Gujarat Ganit Mandal - 59th Annual Conference (Gandhinagar)

Shri A.K.Virani Mathematics Competition - 2022

Conducted by Prof. A. R. Rao Foundation

Time : 5:45 pm. to 6:45 pm.   Date: 05-11-2022   Total Marks : 50

Instructions: - All the questions are compulsory.

- Answers are to be written in a given separate answer sheet.

Que:-1 In the following questions (1 to 5), two statements are given. Write T if the statement is True, and F if it is False. For example, in a question if the first statement is true and the second statement is false, then the option to be chosen is TF. (Each question carries 1 mark. The maximum time you may allot to this question is 6 minutes.)   **[5]**

(1) (i) If the decimal representation of the numbers a and b are infinite, then the decimal representation of $a + b$ is also infinite. (ii) If the decimal representation of ab is finite, then the decimal representation of both a and b are finite.

(A) TT   (B) TF   (C) FT   (D) FF

(2) (i) $\csc^2 x - \cot^2 y = 1$ if and only if $x = y$, where $x, y \in [0, \pi]$.
(ii) $|\cos(-\theta)| = \cos\theta$, where $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(A) TT   (B) TF   (C) FT   (D) FF

(3) (i) Remainder after dividing the square of any odd integer by 16 is 1 or 9.
(ii) If last two digits of a number is 66, then it is not a perfect square.

(A) TT   (B) TF   (C) FT   (D) FF

(4) (i) If the intersection of two lines is an empty set, then these two lines are parallel.
(ii) If two lines lie in distinct planes, then they are non-coplanar.

(A) TT   (B) TF   (C) FT   (D) FF

(5) For $\triangle ABC$, if incentre is I and orthocentre is H, then
(i) $\angle BIC$ is always an obtuse angle. (ii) $\angle BHC$ is always an obtuse angle.

(A) TT   (B) TF   (C) FT   (D) FF

Que:-2 In the following questions(1 to 5), four options A,B,C,D are given. Choose the correct option. (Each question carries 2 marks. The maximum time you may allot to this question is 12 minutes.)   **[10]**

(1) If the fraction $\frac{a}{b} = 0.456456456\cdots = 0.\overline{456}$ is in the lowest form, then $a + b =$ _____.

(A) 485   (B) 1455   (C) 144   (D) 162

(2) $\sqrt{2012\sqrt{4 + 2028 \times 2032} + 81} =$ _____.

(A) 2023   (B) 2022   (C) 2021   (D) 2020

(3) If (Number of subsets of A)+(Number of subsets of B)= 320, then the maximum number of elements in $A \cap B$ can be _____.

(A) 5   (B) 6   (C) 7   (D) 8

(4) If the roots of the equation $(x + 4)(x + 5) = 2021 \times 2022$ are α and β, then $|\alpha + \beta| =$ _____.

(A) 10   (B) 9   (C) 11   (D) 4043

(5) In a $\triangle ABC$, E is the mid point of $\overline{CA}$ and $\overline{AL}$ is the altitude such that $B - L - C$. If $m\angle C = 65°$, then $m\angle ELB = $ _____.

(A) 105° (B) 110° (C) 125° (D) 115°

Que:-3 In the following questions(1 to 5), four options A,B,C,D are given. Choose the correct option (Each question carries 3 mark. The maximum time you may allot to this question is 18 minutes.) [15]

(1) The remainder is 2, after dividing 2022 by a positive integer p. The number of such positive integers p is _____.

(A) 10 (B) 11 (C) 12 (D) 13

(2) If $\log_8 a + \log_4 b^2 = 5$ and $\log_8 b + \log_4 a^2 = 7$, then $ab = $ _____.

(A) 64 (B) 128 (C) 256 (D) 512

(3) Roots of the quadratic equation $x^2 + px + q = 0$ are $\alpha(\neq 0, 1)$ and $\beta(\neq 0, 1)$. If $\alpha^2 = \beta$ and $\beta^2 = \alpha$, then $|p + q| = $ _____.

(A) 0 (B) 1 (C) 2 (D) 3

(4) Let $x \neq y$ be two real numbers. If x, a_1, a_2, a_3, y and b_1, x, b_2, b_3, y, b_4 are two arithmetic progressions, then $\dfrac{b_4 - b_3}{a_2 - a_1} = $ _____.

(A) $\dfrac{8}{3}$ (B) $\dfrac{3}{2}$ (C) $\dfrac{2}{3}$ (D) 6

(5) How many even integers between 4000 and 7000 have four different digits?

(A) 546 (B) 728 (C) 936 (D) 672

Que:-4 In the following questions (1 to 5), four options A,B,C,D are given. Choose the correct option (Each question carries 4 marks. The maximum time you may allot to this question is 24 minutes.) [20]

(1) If $\sin\theta, \cos\theta, \tan\theta$ are in arithmetic progression, then the value of $\sqrt{5\cos^4\theta + 2\cos^3\theta - 2\cos\theta} = $ _____.

(A) 0 (B) 1 (C) 2 (D) 3

(2) If α and β are non zero real roots of the equation $3\alpha x^2 + 3\beta x - (\alpha - \beta) = 0$, then the value of $\left(\dfrac{\alpha}{\beta}\right)^2 - \left(\dfrac{\alpha}{\beta}\right) = $ _____.

(A) 6 (B) 4 (C) 2 (D) 8

(3) If a, b, c, d, e are all zeros of the polynomial $6x^5 + 5x^4 + 4x^3 + 3x^2 + 2x + 1$, then the value of $(1 + a)(1 + b)(1 + c)(1 + d)(1 + e) = $ _____.

(A) $-\dfrac{1}{2}$ (B) $\dfrac{1}{2}$ (C) 2 (D) -2

(4) If all the sides of a convex pentagon $ABCDE$ are equal and $m\angle BCD = 2m\angle ACE$, then $m\angle ACE = $ _____.

(A) 60° (B) 45° (C) 30° (D) 54°

(5) The length of sides of the square $ABCD$ is 1 unit. If P and Q are two points lying on $\overline{BC}$ and $\overline{CD}$ respectively such that $m\angle PAQ = 45°$, then the perimeter of $\triangle PCQ = $ _____.

(A) 2.5 (B) 1.5 (C) 3 (D) 2

Best of Luck