

ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA**Screening Test – Ramanujan Contest**
(NMTC INTER LEVEL)**Saturday, the 15th October 2022**

Note:

1. Fill in the Response Sheet with your Name, Class and the Institution through which you appear, in the specified places, in CAPITAL letters.
 2. Diagrams given are only Visual aids; they are not drawn to scale.
 3. You may use separate sheets to do rough work.
 4. Use of Electronic gadgets such as Calculator, Mobile Phone or Computer is not permitted.
 5. Duration of the Test: 2 pm to 4 pm (2 hours).
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PART – A

1. The number of digits in the answer to the product $16^8 \times 5^{25}$ is
A. 25 **B. 26** **C. 27** **D. 28**
2. This year, Lila's age is the sum of the digits in her Mathematics teacher's age and in five years, Lila's age will be the product of the digits in her Mathematics teacher's age at that time. How old is Lila now?
A. 11 **B. 13** **C. 14** **D. 15**
3. The teacher wrote the numbers 1 to 30 in order as follows:
1 2 3 4 5 6 7 8 9 10 11 12 13...29 30
She then erased 45 of these 51 digits leaving 6 in their original order so that the number formed by them is the largest 6-digit number possible. What is the sum of the digits of this number?
A. 33 **B. 38** **C. 41** **D. 43**
4. Starting with 0 on my calculator, I do a calculation in five steps. At each step, I either add 1 or multiply by 2. What is the smallest number that can not be the final result?
A. 8 **B. 9** **C. 10** **D. 11**
5. What is the smallest positive integer that can be expressed as a sum of nine consecutive integers, the sum of ten consecutive integers and the sum of eleven consecutive integers?
A. 425 **B. 475** **C. 495** **D. 525**

6. The centers of the six faces of a cube are joined to form an octahedron. The centers of the faces of this octahedron are joined to form a smaller cube. What is the ratio of the length of an edge of the smaller cube to that of the larger cube?

A. $1:\sqrt{2}$ B. $1:\sqrt{3}$ C. $1:2$ **D. $1:3$**

7. In a game show, contestants are given a number. They can increase or decrease a digit in the number by the click of a button. They are required to make all the digits in the number equal in as few clicks as possible. For example, if the number given is 114, they can make it 111 by three clicks, decreasing the number 4 by 1 at each click. For the number 5559993321, what is the fewest number of clicks required to make all digits equal?

A. 21 **B. 23** C. 39 D. 41

8. A rectangular room of size $12\text{ft.} \times 17\text{ft.}$ is to be tiled with the greatest number of trapezoidal tiles. The nonparallel sides of the tile are 4 inches and 5 inches (1 ft. = 12 inches). If all sides of the tile have integer length and no tile is cut, how many tiles are needed to complete the task?

A. 1600 B. 1616 **C. 1632** D. 1648

9. Complete the following square with numbers so that the equations stated are correct. The operation in \mathbb{Z} and the value of X are respectively

A. $\times, 12$ B. $\div, 12$
C. $+, 12$ D. $-, 12$

$\sqrt{18}$	$-$	$\sqrt{2}$	$=$	
\div		\times		\div
	$+$		$=$	$\sqrt{2}/6$
$=$		$=$		$=$
54	z		$=$	X

10. We write a date as $mm/dd/yyyy$. For example December 1, 2021 will be written as 12/01/2021. A date is palindromic, if it reads the same left to right and right to left. November 2, 2011 is a palindromic date since it is written as 11/02/2011. What day of the week is the next palindromic date to 11/02/2011?

A. Sunday B. Tuesday C. Thursday D. Saturday

11. A non-standard dice has the numbers 2, 3, 5, 8, 13 and 21 on its faces. The dice is rolled twice and the numbers are added together. What is the probability that the resulting sum is also a value on the dice?

A. $\frac{1}{9}$ **B. $\frac{2}{9}$** C. $\frac{1}{3}$ D. $\frac{1}{6}$

12. A cube has an internal point P such that perpendicular distances from P to the faces of the cube are 2, 4, 6, 8, 10, 12 *cms*. How many other internal points of this cube have this property?

A. 7 B. 13 C. 23 **D. 47**

13. The sum of the lengths of the 12 edges of a cuboid (rectangular parallelepiped) is a *cm*. The distance from one corner of the cuboid to the farthest corner is b *cm*. What, in cm^2 , is the total surface area of the cuboid?

A. $\frac{a^2 - 2b^2}{2}$ **B. $\frac{a^2 - 16b^2}{16}$** C. $\frac{a^2 - 4b^2}{4}$ D. $a^2 + b^2$

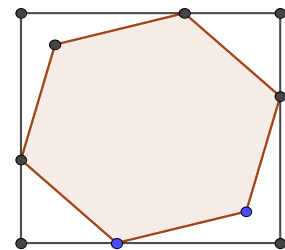
14. A function f , defined on the set of positive integers, is such that $f(xy) = f(x) + f(y)$ for all x and y .

Given that $f(10) = 14$ and $f(40) = 20$, what is the value of $f(500)$?

A. 29 B. 30 **C. 39** D. 48

15. The diagram shows a regular hexagon, with sides of length 1, inside a square. Two vertices of the hexagon lie on a diagonal of the square and the other four vertices lie on the edges.

What is the area of the square?



A. $2 + \sqrt{3}$ B. $3 + \sqrt{2}$ C. 4 D. $1 + \frac{3\sqrt{3}}{2}$

PART - B

16. If n is a positive integer such that

$n^5 = 133^5 + 110^5 + 84^5 + 27^5$,
the value of n is 144.

17. The real root of the equation $8x^3 - 3x^2 - 3x - 1 = 0$ can be written in the form

$$\frac{\sqrt[3]{a} + \sqrt[3]{b} + 1}{c}$$

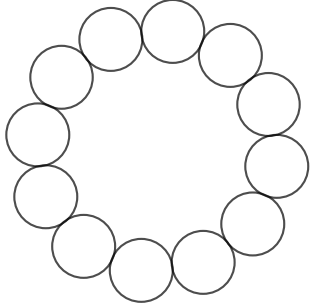
where a , b , and c are positive integers. Then $a + b + c$ is = 98

18. A sequence is defined as follows

$a_1 = a_2 = a_3 = 1$, and, for all positive integers n , $a_{n+3} = a_{n+2} + a_{n+1} + a_n$.

Given that $a_{28} = 6090307$, $a_{29} = 11201821$, and $a_{30} = 20603361$.

Then the remainder when $S = a_1 + a_2 + \dots + a_{28}$ is divided by 1000 is 834

19. A convex hexagon is inscribed in a circle. If its successive sides are 2, 2, 7, 7, 11 and 11, the diameter of the circumscribed circle is 14.
20. The unique integer N , that lies between 2000 and 3000 such that N divides $85^9 - 21^9 + 6^9$ is 2240.
21. Consider the sequence of numbers 24, 2534, 253534, 25353534,....
Let N be the first number in the sequence that is divisible by 99. Then the number of digits in the base ten representation of N is 176.
22. Consider the increasing list of positive integers that do not contain the digit zero, 1, 2, 3... 9, 11, 12,.... The 2022nd number in this sequence is 2685.
23. The points $A(17,13), B(a,13), C(b,59)$ are in the rectangular coordinate plane. If $\sqrt{[AB^2 - (BC + AC)^2][(BC - AC)^2 - AB^2]} = 8096$, the positive value of a is 105.
24. A ring of twelve identical circles is arranged on top of a unit circle C so as to cover C completely, each disk being tangent to its two neighbors as in the Figure. If the sum of the areas of the twelve disks can be written as $\pi(a - b\sqrt{c})$, where a, b, c are positive integers and c is a prime, the value of $a + b + c$ is 135.
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25. The number of three digit numbers in which one of the digits is the sum of the other two is 126.
26. When the three digit number \overline{XYZ} is divided by 8, the quotient is the two digit number \overline{ZX} and the remainder is Y . Then the number \overline{XYZ} is 435.
27. The smallest natural number N for which the number
- $$X = 100000 \times 100002 \times 100006 \times 100008 + N$$
- is a perfect square is 36.
28. The number of distinct powers of x that appear in the expansion of $(x^7 + x^{11} + x^{14})^{10}$ is 56.
29. Five spherical balls of diameter 10 cms fit inside a closed cylindrical tin with internal diameter 16 cms. Then the smallest height of the tin in cms is 42.
30. One hundred people each with either all white hair or all black hair are arbitrarily seated in four rooms. Then the largest number of people with the same color hair we can be sure of finding in at least one of these rooms is 13.