac{a-b}{1+ab}\right)$ (C) $f\left(\frac{a-b}{1-ab}\right)$ (D) none of these

24. How many solutions are there for (a, b) if 7ab73 is a five digit number divisible by 99 ?

(A) 3 (B) 2 (C) 0 (D) 1

25. The number $107^{90} - 76^{90}$ is divisible by

(A) 61 (B) 62 (C) 64 (D) none of these

26. A sequence $a_0, a_1, a_2, a_3, \ldots, a_n \ldots$ is defined such that $a_0 = a_1 = 1$ and $a_{n+1} = (a_{n-1}.a_n) + 1$ for $n \geq 1$. Which of the following is true ?

(A) $4 \mid a_{2004}$ (B) $3 \mid a_{2003}$ (C) $5 \mid a_{2004}$ (D) $2 \mid a_{2003}$

27. A solid cuboid has edges of length a, b, c. What is the surface area ?

(A) $(a + b + c)^2 - (a^2 + b^2 + c^2)$ (B) abc

(C) $2(a^2 + b^2 + c^2)$ (D) ab + bc + ca

28. A circle and a parabola are drawn on a piece of paper. The number of regions they divide the paper into is at most

(A) 3 (B) 4 (C) 5 (D) 6

29. A cubic polynomial P is such that P(1) = 1, P(2) = 2, P(3) = 3 and P(4) = 5. Then P(6) is

(A) 7 (B) 10 (C) 13 (D) 16

30. Which of the following is the best approximation to $\frac{(2^3-1)(3^3-1)(4^3-1)\ldots(1000^3-1)}{(2^3+1)(3^3+1)(4^3+1)\ldots(1000^3+1)}$?

(A) $\frac{3}{5}$ (B) $\frac{33}{50}$ (C) $\frac{333}{500}$ (D) $\frac{3333}{5000}$

PART - A

1. n is a natural number greater than 1, and $A=\frac{\sqrt{n+1}}{n}+\frac{\sqrt{n+4}}{n+3}+\frac{\sqrt{n+7}}{n+6}+\frac{\sqrt{n+10}}{n+9}+\frac{\sqrt{n+13}}{n+12}$

$$B=\frac{1}{\sqrt{n-1}}+\frac{1}{\sqrt{n+2}}+\frac{1}{\sqrt{n+5}}+\frac{1}{\sqrt{n+8}}+\frac{1}{\sqrt{n+11}}$$

then

(A) A = B (B) A = 2B (C) A < B (D) A > B

2. How many distinct rational numbers (a, b, c, d) are there with $a\log_{10}2 + b\log_{10}3 + c\log_{10}5 + d\log_{10}7 = 2011$.

(A) 0 (B) 1 (C) 5 (D) 2011

3. ABCD is a rectangle in which AB = 8, AD = 9. E is on AD such that DE = 4. H is on BC such that BH = 6. EC and AH cut at G. GF is drawn perpendicular to AD produced. Then GF =

(A) 20 (B) 22 (C) 18 (D) 15

4. The sides of the base of a rectangular parallelepiped are a and b. The diagonal of the parallelepiped is inclined to the base plane at an angle θ. Then the lateral surface area of the solid is

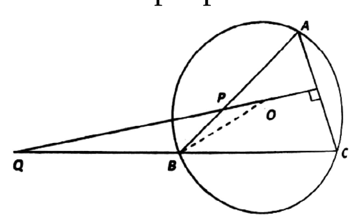
(A) $2(a+b)\sqrt{a^2+b^2}\tan\theta$ (B) $(a+b)\sqrt{a^2+b^2}\tan\theta$

(C) $(a^2+b^2)\sqrt{a+b}\tan\theta$ (D) $2(a^2+b^2)\sqrt{a+b}\tan\theta$

5. The number of integers n which satisfy $(n^2 - 2)(n^2 - 20) < 0$ is

(A) 3 (B) 4 (C) 5 (D) 6

6. In the adjoining figure. O is the circum centre of the triangle ABC. The perpendicular bisector of AC meets AB at P and CB produced at Q. Then



(A) 2∠PQB = 3∠PBO (B) 3∠PQB = 2∠PBO

(C) 4∠PQB = 5∠PBO (D) None of these

7. There are 15 radial spokes in a wheel, all equally inclined to one another. Then there are two spokes which

(A) lie along a diameter of the wheel (B) are perpendicular to each other

(C) are inclined at an angle of 120^0 (D) include an angle less than 24^0

8. Three teams of wood-cutters take part in a competition. The first and the third teams put together produced twice the amount cut by the second team. The second and the third team put together yielded a three-fold output as compared with the first team. Which of the teams won the competition ?

(A) first team (B) second team (C) third team (D) there is a tie

9. The number of positive integral values of n for which $(n^3 - 8n^2 + 20n - 13)$ is a prime number is

(A) 2 (B) 1 (C) 3 (D) 4

10. The value of the expression $\frac{\sqrt{(x+2)^2-8x}}{(\sqrt{x}-2/\sqrt{x})}$ is equal to

(A) $\sqrt{x}$ for all x > 0 (B) $-\sqrt{x}$ for 0 < x < 2

(C) $\sqrt{x}$ for 0 < x < 2 (D) $-\sqrt{x}$ for all x > 0

11. The number of positive integers 'n' for which $3n - 4$, $4n - 5$ and $5n - 3$ are all primes is
(A) 1 (B) 2 (C) 3 (D) infinite

12. a and b are the roots of the quadratic equation $x^2 + \lambda x - \frac{1}{2\lambda^2} = 0$ where x is the unknown and λ is a real parameter. The minimum value of $a^4 + b^4$ is
(A) $2\sqrt{2}$ (B) $\frac{1}{1+\sqrt{2}}$ (C) $\sqrt{2}$ (D) $2+\sqrt{2}$

13. When $b \geq 0$, then $12a^2b^3 - a^6 - b^9$
(A) always is less than or equal to 64 (B) always greater than 64
(C) always negative (D) always lies in the interval [60, 64]

14. There are three natural numbers. The second is greater than the first by the amount the third is greater than the second. The product of the two smaller numbers is 85 and the product of the two larger numbers is 115. If the numbers are x, y, z with $x < y < z$ then the value of $(2x + y + 8z)$ is
(A) 117 (B) 119 (C) 121 (D) 78

15. The number of digits in the sum $100 + 100^2 + 100^3 + ... + 100^{2011}$ is
(A) 4023 (B) 4022 (C) 4024 (D) none of these

PART - B

1. In the sequence $a_1, a_2, ..., a_n$ the sum of any three consecutive terms is 40. If the third term is 10 and the eight term in 8 then the 1000th term is ________.
2. ABCDEF is a non-regular hexagon where all the six sides touch a circle and all the six sides are of equal length. If $\angle A = 140^0$ then $\angle D$ = ______.
3. The difference between the largest 6 digit number with no repeated digits and the smallest six digit number with no repeated digits is _______.
4. Three consecutive integers lying between 1000 and 9999, both inclusive, are such that the smallest is divisible by 11 and the middle one by 9 and the largest by 7. The sum of the largest such four digit numbers is _______.
5. f(x) is a quadratic polynomial with f(0) = 6, f(1) = 1 and f(2) = 0. Then f(3) = _____.
6. While multiplying two numbers a and b Renu reverted the digits of a two digit number and obtained the product to be 391. Renu realized that she made a mistake as her correct answer must be even. The correct product is _______.
7. In a chess tournament players get 1 point for a win 0 for a loss and $\frac{1}{2}$ point for a draw. I a tournament where every player plays against every other player exactly once, the top four scores were $5\frac{1}{2}, 4\frac{1}{2}, 4$ and $2\frac{1}{2}$. The lowest score in the tournaments was ______.
8. When x is real, the greatest possible value of $10^2 - 100^x$ is _______.
9. The diagonals of a convex quadrilateral are perpendicular. If AB = 4, AD = 5, CD = 6, then length of BC is _______.
10. Three circles, each of radius one, have centres at A, B and C. Circles A and B touch each other and circle C touches AB at its midpoint. The area inside circle C and outside circles A and B is ______.
11. The number of rectangles that can be obtained by joining four of the 11 vertices of a 11-sided regular polygon is _______.
12. Let D, E, F be the midpoints of the sides BC, CA and AB respectively of triangle ABC. AB = 16, BC = 21 and CA = 19. The circum-circles of the triangles BDF and CDE cut at P other than D. Then $\angle BPC$ = ________.
13. Let x and y be two distinct three digit positive integers such that their average is 600. Then the maximum value of $\frac{x}{y}$ is _______.
14. Let N = 101010...101 by a 2011 digit number with alternating 1's and 0's. The sum of the digits of the product of N with 2011 is _______.
15. x_1, y_1, x_2, y_2 are real numbers. If $x_1^2 + x_2^2 \leq 2$ and $y_1^2 + y_2^2 \leq 4$, the maximum value of the expression $x_1y_1 + x_2y_2$ is _______.

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PART - A

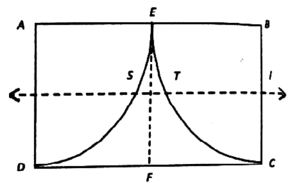
1. Two regular polygons of same number of sides have sides 40 cm and 9 cm is length. The lengths of the side of another regular polygon of the same number of sides and whose area is equal to the sum of the areas of the given polygons is (in cm)

(A) 49 (B) 31 (C) 41 (D) 360

2. If x > y > 0 and $\frac{x+y}{x-y}=\sqrt{2}$, the value of $\frac{x^2+y^2}{xy}$ is

(A) 5 (B) 4 (C) 1 (D) 6

3. In rectangle ABCD, AB = 2BC = 4 cm E and F are midpoints of AB and CD respectively. ESD and ETC are arcs of the circles centred at A and B respectively. If the perpendicular bisector line *l* of EF cuts the arcs at S and T as in the diagram, then ST is equal to (in cm)



(A) $(4-2\sqrt{3})$ (B) $(3+\sqrt{3})$ (C) $(2+2\sqrt{3})$ (D) $(4\sqrt{3}-2)$

4. The value of 'a' for which the expression $\left\{\left(a^{\frac{1}{4}}-a^{\frac{1}{8}}+1\right)^{-1}+\left(a^{\frac{1}{4}}+a^{\frac{1}{8}}+1\right)^{-1}-\frac{2\sqrt[4]{a}-2}{\sqrt{a}-\sqrt[4]{a}+1}\right\}^{-1}$ $-2^{\log_4 a-2}$ takes the value 2012 is

(A) 4048 (B) 6036 (C) 6037 (D) 4047

5. C_1 and C_2 are two non-intersecting circles whose radii are in the ratio 1 : 2. A third circle cuts the smaller circle at A and B and the bigger one at C and D. AB and CD intersect at P. The ratio of the lengths of the tangents from P to the circles C_1 and C_2 is

(A) 1:4 (B) 1:8 (C) 1:1 (D) 1:2

6. a, b, c are real numbers and none of them zero and

E = $\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2+\left(ab+\frac{1}{ab}\right)^2-\left(a+\frac{1}{a}\right)+\left(b+\frac{1}{ab}\right)\left(ab+\frac{1}{ab}\right)^2$. Then E is equal to

(A) 2012 when a = b = 2012 (B) 2012 when ab = 2012
(C) 4 for all real values of a and b (D) 2012 for all real values of a and b

7. The angles of a triangle are in the ratio 2 : 3 : 7. The length of the smallest side is 2012 cm. The radius of the circum circle of the triangle (in cm) is

(A) 2013 (B) 2011 (C) 4024 (D) 2012

8. If a = 2012, b = –1005, c = –1007, then the value of $\frac{a^4}{b+c}+\frac{b^4}{c+a}+\frac{c^4}{a+b}+3abc$ is

(A) 2012 (B) 1 (C) 0 (D) $(2012)^3$

9. ABC is a triangle with AB = 13 cm, BC = 14 cm and CA = 15 cm. AD and BE are the altitudes from A and B to BC and AC respectively. H is the point of intersection of AD and BE. Then the ratio $\frac{HD}{HB}$ =

(A) $\frac{3}{5}$ (B) $\frac{12}{13}$ (C) $\frac{4}{5}$ (D) $\frac{5}{9}$

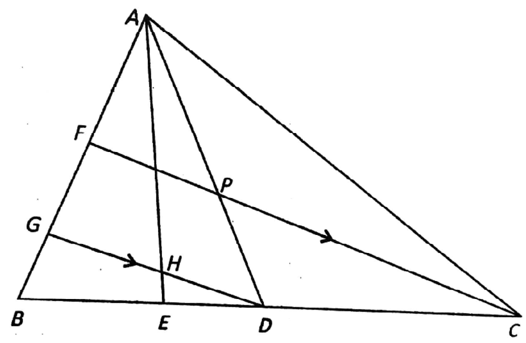
10. For how many positive integrals values of x $\le$ 100 is $(3^x - x^2)$ divisible by 5 ?

(A) 16 (B) 20 (C) 24 (D) 36

11. If one root of $\sqrt{a-x}+\sqrt{b+x}=\sqrt{a}+\sqrt{b}$ is 2012, then a possible value of a, b is

(A) (2000, 2012) (B) (4024, 2012)
(C) (1000, 1012) (D) (1012, 1000)

12. In the figure shown, BD = CD, BE = DE, AP = PD and DG||CF. Then $\dfrac{area\ of\ \Delta ADH}{area\ of\ \Delta ABC}$ is equal to



(A) $\frac{1}{5}$ (B) $\frac{2}{9}$ (C) $\frac{3}{13}$ (D) $\frac{1}{6}$

13. If a = 2012, b = 2011, c = 2010 then the value of $a^2 + b^2 + c^2 - ab - bc - ca$ is

(A) 0 (B) 2012 (C) 3 (D) 4024

14. AX and BX are two adjacent sides of a regular polygon. If $\angle ABX = \frac{1}{3}\angle AXB$, then the number of sides of the polygon is

(A) 6 (B) 7 (C) 9 (D) 5

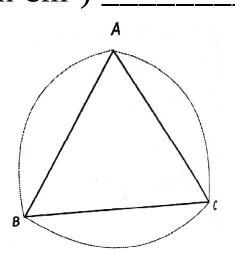
15. A rectangular block of dimensions 16 x 10 x 8 units is painted. It is cut in to cubes of dimensions 1 x 1 x 1. The number of cubes which are not painted at all is

(A) 945 (B) 672 (C) 812 (D) 796

PART - B

1. The value of $\sqrt[3]{5+2\sqrt{13}} + \sqrt[3]{5-2\sqrt{13}}$ is = _______.

2. Triangle ABC is equilateral of side length 8 cm. Each arc shown in the diagram in an arc of a circle with the opposite vertex of the triangle as its centre. The total area enclosed within the entire figure shown (in cm^2) ________.



3. If a, b, c, d, satisfy the equations a + 7b + 3c + 5d = 0, 8a + 4b + 6c + 2d = –16, 2a + 6b + 4c + 8d = 16, 5a + 3b + 7c + d = –16 then the value of (a + d) (b + c = ______.

4. Lines $L_1, L_2, L_3, \ldots, L_{20}$ are distinct. All the lines L_4, L_8, L_{12}, L_{16} and L_{20} are parallel. All the lines $L_1, L_5, L_9, L_{13}, L_{17}$ pass through a given point A, the maximum number of points of intersection of these 20 lines is _______.

5. x, y, z are real numbers such that $(x + y)^2 = 16$, $(y + z)^2 = 36$, $(z + x)^2 = 81$, $x + y + z > 3$. The number of possible values of (x + y + z) is ______.

6. ABCD is a square. A line AX meets the diagonal BD at X and AX = 2012 cm. the length of CX (in cm) is _______.

7. A two digit number is 6 times the sum of its digits. The number formed by interchanging the digits is k times the sum of the digits. Then values of k is _______.

8. O is the centre of a circle of radius 15 cm. M is a point at a distance of 5 cm from O. AMB is any chord of the circle through M, then the value of AM x MB is _______.

9. The combines age of a man and his wife is six times the combined ages of their children. Two years ago their united ages were ten times the combined ages of their children. Six years hence their combined age will be there times the combined age of the children. The number of children they have is ________.

10. A two digit number is less than the sum of the squares of its digits by 11 and exceeds twice the product of its digits by 5. The two digit number is _______.

11. An isosceles trapezoid is circumscribed about a cirlce of radius 2 cm and the area of the trapezoid is 20 cm2. The equal sides of the trapezoid have length ________.

12. A triangle has sides with lengths 13 cm, 14 cm, 15 cm. A circle whose centre lies on the longest side touches the other two sides. The radius of the circle is (in cm) _______.

13. The sum of the roots of the equation $x\sqrt[3]{x^2} = (\sqrt{x})^x$ is _______.

14. ABC and ADE are two secants of a circle of radius 3 cm. A is at a distance of 5 cm from the centre of the circle. The secants include an angle of 30^0. The area of the ΔACE is 10 cm^2. Then the area of the ΔADB (in cm^2) is _______.

15. The value of x which satisfies the equation $5^2 \cdot 5^4 \cdot 5^6 \ldots . 5^{2x} = (0.04)^{-28}$ is ______.

PART - A

1. Two sides of a triangle are 10 cm and 5 cm in length and the length of the median to the third side is $6\frac{1}{2}$ cm. The area of the triangle is $6\sqrt{x}$ cm^2. The value of x is

(A) 12 (B) 13 (C) 14 (D) 15

2. x and y are real numbers such that 7x – 16y = 0 and 4x – 49y = 0, then the value of (x – y) is

(A) $\frac{5}{2}$ (B) $\frac{19}{2}$ (C) $\frac{4115}{2013}$ (D) $\frac{1569}{784}$

3. a, b are positive integers such that

(i) the sum of their squares is S

(ii) the sum of their cubes is C times the sum of the numbers

(iii) S – C = 28

The number of such pairs (a, b) is

(A) 1 (B) 2 (C) 3 (D) 6

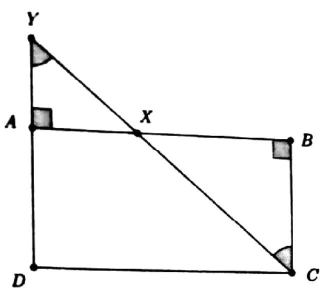
4. The number of numbers of the form 30a0b03 that are divisible by 13, where a,b are digits, is

(A) 5 (B) 6 (C) 7 (D) 0

5. The number of positive integral values of (x, y) which satisfy the equations $\sqrt[3]{x}+\sqrt[3]{y}=4, x+y=28$ simultaneously is

(A) 1 (B) 2 (C) 0 (D) 3

6. ABCD is a rectangle. Through C a variable line is drawn so as to cut AB at X and DA produced at Y. Then BX x DY is



(A) twice the area of the rectangle ABCD.

(B) equal to the area of the rectangle ABCD.

(C) a variable quantity which lies between the area of rectangle ABCD and twice the area of the rectangle ABCD.

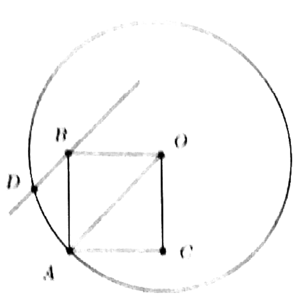
(D) always a constant less than the area of rectangle ABCD.

(A) 1 (B) 2 (C) 0 (D) 3

7. The number of ordered triples (x, y, z) such that x, y, z are primes and $x^y + 1 = z$ is

(A) 0 (B) 1 (C) 2 (D) infinitely many

8. In the adjoining figure, O is the centre of the circle. ACOB is a square with A on the circle. Through B a line parallel to OA is drawn to cut the circle at D nearer to A. Then ∠BOD =



(A) 20^0 (B) 18^0 (C) 15^0 (D) $22\frac{1}{2}^0$

9. The number of real solutions of the equation $x+\sqrt{x^2+\sqrt{x^3+1}}=1$ is

(A) 1 (B) 2 (C) 3 (D) 0

10. In the figure below PQRS is a square of side 2 units. PTR and QTS are quadrants of circles of radius 2 units. With SR as diameter, a semicircle is drawn. A, B denote the areas of the portions shaded. Then (A – B) =

(A) $\frac{3\pi}{2}-4$ (B) $\frac{\pi-1}{3}$ (C) $\frac{3}{2}-\frac{\pi}{4}$ (D) $4-\pi$

11. a, b, c are digits of a 3–digit number such that 64a + 8b + c = 403, then the value of a + b + c + 2013 is

(A) 2024 (B) 2025 (C) 2034 (D) 2035

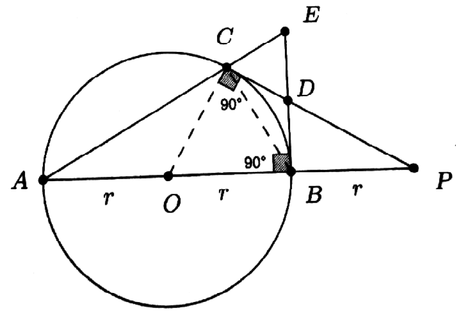
12. What is the sum of the digits of $(9999999999)^3$

(A) 99 (B) 108 (C) 180 (D) 199

13. The number of three digit numbers which are divisible by 3 and have the additional property that the sum of their digits is 4 times their middle digit is

(A) 7 (B) 4 (C) 11 (D) 10

14. In the figure below, AB is a diameter of a circle. AB is produced to P such that BP = radius of the circle. PC is a tangent to the circle. The tangent at B and AC produced cut at E. Then ΔCDE is



(A) isosceles with EC = ED (B) isosceles with EC = CD

(C) equilateral (D) a scalene triangle

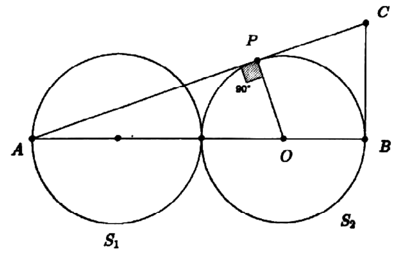
15. Nine numbers are written in ascending order. The middle number is the average of the nine numbers. The average of the five largest numbers is 68 and the average of the five smallest numbers is 44. The sum of all numbers is

(A) 560 (B) 504 (C) 112 (D) 122

PART - B

16. The least value of the positive integer n such that (n + 20) + (n + 21) + (n + 22) + ... + (n + 100) is a perfect square is ______.

17. In the figure below two equal circles S_1, S_2 of radii 2 units each touch each other. AB is the common diameter. The tangent at B meets the tangent from A to be circle S_2 at C as shown. If BC = $K\sqrt{2}$ then the value of K is ______.

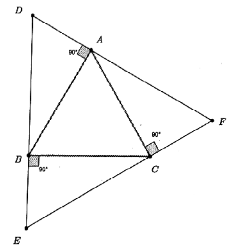


18. When the number $33333^2 + 22222$ is written as a single decimal number, the sum of its digits is ______.

19. The number of three digit numbers such that the product of their digits is a prime number is ______.

20. The number of real values (x, y) for which $2^{x+1} + 3^y = 3^{y+2} - 2^x$ is ______.

21. In the figure below, ΔABC is equilateral. AD, BE and CF are respectively perpendicular to AB, BC and AC. Then $\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta ABC}$ = ______.



22. If f(x) = ax + b and f(f(f(x))) = 27x + 26 then a + b = _______.

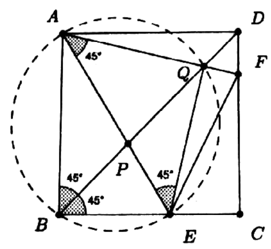
23. The eight digits 6, 5, 5, 4, 4, 3, 2 and 1 are used to form two 3-digit numbers and one 2-digit numbers. The largest possible sum of these numbers is _______.

24. $a \neq 0$, $b \neq 0$. The number of real number pairs (a, b) which satisfy the equation $a^4 + b^4 = (a + b)^4$ is _______.

25. The number of integers greater than 2 and less than 70 that can be written as a^b (where $b \neq 1$) is _______.

26. ABCD is a parallelogram P is a point on AD such that $\frac{AP}{AD} = \frac{1}{2013}$. Q is the point of intersection of AC and BP. Then $\frac{AQ}{AC}$ = _______.

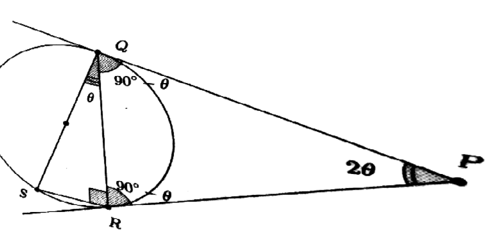
27. ABCD is a square. E and F are points respectively on BC and CD such that $\angle EAF = 45^0$. AE and AF cut the diagonal BD at P, Q respectively. Then $\frac{Area\ of\ \Delta AEF}{Area\ of\ \Delta APQ}$ = _______.



28. m, n are natural numbers. The number of pairs (m, n) for which $m^2 + n^2 = 2mn - 2013m - 2013n - 2014 = 0$ is _______.

29. In the adjoining figure BAC is $30^0 - 60^0 - 90^0$ triangle with AB = 20. D is the midpoint of AC. The perpendicular at D to AC meets the line parallel to AB through C at E. The line through E perpendicular to DE meets BA produced at F. If DF = $5\sqrt{x}$ then x = ____.

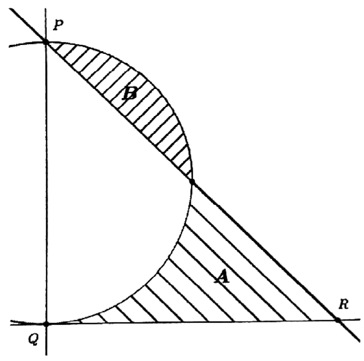
30. PR and PQ are tangents to a circle and QS is a diameter. Then $\frac{\angle QPR}{\angle RQS}$ = _______.



PART - A

1. For some natural number n, the sum of the first n natural numbers is 240 less than the sum of the first (n + 5) natural numbers. Then n itself is the sum of how many natural numbers starting with 1.
(A) 7 (B) 6 (C) 9 (D) 10

2. In the figure, PQ = 42 cm. QR is the tangent to the semicircle at Q. If the difference of the areas of regions A, B is 357, then the base QR of the right triangle PQR is (in cm).
Take $\pi = \frac{22}{7}$.



(A) 42 (B) 48 (C) 52 (D) 50

3. By rearranging the digits of the integer 1288, we get a total of 12 different integers including 1288. The sum of all these twelve integers is
(A) 577727 (B) 63327 (C) 72227 (D) 466627

4. The sides of a right angled triangle are all integers. Two sides are primes that differ by 50. The smallest possible value of the third side is
(A) 60 (B) 57 (C) 53 (D) 49

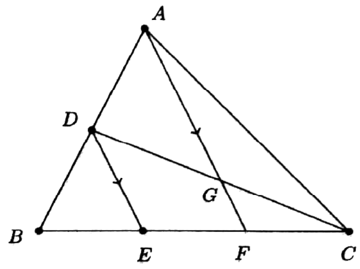
5. In ΔABC, D is on BC such that BD : DC = 3 : 1. E is on AC such that AE : EC = 2 : 3. AD and BE cut at F. If Area of $\Delta AFE = 4\ cm^2$ and area of $\Delta BFD = 30\ cm^2$, area of the triangle ABC (in cm^2) is
(A) $\frac{185}{3}$ (B) $\frac{225}{3}$ (C) $\frac{241}{3}$ (D) $\frac{230}{3}$

6. Observe the angle α_1, α_2, α_3, α_4, α_5 and α_6 in the square grid shown below. The measure of $(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6)$ is
(A) 180^0 (B) 270^0 (C) 360^0 (D) 225^0

7. N is a five digit number. 1 is written after the 5 digits of N to make it a six digit number, which is three times the same number with 1 written before N. (If N = 23456 it means 234561 and 123456). Then the middle digit of the number N is
(A) 2 (B) 4 (C) 6 (D) 8

8. The five digit number a679b is a multiple of 72. Then the value of a + b is
(A) 3 (B) 5 (C) 6 (D) 7

9. In the adjoining diagram D is the midpoint of AB. A line DE is drawn to cut BC at E. AF is parallel to DE. It is given that EF = FC, AF = 10 cm and area of ΔBDE is $5\ cm^2$, the area of ΔAGD (in cm^2) is



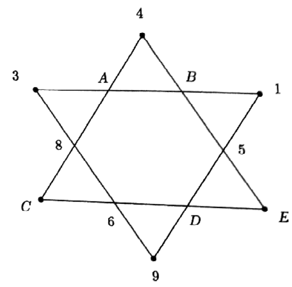
(A) 8 (B) 8.5 (C) 7.5 (D) 7

10. The number of two digit numbers having the property that when they are divided by the sum of their digits, the quotient is 7 without remainder is
(A) 0 (B) 1 (C) 3 (D) 4

11. The sum of all integers n for which $\frac{n^2-9}{n-1}$ is also an integer is

(A) 0 (B) 7 (C) 8 (D) 9

12. The number 1 to 12 are placed in the figure as shown. The sums of the numbers along each line are the same. The number 7 must go to the place marked



(A) A (B) B (C) E (D) D

13. There are 2014 people sitting around a big round table dinner. Each person shakes hands with everybody except the persons sitting on both sides of him. The total number of handshakes that takes place is

(A) 1007 x 2014 (B) 2014 x 2012 (C) 1007 x 2011 (D) 1007 x 2012

14. A, B and C run for a race on a straight road of x meters. A beats B by 30 meters B beats C by 20 meters, A beats C by 48 meters. Then x (in meters) is

(A) 150 (B) 200 (C) 300 (D) 500

15. In triangle ABC, $\angle B = 2\angle C$, AD is the angle bisector of $\angle A$ and DC = AB. Then the measure of $\angle A$ is

(A) 60^0 (B) 72^0 (C) 84^0 (D) 108^0

PART - B

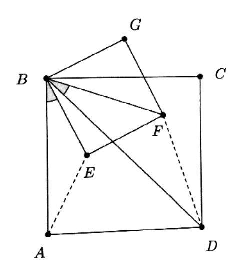
16. The six digit number that becomes 6 times its value when its last three digit are carried to the beginning of the number without their order being changed is _______.

17. If a, b, c are real and a + b + c = 0 the value of $a(b-c)^3 + b(c-a)^3 + c(a-b)^3$ is _____.

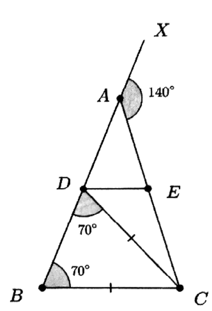
18. The population of a town is 20000. The annual birth rate is 4% and the annual death rate is 2%. The population of the town after 2 years is _______.

19. The symbol $\lfloor x \rfloor$ means the integral part of x. For example, $\lfloor 2,3 \rfloor = 2, \lfloor \sqrt{15} \rfloor = 3$. The value of $E = \lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + ... + \lfloor \sqrt{99} \rfloor + \lfloor \sqrt{100} \rfloor$ is _____.

20. ABCD is a square and BEFG is another square drawn with the common vertex B such that E, F fall inside the square ABCD. If DF = $\sqrt{n}$ AE, then n is ______.



21. In figure, AB = AC. The exterior angle CAX = 140^0. D is the point on AB such that CB = CD. DE is drawn parallel to BC to meet AC at E. The measure of the $\angle DCE$ is ___.



22. m, n are natural numbers. The number of pairs (m, n) such that $(m-8)(m-10) = 2^n$ is _______.

23. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ for $-1 < x < 1$ and if $f\left(\frac{3x+x^3}{1+3x^2}\right) = Kf(x)$, then the value of K is ______.

24. If a, b, c, d are positive integers such that $a^5 = b^4$, $c^3 = d^2$ and c – a = 19, then the numerical value of d – b is ______ (you can express in powers of numbers)

25. The contents of two vessels containing water and milk in the ratio 1 : 2 and 2 : 5 are mixed in the ratio 1 : 4. The resulting mixture will have water and milk in the ratio ____.

26. If n = 560560560560563 and Saket divided n^2 by 8, he will get a remainder _____.

27. The least positive integer by which 396 he multiplied to make the product perfect cube is _______.

28. The value of $\sqrt[3]{\dfrac{1\cdot 2\cdot 4 + 2\cdot 4\cdot 8 + \ldots + n\cdot 2n\cdot 4n}{1\cdot 3\cdot 9 + 2\cdot 6\cdot 18 + \ldots + n\cdot 3n\cdot 9n}}$ is ______.

29. n is a natural number. It is given that (n + 20) + (n + 21) +...... + (n + 100) is a perfect square. The least value of n is _______.

30. ABCD is a rectangle DEFC is a parallelogram. ABEF is a straight line. Area of the quadrilateral CGEF is _______.

PART - A

1. The sum of the values x, y that satisfy the equations $(x + y)2^{y-x} = 1$, $(x + y)^{x-y} = 2$ simultaneously is

(A) 2 (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

2. If $\left(2-\frac{a}{4}-\frac{4}{a}\right)\times\left\{(a-4)\sqrt[3]{(a-4)^{-3}}-\frac{(a^2-16)^{-1/2}(a-4)^{-1/2}}{(a+4)^{-3/2}}\right\}\times\left(\frac{a+4}{a-4}\right)=2016$ the value of a is

(A) $\frac{4}{1007}$ (B) $\frac{3}{2016}$ (C) $\frac{4}{2017}$ (D) none of these

3. ABCD is a square inscribed in a circle of radius 1 unit. The tangent to be circle at C meets AB produced at P. The length of PD is

(A) 2 (B) 3 (C) $\sqrt{13}$ (D) $\sqrt{10}$

4. Quadrilateral ABCD is inscribed in a circle with radius 1 unit. AC is the diameter of the circle and BD = AB. The diagonals cut at P. If PC = $\frac{2}{5}$ then the length of CD is equal to

(A) $\frac{2}{3}$ (B) $\frac{2}{7}$ (C) $\frac{1}{8}$ (D) $\frac{3}{4}$

5. The number of natural numbers n for which the expression $\frac{23n^2+18n+4}{n}$ is also a natural number is

(A) 3 (B) 2 (C) 1 (D) 0

6. The cost price of 16 oranges is equal to the selling price of 12 oranges. Then there is a

(A) 40% profit (B) 20% loss (C) $33\frac{1}{3}$% profit (D) $23\frac{1}{3}$% profit

7. The number of positive integer pairs (a, b) such that ab – 24 = 2b is

(A) 6 (B) 7 (C) 8 (D) 9

8. $A = (2 + 1)(2^2 + 1)(2^4 + 1) \ldots\ldots (2^{2016} + 1)$. The value of $(A + 1)^{1/2016}$ is

(A) 4 (B) 2016 (C) 2^{4032} (D)

9. The sum of two numbers a, b where a < b is 1215 and their H.C.F. is 81. The number of pairs of such pairs (a, b) is

(A) 1 (B) 2 (C) 3 (D) 4

10. The first Republic Day of India was celebrated on 26th January 1950. What was the day of the week on that date ?

(A) Tuesday (B) Wednesday (C) Thursday (D) Friday

11. The 12 numbers $a_1, a_2, \ldots, a_{12}$ are in arithmetical progression. The sum of all these numbers is 354. Let $P = a_2 + a_4 + \ldots + a_{12}$ and $Q = a_1 + a_3 + \ldots + a_{11}$. If the ratio P : Q is 32 : 37, the common difference of the progression is

(A) 2 (B) 3 (C) 4 (D) 5

12. A shopkeeper marks the prices of his goods at 20% higher than the original price. There is an increase in demand of the goods, and he further increases the price by 20%. The total profit % is

(A) 40 (B) 38 (C) 42 (D) 44

13. A circle passes through the vertices A and D and touches the side BC of a square ABCD. The side of the square is 2 cm. The radius of the circle (in cm) is

(A) $\frac{5}{4}$ (B) $\frac{4}{5}$ (C) 1 (D) $\frac{5}{2}$

14. There are four balls – one green, one red, one blue and one yellow and there are four boxes – one green, one red, one blue and one yellow. A child playing with the balls decides to put the balls in the boxes, one ball in each box. The number of ways in which the child can put the balls in the boxes such that no ball is in a box of its own color is

(A) 12 (B) 9 (C) 24 (D) 6

15. The 5 x 5 array of the dots represents trees in an orchard. If you were standing at the central spot marked C, you would not be able to see 8 of the 24 trees. (shown as X). If you were standing at the centre of a 9 x 9 array of trees, how many of the 80 trees would be hidden ?

×	•	×	•	×
•	•	•	•	•
×	•	*C*	•	×
•	•	•	•	•
×	•	×	•	×

(A) 40 (B) 32 (C) 36 (D) 44

PART - B

16. a and b are positive integers such that $a^2 + 2b = b^2 + 2a + 5$. The value of b is ____.

17. After full simplification, the value of the product

$$\left(\sqrt{2+\sqrt{3}}\right)\left(\sqrt{2+\sqrt{2+\sqrt{3}}}\right)\times\left(\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\right)\left(\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}\right).$$

18. ABCD is a rectangle with AD = 1 and AB = 2. DFEB is also a rectangle. The area of DFEB is _______.

19. The two digit number whose units digit exceeds the tens digit by 2 and such that the product of the number and the sum of its digits is 144 is ______.

20. If $x = \frac{p}{q}$ where p, q are integers having no common divisors other than 1, satisfies

$$\sqrt{x+\sqrt{x}} - \sqrt{x-\sqrt{x}} = \frac{3}{2}\sqrt{\frac{x}{x+\sqrt{x}}}.$$

21. AE and BF are medians drawn to the legs of a right angled triangle ABC. The numerical value of $\frac{AE^2 + BF^2}{AB^2}$ is ______.

22. AB is a chord of a circle with center O. AB is produced to C such that BC = OA. CO is produced to E. The value of $\frac{\angle AOE}{\angle ACE}$ is _______.

23. The number of two digit numbers that are less than the sum of the squares of their digits by 11 and exceed twice the product of their digits by 5 is _______.

24. AB is a diameter of circle and CD is a parallel chord. P is any point in AB. The numerical value of $\frac{PC^2 + PD^2}{PA^2 + PB^2}$ is _______.

25. In the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, ... the 2016th term is 2^n. Then n = ______.

26. Each root of the equation $ax^2 + bx + c = 0$ is decreased by 1. The quadratic equation with these roots $x^2 + 4x + 1 = 0$. The numerical value of b + c is ______.

27. The number of integers n such that $\frac{n+2}{n^2+1} > \frac{1}{2}$ is _______.

28. P_1 and P_2 are two regular polygons. The number of sides of P_1 and P_2 respectively are in the ratio 3 : 2 and the respective interior angles are in the ratio 10 : 9. Then the sum of the number of sides of P_1 and P_2 is _______.

29. In triangle ABC, F and E are the mid points of AB and AC respectively. P is any point on the side BC. The ratio $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta FPE}$ is ______.

30. x, y, z are distinct real numbers such that $x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x}$. The value of $x^2y^2z^2$ is _____.

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AMTI (NMTC) - 2017

GAUSS CONTEST - JUNIOR LEVEL

(Standard - IX & X)

Note :

1. Fill in the response sheet with your Name, Class and the institution through which you appear in the specified places.
2. Diagrams are only visual aids; they are NOT drawn to scale.
3. You are free to do rough work on separate sheets.
4. Duration of the test : 2 pm to 4 pm – 2 hours.

PART - A

Note :

- Only one of the choices A, B, C, D is correct for each question. Shade the alphabet of your choice in the response sheet. If you have any doubt in the method of answering, seek the guidance of the supervisor.
- For each correct response you get 1 mark. For each incorrect response you lose $\frac{1}{2}$ mark.

1. If m is a real number such that $m^2 + 1 = 3m$, the value of $\frac{2m^5 - 5m^4 - 2m^3 - 8m^2}{m^2+1}$ is

(A) 1 (B) 2 (C) -1 (D) -2

2. Consider the equation $\frac{7x}{2} - a = \frac{5x}{3} + 9$. The least positive a for which the solution x to the equation is a positive integer is

(A) 1 (B) 2 (C) 3 (D) 4

3. If x = 2017 and y = $\frac{1}{2017}$, the value of $\left\{\frac{x/y + 2}{x/y + 1} + \frac{x}{y}\right\} \div \left\{\frac{x}{y} + 2 - \frac{x/y}{x/y + 1}\right\}$ is

(A) 2017 (B) 2017^2 (C) $\frac{1}{2017^2}$ (D) 1

4. The ratio of an interior angle of a regular pentagon to an exterior angle of a regular decagon is

(A) 4 : 1 (B) 3 : 1 (C) 2 : 1 (D) 7 : 3

5. The smallest integer x which satisfies the inequality $\frac{x-5}{x^2+5x-14} > 0$ is

(A) -8 (B) -6 (C) 0 (D) 1

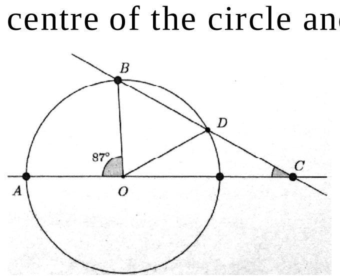
6. If x and y satisfy the equations $\sqrt{\frac{20y}{x}} = \sqrt{x+y} + \sqrt{x-y}, \sqrt{\frac{16x}{5y}} = \sqrt{x+y} - \sqrt{x-y}$ the value of $x^2 + y^2$ is

(A) 2 (B) 16 (C) 25 (D) 41

7. 125% of a number x is y. What percentage of 8y is 5x ?

(A) 30 % (B) 40 % (C) 50 % (D) 60 %

8. If the adjoining figure, O is the centre of the circle and OD = DC. If ∠AOB = 87^0, the measure of the angle ∠OCD is



(A) 27^0 (B) 28^0 (C) 29^0 (D) 19^0

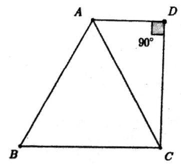
9. a, b, c, d, e are real numbers such that $\frac{a}{b}=\frac{2}{3},\frac{b}{c}=\frac{1}{3},\frac{c}{d}=\frac{1}{4},e=\frac{ac}{b^2+c^2}$. The value of e is

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$

10. The length and breadth of a rectangular field are integers. The area is numerically 9 more than the perimeter. The perimeter is

(A) 24 (B) 32 (C) 36 (D) 40

11. ABCD is a trapezium in which ABC is an equilateral triangle with area $9\sqrt{3}$ square units. If $\angle ADC = 90^0$, the area of the trapezium in square units is



(A) $12\sqrt{3}$ (B) $\frac{15\sqrt{3}}{2}$ (C) $\frac{27\sqrt{3}}{2}$ (D) $\frac{35\sqrt{3}}{2}$

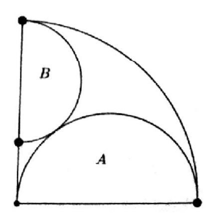
12. p is a prime number such that $p^2 - 8p - 65 > 0$. The smallest value of p is

(A) 7 (B) 11 (C) 13 (D) 17

13. The least positive integer n such that $2015^n + 2016^n + 2017^n$ is divisible by 10 is

(A) 1 (B) 3 (C) 4 (D) None of these

14. In a quadrant of a circle of diameter 4 units semicircles are drawn as shown. The radius of the smaller circle (B) is



(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

15. The product of two positive integers is twice their sum; the product is also equal to six times the difference between the two integers. The sum of these integers is

(A) 6 (B) 7 (C) 8 (D) 9

PART - B

Note :

- Write the correct answer in the space provided in the response sheet.
- For each correct response you get 1 mark. For each incorrect response you lose $\frac{1}{4}$ mark.

16. n is a natural number such that n minus 12 is the square of an integer and n plus 19 is the square of another integer. The value of n is _______.

17. The number of there digit numbers which have odd number of factors is _______.

18. The positive integers a, b, c are connected by the inequality $a^2 + b^2 + c^2 + 3 < ab + 3b + 2c$ then the value of a + b + c is _______.

19. The sum of all roots of the equation $|3x - |1 - 2x|| = 2$ is _______.

20. PQR is a triangle with PQ = 15, QR = 25, RP = 30. A, B are points on PQ and PR respectively such that $\angle PBA = \angle PQR$. The perimeter of the triangle PAB is 28, then the length of AB is ________.

21. A hare sees a hound 100 m away from her and runs off in the opposite direction at a speed of 12 KM an hour. A minute later the hound perceives her and gives a chase at a speed of 16 KM an hour. The distance at which the hound catches the hare (in meters) is _____.

22. Two circles touch both the arms of an angle whose measure is 60^0. Both the circles also touch each other externally. The radius of the smaller circle is r. The radius of the bigger circle (in term of r) is _______.

23. a, b are distinct natural numbers such that $\frac{1}{a}+\frac{1}{b}=\frac{2}{5}$. If $\sqrt{a+b}=k\sqrt{2}$ the value of k is _______.

24. The side AB of an equilateral triangle ABC is produced to D such that BD = 2AC. The value of $\frac{CD^2}{AB^2}$ is _______.

25. D and E trisect the side BC of a triangle ABC. DF is drawn parallel to AB meeting AC at F. EG is drawn parallel to AC meeting AB at G. DF and EG cut at H. Then the numerical value of $\frac{Area(ABC)}{Area(DHE)+Area(AFHG)}$ is _______.

26. In an examination 70% of the candidates passed in English, 65 % passed in Mathematics, 27 % failed in both the subjects and 248 passed in both the subjects. The total number of candidates is ________.

27. In a potato race, a bucket is placed at the starting point, which is 7 m from the first potato. The other potatoes are placed 4 m a part in a straight line from the first one. There are n potatoes in the line. Each competitor starts from the bucket, picks up the nearest potato, runs back with it, drops in the bucket, runs back to pick up the next potato, runs to the bucket and drops it and this process continues till all the potatoes are picked up and dropped in the bucket. Each competitor ran a total of 150 m. The number of potatotes is ________.

28. A two digit number is obtained by either multiplying the sum of its digits by 8 and adding 1, or by multiplying the difference of its digits by 13 and adding 2. The number is _____.

29. The inradius of a right angled triangle whose legs have lengths 3 and 4 is _______.

30. a, b are positive reals such that $\frac{1}{a}+\frac{1}{b}=\frac{1}{a+b}$. If $\left(\frac{a}{b}\right)^3+\left(\frac{b}{a}\right)^3=2\sqrt{n}$, where n is a natural number, the value of n is ______.

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AMTI (NMTC) - 2004

BHASKARA CONTEST - FINAL - JUNIOR LEVEL

1. Show that there are no integers a, b, c for which $a^2 + b^2 - 8c = 6$.
2. Given that $N = 2^n (2^{n+1} - 1)$ and $2^{n+1} - 1$ is a prime number, show that
   a) Sum of the divisors of N is 2N
   b) Sum of the reciprocals of the divisors of N is 2.
3. Given three non-collinear points A, B, C construct a circle with center C such that the tangents from A and B to the circle are parallel.
4. Given a circle with diameter AB and a point X on the circle different from A and B, let t_a, t_b and t_x be the tangents to the circle at A, B and X respectively. Let Z be the point where the line AX meets tb and Y be the point where the line BX meets t_a. Show that the three lines YZ, t_x and AB are either concurrent or parallel.
5. The polynomial $ax^3 + bx^2 + cx + d$ has integral coefficients a, b, c, d. If ad is odd and bc is even show that at least one root of the polynomial is irrational.
6. Let f be a function from N to R satisfying
   (a) f(1) = 1 and (b) f(1) + 2f(2) + 3f(3) + ... + nf(n) = n(n + 1) f(n). Find f (2004).
7. Consider a permutation $p_1p_2p_3p_4p_5p_6$ of the six numbers 1, 2, 3, 4, 5, 6 which can be transformed to 123456 by transposing two numbers exactly four times. By a transposition we mean an interchange of two places – for example, 1 2 3 4 5 6 to 3 2 1 4 5 6 (positions 1 and 3 are interchanged). Find the number of such permutations.
8. Let $a_1, a_2, a_3, \ldots,$ am be a sequence of real numbers. The sum of k– successive terms is called a k– sum, for example $a_j + a_{j+1} + a_{j+2} + \ldots + a_{j+k-1}$ is a k– sum. In a finite sequence of real numbers every 7-sum is negative and every 11-sum is positive. Find the largest number of terms in such a sequence.

AMTI (NMTC) - 2011

BHASKARA CONTEST - FINAL - JUNIOR LEVEL

1. If $a = 2011^{2010}$, $b = 2010^{2011}$, $c = (2010 + 2011)^{2010 + 2011}$, and $d = 2011$, find the value of $\frac{bc(a+d)}{(a-b)(a-c)} + \frac{ac(b+d)}{(b-a)(b-c)} + \frac{ab(c+d)}{(c-a)(c-b)}$.
2. The internal bisectors of angles A, B, C of triangle ABC meet the circumcircle respectively at P, Q, R. I is the incentre of ABC. PQ meets BC, CI and CA at T, Y, L respectively PR meets BC, BI and AB at S, X, M respectively.

   Prove :

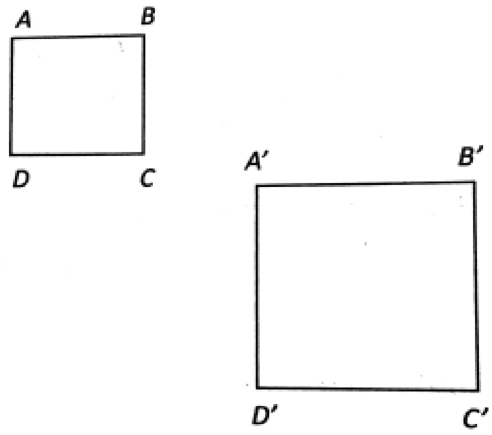
   (i) I is the orthocenter of the triangle PQR.

   (ii) RQTS, RQYX and RQLM are cyclic quadrilaterals.
3. Let $A = \{a^2 + 4ab + b^2 \mid a, b \text{ are positive integers}\}$. Prove that $2011 \neq A$.
4. If $1 \leq x \leq 64$, find the greatest value of the expression. $(\log_2 x)^4 + 12(\log_2 x)^2 \log_2 \left(\frac{8}{x}\right)$.
5. If perpendiculars are drawn from the vertices of a square to a line in the plane of the square (the line is not parallel to any side or diagonal), prove that the sum of the squares of the perpendiculars from one pair of opposite vertices exceeds twice the product of he perpendiculars from the other pair of opposite vertices by the area of the square.
6. x, y, z are real numbes such that $x + y + z = 3$ and $xy + yz + zx = a$ (where a is a real parameter). Determine the value of 'a' for which the difference between the maximum and minimum possible value of x is equal to 8.
7. abc is a three digit number. ab, bc, ca are two digit numbers. Determine all three digit numbers abc such that abc = ab + bc + ca.
8. If a, b, c, d are four real numbers such that $a + 2b + 3c + 4d \geq 30$, prove that $a^2 + b^2 + c^2 + d^2 > 30$.

AMTI (NMTC) - 2012

BHASKARA CONTEST - FINAL - JUNIOR LEVEL

1. H is the orthocenter of an acute angled triangle ABC, with circumcentre O. Let P be a point on the arc BC not containing A of the circumcircle different form B and C. Let D be a point such that AD = PC and AD parallel to PC. Let K be the orthocentre of the triangle ACD. Prove that K lies on the circumcircle of triangle ABC.
2. Find all positive integer solution of the equation $4x^3 - 3x - 1 = 2y^2$.
3. Consider the set A of numbers $\left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{2012}\right\}$. We delete two of them say 'a' and 'b' and in their place we put only one number a + b + ab. After performing the operation 2011 times what is the number that is left over.
4. Seven digit numbers are formed by the digits 1, 2, 3, 4, 5, 6 and 7. In each number no digit is repeated. Prove that among all these numbers there is no number which is a multiple of another number.
5. ABCD and A'B'C'D' are two unequal squares in a plane placed as in the figure (A'B' parallel to AB etc.)



6. Find integers x, y, z such that $x^2z + y^2z + 4xy = 40$, $x^2 + y^2 + xyz = 20$.
7. There are two natural numbers whose product is 192. It is given that the quotient of the arithmatic mean to the harmonic mean of their greatest common measure and the least common multiple is $\frac{169}{48}$. Find the numbers.
8. Find all the positive integral solution of the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{2013}$.

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1. Two circles S_1 and S_2 intersect at points A and B. The tangent at A to S_1 meets S_2 at C and the tangent at A to S_2 meets S_1 at D. A line through A interior to the angle CAD meets S_1 at M and S_2 at N and meets the circumcircle of triangle ACD at P. Prove that AM = NP.

2. Let a, b, c, d be positive real numbers. Show that

$$\frac{ab+bc+ca}{a^3+b^3+c^3}+\frac{ab+bd+da}{a^3+b^3+d^3}+\frac{ac+cd+da}{a^3+c^3+d^3}+\frac{bc+cd+db}{b^3+c^3+d^3}$$
$$\le \min\left\{\frac{a^2+b^2}{(ab)^{3/2}}+\frac{c^2+d^2}{(cd)^{3/2}},\frac{a^2+c^2}{(ac)^{3/2}}+\frac{b^2+d^2}{(bd)^{3/2}},\frac{a^2+d^2}{(ad)^{3/2}}+\frac{b^2+c^2}{(bc)^{3/2}}\right\}.$$

3. Find prime numbers p such that $4p^2 + 1$ and $6p^2 + 1$ are also prime numbers.

4. a) Find all positive integral solutions x, y, z of the equation xy + yz + zx = xyz + 2.

   b) ABC is an equilateral triangle. D is a point inside the triangle such that DA = DB. E is a point that satisfies the two conditions (i) ∠DBE = ∠DBC and (ii) BE = AB.

5. a) Show that the numbers 1 to 15 cannot be divided into a group A to 2 numbers and a group B of 13 numbers in such a way that the sum of the numbers in B is equal to the product of the numbers in A.

   b) Squares ABCD and BCFG are drawn outside of a triangle ABC. Prove that if DG is parallel to AC then the triangle ABC is isosceles.

6. There are 13 white, 15 black and 17 red beads on a table. You have many number of beads of these colours with you. In one step 2 beads on the table of different colours are closen by you and you replace each one by a bead of the third colour from you. After how many such steps you will have all the beads of the same colour ?

7. a) If a, b, c, d are positive real numbers such that a + b + c + d = 1, show that $\frac{a^3}{b+c}+\frac{b^3}{c+d}+\frac{c^3}{d+a}+\frac{d^3}{a+b}\ge\frac{1}{8}$.

   b) A 4-digit number n not containing the digit 9 is a square of an integer. If we increase every digit of n by 1 we get a square of another integer again. Find all such n.

8. a) Find all positive real numbers x, y, z which satisfy the following equations simultaneously.
$$x^3+y^3+z^3 = x+y+z$$
$$x^2+y^2+z^2 = xyz$$

   b) Do there exist 10 distinct integers such that the sum of any 9 of them is a perfect square ?

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1. a) If $p_1, p_2, \ldots, p_{2014}$ is an arbitrary rearrangement of 1, 2, 3, ..., 2014. Show that $\frac{1}{p_1+p_2}+\frac{1}{p_2+p_3}+\ldots+\frac{1}{p_{2013}+p_{2014}} > \frac{2013}{2016}$.

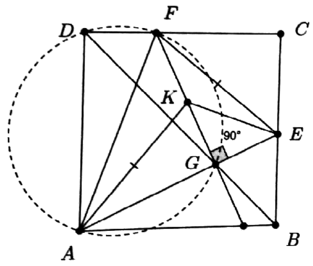
   b) Find positive integers n such that $\sqrt{n-1}+\sqrt{n+1}$ is rational.

2. ABCD is a quadrilateral inscribed in a circle of center O. Let BD bisect OC perpendicularly. P is a point on the diagonal AC such that PC = OC. BP cuts AD at E and the circle ABCD at F. Prove that PF is the geometric mean of EF and BF.

3. a) The Fibonacci sequence is defined by $F_0 = 1$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$, $n \geq 2$. Show that $7F^3_{n+2} - F^3_n - F^3_{n+1}$ is divisible by F_{n+3}.

   b) If x, y, z are each greater than 1, show that $\frac{x^4}{(y-1)^2}+\frac{y^4}{(z-1)^2}+\frac{z^4}{(x-1)^2} \geq 48$.

4. ABCD is square E and F are points on BC and CD respectively such that AE cuts the diagonal BD at G and FG is perpendicular to AE. K is a point on FG such that AK = EF. Find the measure of the angle EKF.



5. a) If none of a, b, c, x, y, z is zero, and $\frac{x^2(y+z)}{a^3}=\frac{y^2(z+x)}{b^3}=\frac{z^2(x+y)}{c^3}=\frac{xyz}{abc}=1$ prove that $a^3 + b^3 + c^3 + abc = 0$.

   b) Solve for x, y, z : $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}=\frac{y}{x}+\frac{z}{y}+\frac{x}{z}=x+y+z=3$.

6. In the dog language BOW, the alphabet consists of the letters B, O, W only. Independently of the choice of the BOW and of length n (i.e.) number of alphabets in the word is n) from which to start, one can construct all the BOW words with length n using iteratively the following rules.

   i) reverse the order of the letters of the word (if BOWW is a word then if we reverse the order of letters we get WWOB)

   ii) replace two consecutive letters as follows :

   BO $\to$ WW, WW $\to$ BO,
   WB $\to$ OO, OO $\to$ WB,
   OW $\to$ BB, BB $\to$ OW

   Given that BBOWOBOWWOBOWWWOBOWWWWOBB is a BOW word, does the BOW language have the following words ?

   a) BWOBWOBWOBWOBOWBOWBOWBOWB

   b) OBWOBWOBWOBWOBWOBWBOWBOWBO

7. A merchant bought a quantity of cotton; he exchanged this for oil and he sold the oil. He observed that the number of kg of cotton, the number of liters of oil obtained for each kg and the number of rupees for which he sold formed a decreasing geometric progression. He calculate that if he had obtained 1 kg more of cotton, one liter more of oil for each kg and Rs.1 more for each liter, he would have obtained Rs. 10169 more, whereas if he had obtained one kg less of cotton and one liter less of oil for each kg and Rs. 1 less for each liter, he would have obtained Rs. 9673 less. How much did he actually receive ?

8. There are three towns A, B and C. A person walking from A to B, driving from B to C and riding a horse from C to A completes the journey is $15\frac{1}{2}$ hours. By driving from A to B, riding a horse from B to C and walking from C to A, he could make the journey in 12 hours. On foot he could make the journey in 22 hours, on horseback in $8\frac{1}{4}$ hours and driving in 11 hours. To walk 1 KM, ride 1 KM and drive 1 KM, he takes altogether half an hour. Find the rates at which he travels and the distance between the towns.

AMTI (NMTC) - 2017

BHASKARA - FINAL - JUNIOR LEVEL

1. (a) If a, b, c are positive reals and a + b + c = 50 and 3a + b – c = 70. If x = 5a + 4b + 2c, find the range of values of x.

   (b) The sides a, b, c of a triangle ABC satisfy the equation $a^2 + 2b^2 + 2016c^2 - 3ab - 433bc + 2017ac = 0$. Prove that b is the arithmetic mean of a, c.

2. In an isosceles triangle ABC, AB = BC. The bisector AD of ∠A meets the side BC at D. The line perpendicular to AD through D meets AB at F and AC produced at E. Perpendiculars from B and D to AC are respectively BM and DN. If AE = 2016 units, find the length MN.

3. (a) Two circles with centres at P and Q and radii $\sqrt{2}$ and 1 respectively intersect each other at A and D and PQ = 2 units. Chord AC is drawn to the bigger circle to cut it at C and the smaller circle at B such that B is the midpoint of AC. Find the length of AC.

   (b) Find the greatest common divisor of the numbers $n^n - n$, n = 3, 5, 7, 9, ...

4. (a) A book contained problems an Algebra, Geometry and Number theory. Mahadevan solved some of them. After checking the answers, he found that he answered correctly 50% problems in Algebra, 70% in Geometry and 80% in Number theory. He further found that the solved correctly 62% of problems in Algebra and Number theory put together, 74% questions in Geometry and Number theory altogether. What is the percentage of correctly answered questions in all the three subjects ?

   (b) Find all pairs of positive integers (a, b) such that $a^b - b^a = 3$.

5. a, b, c are positive real numbers. Find the minimum value of $\frac{a+3c}{a+2b+c}+\frac{4b}{a+b+2c}-\frac{8c}{a+b+3c}$.

6. (a) Show that among any n + 1 whole numbers, one can find two numbers such that their difference is divisible by n.

   (b) Show that for any natural number n, there is a positive integer all of whose digits are 5 or 0 and is divisible by n.
